Real Analysis

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Description of Analysis

The area of undergraduate analysis includes introductory and advanced courses. First courses focus on analysis on the real line including properties of real numbers, basic point-set topology, and the theory of single variable calculus. At some larger institutions, the introductory analysis course is offered at several different levels, to allow well-prepared students to advance immediately through the material, and less-prepared students more time to navigate the topics. Advanced courses in analysis include topics such as measure theory, Lebesgue integration, metric space theory, functional analysis, and complex analysis. Most programs offer at least the courses in introductory analysis and complex analysis. Analysis courses are considered among the most difficult courses in the mathematics curriculum. Some courses in analysis are the first higher-level mathematics courses students take. The logic, even in introductory courses, is much more sophisticated than what students have encountered in previous courses of calculus and introduction to proof. Finally, the introductory analysis course is the first course in which students undertake the serious study of sequences and series of functions, and for many students it is the first serious study of sequences and series of constants.

Student Audience

It is standard that undergraduate mathematics majors, including majors intending a career in high school teaching, complete at least one course in introductory analysis, though some programs do not require this. Given the applicability of the results and methods of analysis to most areas of mathematics, we recommend that an introductory course in analysis be required of or at least strongly recommended to all mathematics majors. Many departments report physics, economics and computer science majors as audiences for their courses. Some engineering students also take these introductory courses in analysis. More advanced analysis courses typically enroll only those students intending graduate study in mathematics or in some cases economics.

History

The topics covered in most introductory analysis courses have not changed significantly in at least the past 30 years. While there are dozens of available new textbooks for the course, many of the textbooks that were popular in the 1980s are still in use today (e.g. Rudin, (2)), some with additional authors and updates. Some programs offer more advanced courses in analysis that include topics such as measure theory and functional analysis, material that had been the purview of graduate mathematics programs. The primary mode of delivery for introductory courses appears to be lecture with class discussion. However, the approach is varied with some institutions creating student-centered environments such as inquiry-based learning.

Use of technology

Tools such as graphing calculators and computer algebra systems can improve student understanding of convergence, continuity, and differentiability, and can illustrate interesting cases such as $f(x) = \begin{cases} x^n \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$ being a function that is $n - 1$ times differentiable.

There are readily available Java applets that provide visualizations of the δ-ϵ definition of limit. Appropriate integration of such resources can help students gain deeper understanding of the complicated definitions and results in real analysis.

Prerequisites

At most institutions, the first course in analysis requires completion of the standard single variable calculus sequence and at least one "transitions" course that introduces students to methods of proof. A background in proof including an introduction to quantified statements, proofs by contradiction, contraposition, and mathematical induction, is extremely helpful to students working with the multiply-quantified statements that abound in analysis (e.g., "for every $\epsilon > 0$, there exists $\delta > 0$, . . ."). However, at some institutions Introductory Real Analysis or Advanced Calculus is used as the first introduction to proof course. Some textbooks are carefully and deliberately written to serve this purpose (See Lay (1) or Zorn (2), for examples). When introductory analysis is used as the topic for a first proofs course, the scope of the course is usually narrowed when compared to an introductory analysis course that has a proofs course as prerequisite.

With few exceptions, higher-level analysis courses require the introductory analysis course as a prerequisite.

Cognitive goals addressed

An introductory analysis course typically focuses on the rigorous development of properties of the set of real numbers, and the theory of functions on the real line. This includes the study of the topology of the real numbers, sequences and series of real numbers, continuity, sequences of functions, differentiability, and Riemann integration. While the specific content of this course is important, of equal importance is helping students develop their analytical reasoning and critical thinking skills. Students must be creative in analysis, and must learn to persevere in solving complicated problems. They should develop the ability to read, understand, and write proofs, and to accurately and concisely communicate mathematics.

To understand concepts including limits, convergence of sequences and series, subsequences, the completeness of $\mathbb{R}$ in contrast with the set $\mathbb{Q}$ of rational numbers, and definitions of the derivative and the Riemann integral, students must grapple with infinite processes. They must become comfortable with imagining things that are difficult to visualize. Students must think creatively to achieve this. Working with the delta-epsilon definition of limit is one of the major stumbling blocks to student success in the course. Constructing delta-epsilon proofs often requires the student to work backwards: they first identify their target (what they need to conclude in their proof), work backwards toward what they know (their hypotheses), and then reverse these steps to write a logical proof that is clear, concise, and correct. All of this takes deep thought, organization, creativity, and careful analysis.

Communicating mathematics, both in written and verbal form, should be integral to any analysis course. Analysis proofs are sophisticated and students must develop strong communication skills to successfully write and communicate them. An introductory analysis class is the perfect setting for students to begin learning to write carefully about mathematics and to present their proofs to their peers. It is also an ideal time to introduce students to professional mathematical typesetting programs such as LaTeX. In many ways, the first course in analysis introduces students to the profession of mathematics.

Student-centered courses (for example, inquiry based learning, problem based learning, Moore Method) tend to focus more on these general skills than lecture-based courses, but sometimes cover fewer topics. In any case, class enrollment limits should be small enough to assure each student is given the opportunity to develop these skills.

Other Mathematical Outcomes

- Students should demonstrate understanding and facility with proof techniques including direct proof, proof by contradiction, and proof by contraposition.
- Students should improve both written and oral mathematical communication skills.
- Students should learn the mathematics that forms the rigorous foundation of calculus of a single real variable. They should develop facility with the major theorems of calculus.
- In order to help them understand the ideas better, students should be able to construct appropriate examples and counterexamples. To this end, the course should present the students with statements whose veracity is in doubt and asked to determine whether or not those statements are true.
- Among the topics typically included in a first course in analysis, students should have an understanding of
    - Properties of sets of real numbers including open and closed sets, density and compactness,
    - Sequences and series of real numbers, the definition of convergence, Cauchy sequences, limit theorems (such as the monotone convergence theorem), and relationships between the topology of the real line and sequences (such as the Heine-Borel theorem)
    - Functions of a real variable, including continuity, differentiability and important theorems (e.g. intermediate-value theorem, mean-value theorem),
    - Sequences and series of functions of a real variable, pointwise convergence, uniform convergence, power series and Taylor series,
    - Riemann integration and the fundamental theorem of calculus.
    - The historical development of the theory of analysis.

Ideal Course

The ideal course in introductory analysis should help students understand the underpinnings of calculus and to prepare them to dive further into important topics such as measure and probability theory, metric and normed linear spaces, differential geometry, and functional analysis, and to prepare for advanced excursions in differential equations.

Students in this course should learn to work independently, to read and understand theorems, and to construct their own arguments and explain them to others. The course should also introduce students to mathematical writing.

Sample Course Syllabi

Introductory Analysis

The following are suggested themes and topics for a standard one-semester course in introductory analysis that has a proofs-based prerequisite. Topics marked with # can be considered optional topics to include given sufficient time. We list the themes and topics in one logical order, but there are other orders in which these topics can be covered. For example, some may cover integration before covering series of functions and function approximation.

- Properties of the real numbers
  - Completeness
  - Open and closed sets
  - Properties of absolute value
  - Supremum and infimum of a set
- Sequences and series of real numbers
  - Limit of a sequence
  - Convergence of bounded monotone sequences
  - Subsequences and the Bolzano-Weierstrass theorem
  - Cauchy sequences
  - Series convergence
  - Tests for convergence
  - Absolute convergence versus conditional convergence and corresponding properties
- More topology of the real line
  - Compactness
  - The Heine-Borel theorem
  - Dense sets
  - The Baire category theorem #
- Functions on the real line
  - Continuity
    - Continuity and compact sets
    - The extreme value theorem
    - Uniform continuity
    - The intermediate value theorem
    - Lipschitz continuity #
  - Differentiability
    - Properties of differentiable functions
    - Darboux's Theorem and the intermediate value property of derivatives
    - Fermat's Theorem and the location of extrema for a differentiable function
    - The mean value theorem
  - Functions illustrating important ideas, for example
    - Nowhere continuous functions
    - Functions that are differentiable but with a discontinuous derivative
    - Continuous nowhere differentiable functions
- Sequences of functions
  - Pointwise convergence
  - Uniform convergence and its relationship with continuity
  - Weierstrass M-test
- Series of functions and function approximation

    - Power series
    - Taylor series
    - Fourier Series #
    - Bernstein polynomials and Weierstrass approximation #
    - Lagrange interpolating polynomials #
- Riemann integration
    - Criteria for integrability
    - Properties of the Riemann integral
    - Uniform convergence and integration (when can the limit move inside the integral)
    - The fundamental theorem of integral calculus
- Metric spaces #
    - Topology of metric spaces #
    - Equivalent metrics #
    - Completeness #
    - Continuous mappings between metric spaces #
- Normed linear spaces #
    - Topology of normed linear spaces #
    - Equivalent norms #
    - Completeness #
    - Continuous mappings between normed linear spaces #
- Introduction to Lebesgue Measure #

Introductory Analysis with Introduction to Proof
The following are suggested themes and topics for a one-semester course in introductory analysis that does not have a proofs-based prerequisite. Topics marked with # can be considered optional topics to include given sufficient time. We list the themes and topics in one logical order, but there are other orders in which these topics can be covered.

- Logic, connectives, quantifiers
- Set notation and operations
- Proof techniques
    - Direct proof
    - Inductive proofs
    - Proof by contradiction
- Functions
- Cardinality
- Properties of the real numbers
    - Completeness
    - Open and closed sets
    - Properties of absolute value
    - Supremum and infimum of a set
- Sequences and series of real numbers
    - Limit of a sequence
    - Convergence of bounded monotone sequences
    - Subsequences and the Bolzano-Weierstrass theorem
    - Cauchy sequences
    - Series convergence
    - Tests for convergence
    - Absolute convergence versus conditional convergence and corresponding properties
- More topology of the real line
    - Compactness
    - The Heine-Borel theorem
    - Dense sets
    - The Baire category theorem #
- Functions on the real line
    - Continuity
        - Continuity and compact sets
        - The extreme value theorem
        - Uniform continuity
        - The intermediate value theorem
    - Differentiability
        - Properties of differentiable functions
        - Darboux's Theorem and the intermediate value property of derivatives
        - Fermat's Theorem and the location of extrema for a differentiable function
        - The mean value theorem
    - Functions illustrating important ideas, for example
        - Nowhere continuous functions
        - Functions that are differentiable but with a discontinuous derivative
        - Continuous nowhere differentiable functions

- Sequences of functions
    - Pointwise convergence
    - Uniform convergence and its relationship with continuity
    - Weierstrass M-test
- Series of functions and function approximation #
    - Power series #
    - Taylor series #
    - Fourier Series #
    - Bernstein polynomials and Weierstrass approximation #
    - Lagrange interpolating polynomials #
- Riemann integration #
    - Criteria for integrability #
    - Properties of the Riemann integral #
    - Uniform convergence and integration (when can the limit move inside the integral) #
    - The fundamental theorem of integral calculus #

Introductory Analysis based on Fourier Series
The following are suggested themes and topics for a one-semester course in introductory analysis that is built around Fourier series and that has a proofs-based prerequisite. We list the themes and topics in one logical order, but there are other orders in which these topics can be covered.

- Fourier series: what they are, why they are important, why they are problematic
- Infinite sequences and series of constants
    - Limit of a sequence
    - Convergence of bounded monotone sequences
    - Subsequences and the Bolzano-Weierstrass theorem
    - Cauchy sequences
    - Series convergence
- Continuity, differentiability, mean value theorem, properties of R
    - Completeness
    - Open and closed sets
    - Compactness
    - Continuity and compact sets
    - The extreme value theorem
    - Uniform continuity
    - The intermediate value theorem
    - Properties of differentiable functions
    - Darboux's Theorem and the intermediate value property of derivatives
    - Fermat's Theorem and the location of extrema for a differentiable function
    - The mean value theorem
- Convergence Tests
    - Absolute convergence versus conditional convergence and corresponding properties
- Uniform Convergence and its implications
    - Sequences of functions
    - Pointwise and uniform convergence
    - Uniform convergence and the relationship to continuity
    - Uniform convergence and the relationship to term-by-term differentiation
- Riemann Integration
    - Criteria for integrability
    - Properties of the Riemann integral
    - Uniform convergence and integration (when can the limit move inside the integral)
    - The fundamental theorem of integral calculus

Bibliography

Remark*: The presence of a text on this list is not meant to imply an endorsement of that text, nor is the absence of a particular text from the list meant to be an anti-endorsement. The texts are chosen to illustrate the sorts of texts that support various types of real analysis courses. Please note that some of the articles listed were written by authors of this report.*

Texts Referenced in Report

1. Lay, Steven R. *Analysis: With an Introduction to Proof*. 5th. ed, Pearson, 2012.
2. Rudin, Walter. *Principles of Mathematical Analysis*. 3rd. ed, McGraw Hill, 1976.
3. Zorn, Paul. *Understanding Real Analysis*. CRC Press, 2010.

Specific Tasks or Activities

4. Amstrong, G. M. Some projects for calculus and analysis. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 1*(3), 275-286, 1991.

   This paper suggests some project appropriate for real analysis students. Project I compares concepts of calculus with the development of infinitesmals. Project II defines three additional types of derivatives and compare them with the ordinary derivative. Project III contrasts integral of Newton, Cauchy, and Riemann. The author provided results of class testing the project.

5. Barnes, J. Teaching real analysis in the land of make believe. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 17*(4), 366-372, 2007.

   This article introduced the authors experience of teaching mathematical definitions in a standard introductory real analysis. The definitions covered in this article are those of convergent sequences, bounded sequences, Cauchy sequences, continuous functions, removable discontinuity, and uniform continuity. Fairy tales describing a few of the definitions in detail were suggested as reading materials to help students understand these definitions.

6. Barnes, J. Feather boas in real analysis. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 21*(2), 130-141, 2011.

   This article described a collection of hands-on activities using feather boas, yarn, and doorknob hangers to help students understand the formal definitions for convergent sequences, continuity, and uniform continuity.

7. Mamona-Downs, J. Letting the intuitive bear on the formal: A didactical approach for the understanding of the limit of a sequence. *Educational Studies in Mathematics, 48,* 259-288, 2001.

   This article provides an analysis of the formal definition of limit via identifying roles of each symbol that occurs to achieve a mental image firmly consonant with the definition and a description of how this mental imagery may be used to reexamine the validity of some intuitive beliefs. The author also provided a series of tasks to encourage students' intuitive images of a sequence as having an ultimate term associated with the limit.

8. Mamona-Downs, J. On introducing a set perspective in the learning of limits of real sequences. *International Journal of Mathematical Education in Science and Technology, 41*(2), 277-291, 2010.

   This article aimed to provide students with opportunities to contrast the convergent behavior of a sequence and the accumulation points of the underlying set of the sequence through their commonalities and differences in structure. The author claims that the more set-oriented perspective provides students with an alternative source for proving propositions concerning properties of the limit of a sequence. Some examples were given and comments were made on the cognitive level in this article.

9. Roh, K. How to help students conceptualize the rigorous definition of the limit of a sequence. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 20*(6), 473-487, 2010a.

   This article presented how real analysis students can be guided to conceptualize the rigorous definition of the limit of a sequence. In particular, this article proposed a hands-on activity, called the ε-strip activity, with a physical device made from translucent paper, as an instructional method to help students understand what it means for a sequence to have a limit via visualization of the ε-N definition of the limit of a sequence. Two statements were suggested to students, each of which was written in terms of a relationship between ε and N, similar to the ε-N definition: One required infinitely many points of the sequence in the ε-strips, which students commonly suggest, but is not compatible with the standard definition. The other required only finitely many points outside the ε-strips, which sounds counterintuitive to students, but is logically equivalent to the standard definition. This article also describes the learning objectives of each step of the activity, and provides detailed instructional methods to guide students to reach the learning objectives.

10. Roh, K., & Y.-H Lee, The Mayan activity: A way of teaching multiple quantifications in logical contexts. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 21*(8), 685-698, 2011.

    This article suggests an instructional intervention to help students understand statements involving multiple quantifiers in real analysis contexts. The authors first focused on students' mis-interpretations of multiple quantifiers related to the ε-N definition of the limit of a sequence and pointed out that they resulted from a lack of understanding of the significance of the order of the quantifiers in the definition. Next, the authors introduced the Mayan activity which was designed to cause and then to help students resolve their cognitive dissonance. In particular, the Mayan stonecutter story in the activity was presented in an understandable and colloquial form so that students can recognize the independence of ε from N in the ε-N definition. The authors illustrate how the Mayan activity can be used in classroom as a useful instructional intervention for student to study proofs of convergent sequences and Cauchy sequences.

Students' Understanding or Perspectives

11. Dawkins, P. Metaphor as a possible pathway to more formal understanding of the definition of sequence convergence. *The Journal of Mathematical Behavior, 31*(3), 331-343, 2012.

This article presented how an instructor's introduction of a metaphor constituted an experientially real context in which an undergraduate real analysis students developed a property-based definition of sequence convergence. The author traced the transformation of the student's conception from non-standard, personal concept definition rooted in the metaphor to a concept definition for sequence convergence compatible with the standard definition. This study also documented the student's progression through the definition-of and definition-for stages of mathematical activity in an interactive lecture classroom context.

12. Edwards, B. *Undergraduate mathematics majors' understanding and use of formal definitions in real analysis*. Unpublished doctoral dissertation, The Pennsylvania State University, 1997.

    The focus of the study was on undergraduate mathematics majors' understanding and use of formal definitions in real analysis. This thesis includes a case-study analysis and a cross-case analysis which focused on students' understanding of the role of mathematical definitions and the logical and conceptual complexities of such definitions as those for pointwise continuity, absolute continuity, infinite decimals, and connectedness. The author found that students did not necessarily share mathematicians' understanding of the role of mathematical definitions as lexical, and they had difficulties with the complexities of wording and logic, especially in the continuity definitions. Also, if students settled prematurely on the meaning of all or part of a given definition they seemed to be unable or unwilling to consider conflicting evidence.

13. Roh, K. Students' images and their understanding of definitions of the limit of a sequence. *Educational Studies in Mathematics, 69*, 217-233, 2008.

    The author explored how students' images of limit influence their understanding of definitions of the limit of a sequence. In a series of task-based interviews, students evaluated the propriety of statements describing the convergence of sequence through a specially designed hands-on activity, called the ε-strip activity (Roh, 2010a). In particular, this paper illustrates how students' understanding of definition of the limit of a sequence can be influenced by their images of limits as asymptotes, cluster points, or true limit points. The implications of the study for the teaching and learning of the concept of limit are also discussed in this paper.

14. Roh, K. An empirical study of students' understanding of a logical structure in the definition of limit via the ε-strip activity. *Educational Studies in Mathematics, 73*, 263-279, 2010b.

    This study provided empirical evidence that students' understanding of the limit of a sequence is closely related to their understanding of a logical structure in defining the limit of a sequence. The subjects of the study were college students who had already encountered the concept of limit and limit symbols, but did not have any experience with rigorous proofs using the ε-N definition. The author addressed three essential components that students must conceptualize in order to understand properly the relationship between ε and N in the ε-N definition of limit: The index N corresponding to ε should be determined before completing the variation of ε, any ε in the definition implies the arbitrariness of error bounds, and the arbitrariness of ε implies that the error bound decreases towards 0.

15. Swinyard, C., Reinventing the formal definition of limit: The case of Amy and Mike. *Journal of Mathematical Behavior, 30,* 93-114, 2011.

    This article focused on the evolution of the students' definition of limit in ten-week teaching experiment. Two students, neither of whom had previously seen the conventional $\varepsilon - \delta$ definition of limit, reinvented a formal definition of limit capturing the intended meaning of the conventional definition. This article provided a detailed account of how student might think about limits formally.

16. Swinyard, C., & S. Larsen, Coming to understand the formal definition of limit: Insights gained from engaging students in reinvention. *Journal for Research in Mathematics Education, 43*(4), 465-493, 2012.

    This article illustrates a model of how student might come to understand the concept of the limit of a function at a point and at infinity, respectively. Analyzing two teaching experiments, the authors proposed two theoretical constructs to account for the students' success in formulating and understanding the formal definition of the limit of a function: (1) the need for students to move away from their tendency to attend first the input variable of the function; and (2) the need for students to overcome the practical impossibility of completing an infinite process.

Instructional Techniques or Classroom Structures

17. Alcock, L., Interactions between teaching and research: Developing pedagogical content knowledge for real analysis. In R. Leikin & R. Zazkis (Eds.), *Learning through teaching mathematics: Development of teachers' knowledge and expertise in practice* (pp. 227-245). DOI: 10.1007/978-90-481-3990-3_12, 2010.

    This chapter describes the author's five practices used in teaching undergraduate real analysis: regular testing on definitions, tasks that involve extending example spaces, tasks that involve constructing and understanding diagrams, use of resources for improving proof comprehension and tasks that involve mapping the structure of a whole course. The author describes the rationale for each of these practices from a teacher's point of view and relate each to results from mathematics education research, focusing on the need for students to develop skills on multiple levels and the question of how best to use available lecture time.

18. Dawkins, P., Concrete metaphors in the undergraduate real analysis classroom. In S. L. Swars, D. W. Stinson, & S. Lemons-Smith (Eds.) *Proceedings of the 31st Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education,* (5, pp. 819-826), Atlanta, Georgia: Georgia State University, 2009.

    The author examined metaphors that an instructor used in a real analysis classroom, and classified them into logical and mathematical metaphors. Logical metaphors were those used to clarify the logical structure of a definition and perhaps guide a proof structure. Mathematical metaphors were those used to demonstrate aspects of the concepts themselves or the structure of the definitions. The instructor's teaching methods in Dawkins' study were different from "definition-theorem-proof" instruction in the sense that the instructor involved students in choosing definitions or theorems and in generating examples, arguments, and counter-examples. The teaching methods were also

different from all-student-discovery styles in the sense that the instructor always gave the class guidance and introduced formal mathematical knowledge (definitions and theorems) once the students arrived at the underlying ideas of the formal mathematical knowledge. The article reported that instructional metaphors were likely to be integrated into the students' concept images, and were likely used for recall, integrated into student language, and potentially introduce misconceptions.

19. Dawkins, P., & K. Roh, Mechanisms for scientific debate in real analysis classrooms. In Weist, L. & Lamberg, T. (Eds.), *Proceedings of the 33rd Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 820-828), Reno, NV: University of Nevada, Reno, 2011.

    This article presents two accounts of successful scientific debate from different undergraduate, real analysis classrooms by combining observations from separate instances of non-traditional, inquiry-oriented real analysis instruction. The first real analysis class was taught at a mid-sized, research university in the spring of 2009. A mathematician who specialized in differential geometry taught the first real analysis course. This article illustrated one day of the first course leading up to the definition of the limit of a function at a point, in which the instructor presented examples that did and did not have limits due to restricted domains so as to motivate the standard requirement that limits only be defined at cluster points of the domain of the function. On the other hand, the second real analysis class was taught at a large-sized, research university in the spring of 2010. The second author of the article served as an instructor of the second real analysis course. Instruction in the course followed an inquiry approach, in which students were often asked to make and justify conjectures and to evaluate argument. This article illustrated one day of the second course dealing with the following task: (1) Does there exist a real number x satisfying that for any $\varepsilon > 0$, $|x| < \varepsilon$? (2) If there does exist an x is, there any other real number x satisfying that for any $\varepsilon > 0$, $|x| < \varepsilon$? This article identified the means by which the debate began and proceeded toward resolution in each class.

20. Dumitraşcu, D., Integration of guided discovery in the teaching of real analysis. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 19*(4), 370-380, 2009.

    This article discusses the author's experience with teaching a undergraduate real analysis class using both lecturing and the small group guided discovery method. It covers descriptions of the organizational and administrative components of the class, examples of successes and difficulties that the students had with the mathematical content of the course, some of the students' comments regarding the instructional approach, and a way to test students' progress by examining students' concept images of real number changed and evolved.

21. Greer, M. L., & B. Reed, Blogs hit classroom: Students start reading. *Problems, Resources, and Issues in Mathematics Undergraduate Studies*, 18(2), 139-148, 2008.

    This article reports how blogs can be successfully used for a conversation between a professor and students as part of a real analysis course. The goals of using a blog included giving students' a motivation to read ahead in their textbooks, providing another means of communication between students and the professor, and creating a space for students to write about mathematics.

22. Maher, R. Step by step proofs and small groups in first courses in algebra and analysis. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 4*(*3*), 235-243, 1994.

This article presented a successful way of teaching proof writing in several different settings. Two components of the approach involved the use of step by step proofs for selected theorems, coupled with small group activities to expand upon and extend these results. This article provides how the two-part approach was implemented to teach a proof of the product of two convergent sequences to be convergent.

23. Peterson, E., Using a wiki to enhance cooperative learning in a real analysis course. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 19*(1), 18-28, 2009.

This article described how the author used a wiki-based website in a real analysis course, and assessed its effectiveness. The wiki was used to post course materials, maintain a forum, enable students to write collaborative projects, and enable students to develop a glossary of important terms. The author claimed that wiki proved to be very successfully by facilitating student collaboration, exposing students to LaTex, and even helping study for examinations.

24. Weber, K., Traditional instruction in advanced mathematics courses: a case study of one professor's lectures and proofs in an introductory real analysis course. *Journal of Mathematical Behavior, 23*, 115-133, 2004.

This paper reports a real analysis classroom taught via conventional lectures, fitting the "definition-theorem-proof" style of instruction. The professor used several distinct teaching styles that differed according to the instructors' perceptions of the mathematical content and students' needs. Weber's work provides insight about how conventional lecture style of advanced mathematics courses can be structured by various teaching methods and how faculty perceptions of student understanding can guide their preparation and decision making within the lecture format.