Carl B. Allendoerfer Awards

Jocelyn R. Bell and Frank Wattenberg

"The Slippery Duck Theorem," *Mathematics Magazine*, 93:2, 91–103. 10.1080/0025570X.2020.1708693

From the compelling title to the surprising and satisfying results, this article is a joy to read. The authors begin with a short history of the dog-and-duck problem that has given rise to the study of pursuit curves in $\mathbb{R}^2$. Imagine a duck paddling along the edge of a pond with a given shape while a dog at the center of the pond starts making its way toward the duck. The dog's path is an example of a pursuit curve.

Bell and Wattenberg put an interesting twist on this classic problem. They assume that the dog is swimming at a constant speed *slower* than the duck, and that the duck is "slippery", that is, consistently gets away the moment the dog captures the duck. By fixing the duck's path in advance, these assumptions produce some surprising results on the limiting behavior of the pursuit curve. The authors rely primarily on results from a standard first course in differential equations and a little analysis, together with Carathéodory's existence theorem and Brouwer's Fixed Point Theorem. With these tools, the authors prove (1) the existence of a cycle in the pursuit curve, (2) that such a cycle is unique, and (3) that such a cycle is independent of the dog's starting position (i.e., a limit cycle).

The proofs are clear, and the authors' use of various common tools, together with short explanations of less-commonly-taught results, makes this an excellent article for undergraduate math majors interested in exploring the next step beyond their differential equations class. In addition, the artistically pleasing examples provided by the authors suggest a plethora of "tweaks" that faculty might make to this particular problem to develop projects for their own students to consider. In this way, anyone reading the work will find something to intrigue and inspire.

Response

We are absolutely delighted that our slippery duck paper has been selected for a Carl B. Allendoerfer award! We were searching for ways to include and engage cadets at West Point in mathematical exploration, and Hathaway's classic dog-and-duck problem from the Monthly fit the bill perfectly. We were ourselves surprised by the generality of our main result, the "slippery duck theorem". As an application of Brower's fixed point theorem, it is a nice reminder that abstract theorems in fields like topology sometimes have practical applications.

We really had fun working on this problem, especially investigating limit cycles for different "duck paths". We used Mathematica but any software that supports graphics and numerical solutions of systems of differential equations should work. This is a really rich source of student projects. There is a lot left to discover.

Biographical Sketches

Jocelyn Bell received her PhD in 2011 from the State University of New York, with a concentration in general topology. From 2011 to 2016 she held a postdoctoral position at West Point, where she also worked on problems in network science. Since 2016 she has been an assistant professor in the department of mathematics at Hobart and William Smith Colleges. She has three little girls who love "playing numbers".

Frank Wattenberg retired at the end of June 2020 after over 50 years as a mathematician and mathematics teacher primarily interested in mathematical modeling for personal and public policy decisions. Like many of us, he has been forced by recent events to question the assumption that good science and good science education by themselves empower us to improve our world. The work of Dan Kahan and others at

the Cultural Cognition Project is particularly important. Frank is hard at work on:

Seeing Stories through Everyday Cellphone Photography

Most of our students always have with them cellphones with remarkably capable cameras. *Seeing Stories* is about developing our students' powers of visual expression and narrative and about developing lifelong habits of everyday visual dairying. Visual narrative can nurture our sense of self and place in history and foster understanding and empathy across cultural divides.

Seeing Stories can become an engaging theme throughout students' academic lives—appearing in units from 15 minute units on "pictures of the day" from the morning's newspapers or Twitter feed to open-ended personal and creative projects. Along the way, students will develop standard material from the STEAM disciplines. As one example, middle school geometry and understanding of proportion and ratios is essential to composing effective photographs.

Seeing Stories is an example of the synergy between the sciences and the arts—for example, da Vinci's study of anatomy as he sought to capture people and animals on canvas—and modern digital image processing powering the creation of new mixed realities. *Seeing Stories* also multiplies the individual powers of images and words. Finally, and perhaps most importantly, students developing their powers of visual and verbal narrative do so by an ascending double helix of intertwined art appreciation and art creation.