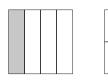
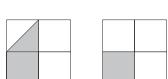
### **AMC 8 Practice Questions** Example

Each of the following four large congruent squares is subdivided into combinations of congruent triangles or rectangles and is partially shaded. What percent of the total area is partially shaded?





The original problem and choices from the 2011 AMC 8 contest

(A)  $12\frac{1}{2}$  (B) 20 (C) 25 (D)  $33\frac{1}{3}$  (E)  $37\frac{1}{2}$ 

2011 AMC 8, Problem #7— "Find the shaded portion of each square separately."

Problem number Hint

Solution

Answer (C): The upper left and the lower right squares are each one-fourth shaded, for a total of one-half square. The shaded portions of the upper right and lower left squares make up one-half square. So the total shaded area is one full square, which is 25% of the total area.

Solution from official solutions

Difficulty: Medium SMP-CCSS: 2, 7

CCSS-M: 6G.1, 6RP.3C

Standards for Math Practice Common Core State Standard Difficulty, Percent correct

100-80% Easy Med Easy 80-60% Medium 60-40% Med Hard 40-20% Hard 20-0%

Danica wants to arrange her model cars in rows with exactly 6 cars in each row. She now has 23 model cars. What is the smallest number of additional cars she must buy in order to be able to arrange her cars in this way?

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

#### 2013 AMC 8, Problem #1-

"What is the smallest multiple of 6 that is at least 24?"

#### Solution

**Answer (A):** The smallest multiple of 6 that is at least 23 is 24, so Danica must buy 1 additional car.

Difficulty: Easy

SMP-CCSS: 1. Make sense of problems and persevere in solving them; 7. Look for and make use of

CCSS-M: 6.NS.B. Compute fluently with multi-digit numbers and find common factors and multiples.

A sign at the fish market says, "50% off, today only: half-pound packages for just \$3 per package." What is the regular price for a full pound of fish, in dollars?

**(A)** 6 **(B)** 9 **(C)** 10 **(D)** 12 **(E)** 15

### 2013 AMC 8, Problem #2—

"How much does a half-pound package cost at the regular price?"

#### Solution

Answer (D): A half-pound package costs \$3 at the sale price, so it would cost \$6 at the regular price. A whole pound would cost \$12 at the regular price.

Difficulty: Medium Easy

**SMP-CCSS:** 1. Make sense of problems and persevere in solving them; 4. Model with mathematics. **CCSS-M:** 7.RP.A. Analyze proportional relationships and use them to solve real-world and mathematical problems.

What is the value of  $4 \cdot (-1+2-3+4-5+6-7+\cdots+1000)$ ?

- **(A)** -10 **(B)** 0 **(C)** 1 **(D)** 500 **(E)** 2000

### 2013 AMC 8, Problem #3—

"Find the sum inside the parentheses."

Answer (E): Inside the parentheses are 500 pairs of numbers, each with a sum of 1. Therefore the expression equals  $4 \cdot 500 = 2000.$ 

Difficulty: Medium

SMP-CCSS: 1. Make sense of problems and persevere in solving them; 7. Look for and make use of

 $\mathbf{CCSS-M:}\ 7.\mathbf{EE.A.}\ Use\ properties\ of\ operations\ to\ generate\ equivalent\ expressions.$ 

A fair coin is tossed 3 times. What is the probability of at least two consecutive heads?

(A) 
$$\frac{1}{8}$$

**(B)** 
$$\frac{1}{4}$$

(C) 
$$\frac{3}{8}$$

(A) 
$$\frac{1}{8}$$
 (B)  $\frac{1}{4}$  (C)  $\frac{3}{8}$  (D)  $\frac{1}{2}$  (E)  $\frac{3}{4}$ 

**(E)** 
$$\frac{3}{4}$$

2013 AMC 8, Problem #8—

"List the number of possible equally likely outcomes."

Answer (C): List the 8 possible equally likely outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT. Only HHH, HHT, THH have at least 2 consecutive heads, so the probability of at least 2 consecutive heads is  $\frac{3}{8}$ .

Difficulty: Medium Hard

SMP-CCSS: 1. Make sense of problems and persevere in solving them; 4. Model with mathematics. CCSS-M: 7.SP.C. Investigate chance processes and develop, use, and evaluate probability models.

What is the ratio of the least common multiple of 180 and 594 to the greatest common factor of 180 and 594?

**(A)** 110 **(B)** 165 **(C)** 330 **(D)** 625 **(E)** 660

# 2013 AMC 8, Problem #10— "Find the prime factorization of 180 and 594."

#### Solution

Answer (C): Because the prime factorizations of 180 and 594 are  $2^2 \cdot 3^2 \cdot 5$  and  $2 \cdot 3^3 \cdot 11$ , respectively, the least common multiple of 180 and 594 is  $2^2 \cdot 3^3 \cdot 5 \cdot 11$ , and their greatest common factor is  $2 \cdot 3^2$ . The ratio of their least common multiple to their greatest common factor is  $\frac{2^2 \cdot 3^3 \cdot 5 \cdot 11}{2 \cdot 3^2} = 2 \cdot 3 \cdot 5 \cdot 11 = 330$ .

Difficulty: Medium Hard

 ${\bf SMP\text{-}CCSS:}\ 1.$  Make sense of problems and persevere in solving them.

CCSS-M: 6.NS.B. Compute fluently with multi-digit numbers and find common factors and multiples.

If  $3^p + 3^4 = 90$ ,  $2^r + 44 = 76$ , and  $5^3 + 6^s = 1421$ , what is the product of p, r, and s?

**(A)** 27 **(B)** 40 **(C)** 50 **(D)** 70 **(E)** 90

### 2013 AMC 8, Problem #15—

"Find the value for p, r and s from the equations given."

#### Solution

**Answer (B):** From the first equation,  $3^p+81=90$ , so  $3^p=9$ , and p=2. From the second equation,  $2^r=32$ , so r=5. From the third equation,  $6^s+125=1421$ , so  $6^s=1296$ , and s=4. The product of p,r, and s is  $2\cdot 5\cdot 4=40$ .

 ${\bf Difficulty:} \ {\rm Medium}$ 

SMP-CCSS: 1. Make sense of problems and persevere in solving them.

CCSS-M: 8.EE.A Work with radicals and integer exponents.

A  $1 \times 2$  rectangle is inscribed in a semicircle with the longer side on the diameter. What is the area of the semicircle?

(A) 
$$\frac{\pi}{2}$$

**(B)** 
$$\frac{2\pi}{3}$$

**(C)** 
$$\pi$$

**(D)** 
$$\frac{4\pi}{3}$$

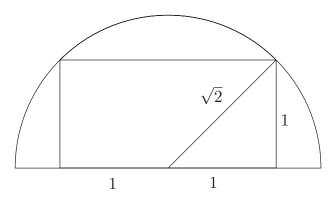
(A) 
$$\frac{\pi}{2}$$
 (B)  $\frac{2\pi}{3}$  (C)  $\pi$  (D)  $\frac{4\pi}{3}$  (E)  $\frac{5\pi}{3}$ 

### 2013 AMC 8, Problem #20—

"Use the Pythagorean Theorem to find the radius of the semicircle."

Solution

Answer (C):

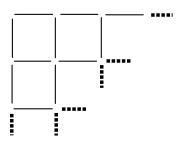


By the Pythagorean Theorem, the radius of the semicircle is  $\sqrt{2}$ , so its area is  $\frac{\pi(\sqrt{2})^2}{2} = \pi$ .

Difficulty: Hard

 ${\bf SMP\text{-}CCSS:}\ 1.$  Make sense of problems and persevere in solving them. CCSS-M: 8.G.B. Understand and apply the Pythagorean Theorem.

Toothpicks are used to make a grid that is 60 toothpicks long and 32 toothpicks high. How many toothpicks are used altogether?



**(A)** 1920 **(B)** 1952 **(C)** 1980 **(D)** 2013 **(E)** 3932

### 2013 AMC 8, Problem #22-

"Find the number of columns and rows for 60 toothpicks long and 32 toothpicks high."

#### Solution

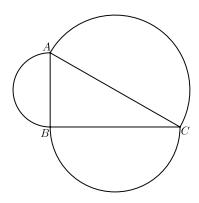
**Answer (E):** A grid 60 toothpicks long and 32 toothpicks high needs 61 columns of 32 toothpicks and 33 rows of 60 toothpicks. Therefore a total of  $(61\times32)+(33\times60)=3932$  toothpicks are needed.

Difficulty: Hard

 $\mathbf{SMP\text{-}CCSS:}\ 1.$  Make sense of problems and persevere in solving them; 7. Look for and make use of structure.

 $\mathbf{CCSS\text{-}M} \text{:}\ \mathrm{None}$ 

Angle ABC of  $\triangle ABC$  is a right angle. The sides of  $\triangle ABC$  are the diameters of semicircles as shown. The area of the semicircle on  $\overline{AB}$  equals  $8\pi$ , and the arc of the semicircle on  $\overline{AC}$  has length  $8.5\pi$ . What is the radius of the semicircle on  $\overline{BC}$ ?



**(A)** 7 **(B)** 7.5 **(C)** 8 **(D)** 8.5 **(E)** 9

### 2013 AMC 8, Problem #23—

"Find the area for the circle with diameter  $\overline{AB}$ ."

#### Solution

Answer (B): The circle with diameter  $\overline{AB}$  has twice the area of the corresponding semicircle; thus the area of the circle is  $16\pi$  and its radius is 4. Consequently AB=8. The circle with diameter  $\overline{AC}$  has circumference  $17\pi$ , so AC=17.  $\overline{AC}$  is the hypotenuse of the right triangle. By the Pythagorean Theorem,  $17^2=8^2+(BC)^2$ . Therefore BC=15, and the radius is 7.5.

Difficulty: Medium Hard

SMP-CCSS: 1. Make sense of problems and persevere in solving them. CCSS-M: 8.G.B. Understand and apply the Pythagorean Theorem.