Paul R. Halmos - Lester R. Ford Awards

Alex Rice

"Reciprocal Sums and Counting Functions," *The American Mathematical Monthly,* 129:10, 903–912. doi.org/10.1080/00029890.2022.2115268

This accessible article explores the relationship between counting functions and sums of reciprocals of positive integers. Counting functions count the number of elements in a subset of the natural numbers up to a threshold $x>1$, for instance the function $\pi(x)$ which counts the number of primes up to x. Reciprocal sum functions add the reciprocals of a subset of the natural numbers, for instance the sum of the reciprocals of the primes, up to a threshold $x>1$. Classical and cutting-edge connections between counting functions and reciprocal sums for sets of positive integers are presented using only basic calculus, a well-known harmonic series estimate, and a partial summation formula that uses a discrete-continuous analog of integration by parts. From the extent to which the counting function and the reciprocal sum function inform each other for a set of positive integers, to an exposition on breakthrough work on the convergence or divergence of reciprocal sums of a set of positive integers and the presence of three term arithmetic progressions in the set (i.e., $\{n, n+d, n+2d\}$, $n, d \in \mathbb{N}$), this engaging article guides the reader through this exciting topic.

Response

I am equal parts surprised and grateful to receive this award. I am especially pleased for the recognition of this article on reciprocal sums and counting functions, my first contribution to the *Monthly*, because it includes components whose origins span the entirety of my career as a mathematical writer. For the earliest such example, a substantial chunk of the article's content can be traced to a project completed for an elementary number theory course during my third year as an undergraduate at the University of Georgia, presented to my peers the day after my 21st birthday. To honor that, I will paraphrase the acknowledgements from that paper: "I would like to express my infinite gratitude for the mathematical as well as personal support provided by all my mentors, friends, colleagues, without whom I may still be a journalism major."

Biographical Sketch

Alex Rice is an associate professor of mathematics at Millsaps College in Jackson, MS. He earned his BS in 2008, and his PhD in 2012, from the University

of Georgia, a short trip from his hometown of Roswell in metro Atlanta. Prior to arriving at Millsaps in 2017, he held visiting positions at Bucknell University in Lewisburg, PA, and the University of Rochester in New York. While his personal research lies primarily in arithmetic combinatorics, he also supervises an annual summer research program for Millsaps undergraduates, producing published work in discrete geometry, Ramsey theory, number theory, and hopefully more to come. Alex lives in Jackson with his partner, Sabrina, a psychology professor at Millsaps, and their beloved 16-year-old cat, Cauchy. Alex adopted and named Cauchy as an undergraduate, and she has accompanied him on his entire winding mathematical journey.