Technology and the Mathematics Curriculum

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Introduction

What role should technology play in teaching and learning undergraduate mathematics? We assume the primary goals for using technology are to deepen student understanding of mathematics and statistics and to increase student motivation and engagement. We believe the informed and intelligent use of technology helps meet these goals. We address teachers who seek to enhance current use of technology and colleagues who do not use technology now. We discuss departments and institutions concerned with the implications of technology and also faculty incentives and rewards for the risks and hard work of creating and implementing technology innovations.

We have a broad and flexible view of what technology means. Wherever feasible we specify technologies by function, referring to computer algebra system (CAS), graphing calculator (or a smart phone with a graphing app), online applet, course management system, spreadsheet, or statistical package, rather than by "brand" names. We include both free software and online utilities and packages sold commercially. For example, a formal assignment could require students to use the software licensed by the school, free online utilities such as WolframAlpha[39], Sage[30], SOCR[35], or freely downloadable software such as R[28] or GeoGebra[11]. Some commercial products are better documented and supported than free ones, but not always. The landscape is changing rapidly. Many technologies work on multiple platforms. Students do (and could be encouraged to) use their smart phones and other hand-held devices to access online resources in addition to computers and calculators. Different platforms will be appropriate in different environments and we do not specify them. Although it is imprecise, we use "mathematics" and "mathematical" to refer to the range of subjects taught by departments of mathematical sciences; in particular, we include statistics. We use the term "teacher" in lieu of "instructor" or "professor".

We discuss five ways technology can be used to enhance teaching and learning:

- **Exploration:** Technology can be used to explore ideas and substantial applications.
- **Computation:** Technology can enable students to work with rich examples, realistic applications, and large data sets.
- **Communication:** Technology can facilitate communication between teacher and student and among students, inside and outside the classroom.
- **Assessment:** Technology can give students more ways to demonstrate knowledge and understanding and improve the effectiveness of both formative and summative assessment.

- **Motivation:** The use of technology can increase students' engagement and motivation.

Using technology for **visualization** could warrant a separate section, but instead we have referred to visualization throughout this document. The coming *Pedagogy Guide* [15] of the MAA Committee on the Teaching of Undergraduate Mathematics will provide some evidence on how well certain pedagogical strategies work.

We also discuss

- **Benefits after college:** Students' experience using technology can help them in the workplace and in graduate school.

- **Issues for departments and institutions:** The use of technology raises a host of issues for departments, colleges and universities.

An annotated bibliography, including digital sources and websites, appears at the end of this report.

Key to the effective use of technology is using it intelligently. Is the technology appropriate for the given problem? Are hypotheses satisfied to guarantee that the process will produce an answer? If numerical, is the answer plausible? Of the right order of magnitude? These questions can be subtle, even for experienced teachers, but they should always be raised. Also the effort required for successful use of a specific technology by students at a particular level should be balanced against the benefits.

1 Exploration

The goal of exploration is for students to engage with examples, deepen understanding, notice patterns, and make conjectures, thus providing a natural setting for recognizing the importance of proof to support correct conjectures. To be effective, exploration must culminate in clear statements of what has been discovered. Teachers must follow up and be sure students understand what the exploration has achieved.

Students in a College Algebra course that emphasizes modeling and numeracy may use a spreadsheet to work examples, build familiarity with ideas, and look for patterns. Any graphing utility can enable students to experiment with the effect of changing parameters of a function, say with the quadratic function $f(x) = a(x-b)^2 + c$. For students in multivariable calculus it might be a first look at the "second derivative test" via the plot of a function $f(x, y)$ and the expression $D(x, y) = f_{xx}(x, y)f_{yy}(x, y) - (f_{xy}(x, y))^2$ at a critical point. Technology can allow students low-cost attempts to identify parameter values to achieve a desired result, and they can learn even from failed attempts. A graphing utility can also change a "procedure," like the method of Lagrange multipliers, into something that is geometrically meaningful. Technology can help Linear Algebra students visualize the effect of a linear transformation of the plane and notice the role of the determinant in comparing the area and orientation of a finite region and its image.

Assignments can also help students think critically about what technology tells them. A Calculus assignment with a twist asks students to analyze/critique the following argument. The function $1/(\cos x + 2)$ is a positive continuous function for all x, so its antiderivatives must all be continuous, strictly increasing functions, but the antiderivative given by a CAS is periodic of period 2π and hence not strictly increasing. Explain this.

In a Transition to Proofs course, students can use a simple program (or a CAS or an applet) to examine the sequence $x_n = f^n(x_0)$ of iterates of a linear function $f(x) = ax + b$ as they vary the initial point x_0 and the parameters a and b. Students observe patterns, formulate conjectures, and try to prove them. Along the way, students have to develop the precise language of convergence.

In a Real Analysis course, students' exploration of the iteration of a function of the form $g_c(x) = cx(1-x)$ introduces them to discrete dynamical systems and the analysis of fixed points. Specialized programs like GAP [10], Magma [16], and Sage [30] permit explorations in Abstract Algebra.

For Differential Equations, the displacement, $y(t)$, at time t of a physical device satisfies a differential equation, $ay''(t) + by'(t) + cy(t) = f(t)$, where $f(t) = \cos(\omega t)$. Students use technology to explore the effects of changes in parameters, discover the relationship between the frequency ω of the function f and the natural frequency of the solution of the differential equation, and experience beats and resonance (especially if their CAS gives output aurally as well as visually). They gather data through numerical or analytic runs of a CAS to create a frequency response plot showing the maximum displacements of the physical device across all input frequencies ω.

Students can explore the online universe, using search engines or online sites dedicated to mathematics, data or computation to look up a mathematical concept or a person/place of mathematical interest. The MAA provides a site [5] to explore resources for a variety of courses. Another source is the site *Demos with Positive Impact* [6]. The American Mathematical Association of Two Year Colleges (AMATYC) has a project called *The Right Stuff: Appropriate Mathematics for All Students* [29]. There are many special-purpose applets available, but they can be difficult to find. As one example, Wolfram Research [38] has many demos.

2 Computation

In courses at all levels, substantial and realistic applications involve "messy" mathematics that makes calculation by hand onerous or infeasible. Using technology opens the door for students to set up solution strategies, justify their analyses, and interpret the results. For example, in Calculus there is no reason to restrict the study of optimization to situations where functions are simple and zeroes of derivatives are easy to find by hand.

Linear Algebra is a good place for using technology to carry out lengthy and/or complex computations, so students can concentrate on the underlying ideas. For example, when using a Leslie matrix to study populations, technology can permit a preliminary "what if" analysis by changing parameters through iterations, before a formal analysis of the model via eigenvalues of the matrix.

In Differential Equations, students can see the form of the solution using a CAS and understand the nature of a solution, say of steady state and transient portions. When analytic solutions are not possible, numerical solutions, even using simple Euler methods in a spreadsheet, can give insight as well as permit a study of the importance of step size in approximating a solution. Moreover, estimating parameters in a differential equation model for which data are available can suggest or confirm both the appropriate modeling mechanism and the form of the solution.

There are many options currently available for accessing and using powerful computer algebra systems. The most widely used packages include Maple[17], Mathematica[19], MATLAB[22], and Sage[30].

Introductory Statistics is another course where the use of technology for computation is essential. In this *2015 CUPM Guide*, the report on statistics has six overall recommendations for all introductory statistics courses, including statistics for mathematics majors. These recommendations include: "use real data" and "use technology for developing concepts and analyzing data."

Besides technology for exploration and analysis, students can also use technology for simulation to build understanding of probability and distributions, including such basic concepts as the distribution of sample means and the Central Limit Theorem. Many introductory statistics texts come with online resources. Other statistical packages (some commercial, some free) are independent of a text. The free *OpenIntro* [24] offers text plus supporting technology. The SIGMAA on Statistics Education has compiled a list of technology resources for the teaching of statistics [33].

3 Communication

We don't minimize the importance of communication that does not use technology. Indeed, technology-aided communication, using social media or structured platforms for the special purpose of course communication, often leads to face-to-face communication.

Communication is an important element of exploration, as students write up their findings. In fact, in an Inquiry-Based Learning class, student writing — facilitated by using a wiki — can produce a text for the class by the end of the term [13].

Alternatives to traditional textbooks in mathematics using technology are emerging. Free online and open source electronic textbooks are available in growing number. Examples include [2] and [36]. The American Institute of Mathematics [1] provides a clearing house for such materials.

There is considerable interest in using flipped classrooms. In the "forward" flip, students watch videos, read, and use technology before class to prepare for practice and exploration in class. In the "reverse" flip, students first explore a new idea in class and then use technology as well as watch or read material after class.

Videos can be used to add a coda to a class to be posted for student use before the next class. Some teachers use existing videos. Others use many levels and kinds of technology to produce their own videos in order to personalize the message. In this regard we note the apps EduCreation [7] and Screencast-o-matic [32].

Many teachers use course management software to monitor student success in online homework. Asking students to solve problems and/or submit questions before class informs the teacher about student successes and struggles and guides planning for the class, e.g., to determine what concepts to emphasize and/or how to group students for collaborative work.

The "ink technology" of a tablet or a so-called smart-pen (such as Livescribe [14]), can be used in addition to an online homework system. Students work on the online homework problems, but rather than writing on paper, they write using ink technology. There is no need to type, nor is the students' writing "interpreted," because the ink technology captures each student's process as the image of what they wrote and how they wrote it in real time. Each student can enter the final answer in the online homework. The teacher can review the tabulated online homework results and also the ink technology work and can choose the work of one or more students to share with the class. In the classroom the teacher can pause the display at any point to open discussion, give the solver feedback on his or her approach, and offer suggestions for next steps as well as engage other students in "what would you do here?" discussions. Ink technology solutions can also be posted anonymously using the course management system, and the teacher can ask students to comment and/or perhaps answer specific questions. The students' comments can be posted, or the discussion can continue in class.

There are many ways to use technology to capture what is done in class so it can be posted for students to review (or to preview). In addition, there might be a network of mobile devices used in the classroom, so the work in class can be preceded or followed by work online. The teacher can also use technology to present material in or out of the classroom, using, e.g., PowerPoint, Beamer, or PREZI [26]. The latter permits the teacher to jump to smaller pieces and can be stored online.

Students and teachers can use electronic discussion boards, wikis (from publishers or via course management systems), and commercial social media for collaboration and communication. An advantage of social media is that teacher-student communication can occur where the student already spends time. If some students find the teacher's presence in their social world intrusive, there are alternative cloud-based tools to reach students, such as Google+.

Students' smart phones can be an important platform for communication. Some course management systems let students choose how they prefer to receive communications: email, text messages, listserves, social media, etc. Some teachers use course reminder features on phones, e.g., text messaging, when they do not have course management systems. In some instances students do almost all their course work on their phones, including dictating writing to a voice-to-text system, proof reading in a word processor, and sending to a wireless printer.

Massive Open Online Courses, MOOCs, are a relatively new and growing presence on the landscape. Individual institutions are making their own decisions about whether their students can earn credit for taking certain MOOCs, possibly with local certification. In this way, existing MOOCs can expand their offerings. Others can use parts of MOOCs to enhance their own courses and widen their own students' experiences, perhaps recommending that their students watch selected lectures or demonstrations. Consider one example [23] in Robert Ghrist's Calculus Single Variable through Coursera.

Some schools are producing courseware to reach beyond their own campuses. This can result in a MOOC reaching a very large audience or a course shared among just a few institutions. Technology that captures the entire classroom (aurally and visually) can be used for "blended" classes that have both in-person and virtual participants. This can permit small institutions to team up and take turns offering a specialized course in real time. At Mount Holyoke College, the Mathematics Leadership Program uses a technology called Zoom [41] that unifies cloud video conferencing, online meeting and cross-platform group chat, to offer a hybrid course for in-service teachers with meetings simultaneously for on-campus participants and those from around the world [20].

Writing mathematics is an important form of communication for all students and at all levels. Technology enables a thorough mix of text and graphics for enhanced communication. Students bound for graduate school will be especially helped by learning to use TEX. See the next section for the use of technology to enable and enhance the assessment of writing assignments.

4 Assessment

Assessment can indicate whether students know facts, understand ideas, or can do a more complex activity, such as modeling or problem-solving. Technology can inform both the teacher and the student of what needs improvement. When students are active in the classroom (often facilitated by technology), the teacher can learn more about what students know and what they need to work on by listening to student conversations, live or captured.

Online homework systems (including WeBWorK [37]) are becoming more widely used. These systems collect extensive data that can help teachers and their departments refine their course offerings. Online homework systems can also be adapted for use in course placement. Some online systems incorporate tutorials that can extend the capacity of on-site tutors.

Discussing problem-solving using authentic student work, through video and ink technology, helps both the teacher and the students gain insight into what is understood. This technology helps the teacher plan classroom activities that are appropriate and effective. In another use, students can upload their solutions or essays, and their writing can be sent to other students for comments. The quality of a student's editorial comments contributes to his or her grade on the assignment.

Students posting questions and comments reveal their understanding. In the real or virtual classroom, "clickers" [3] and their smart phone, calculator, and computer analogs can provide useful feedback. For example, texting to answer with a phone can provide a teacher with recordable and instant assessment results. Index cards used as low-tech substitutes for clickers can provide the same immediate feedback, but not the recordability nor the anonymity.

Technology also affects assessment practices by challenging teachers to construct examination questions that cannot be entirely "answered" by technology. We say more about this in the section on department issues.

5 Student Motivation

When students take ownership of their learning, they are motivated to persevere and succeed. Pedagogical strategies that engage students actively in exploring and discovering ideas, solving problems, communicating their ideas to others, and reflecting on their own thinking can foster this ownership.

For many students, seeing the mathematics they are studying applied to real-world situations is an important motivator. Students then see how the mathematics they are learning will re-appear in subsequent courses. Technology can enable students to work with more realistic applications of mathematics and with real data. The scale can vary from a simple activity in class to a substantial student project. Indeed, there is growing interest in using applications, coupled with technology, to introduce and motivate the learning of mathematics. For example, SIMIODE [34] is a site for learning and teaching differential equations using realistic modeling and technology in this way. In addition, when students work with large data sets or complex calculations in their mathematics classes, they can draw on those classes for projects in courses offered in other departments.

When using technology, students seem to be more apt to try things and try again, to be playful and less fearful of error. Students may see by-hand solutions as laborious and error-prone and thus be more cautious working by hand. Perhaps the technology environment is less threatening because it can be more private. The experimentation and risk that accompany game playing have taught many current students to explore more boldly with technology. The immediate feedback offered by technology may make students more adventurous and more persistent. Visual images provided by technology can also be a powerful inducement to persist and search for understanding.

Some teachers have used game environments and rewards to motivate students to do activities that build understanding of mathematical concepts. For example, games in mathematical ecology can help students understand the elements of the mathematical models being used to generate the simulations. In fact, some recent books consider the use of games in learning; see for example [31].

Another use of games for teaching has been devised by George Woodbury [40] who uses a grading system that involves game-design features. Although students earn no points for their online homework scores, the students who "level up" (earn essentially perfect homework scores and a mastery score on the online practice quiz) can earn more points for their in-class exam scores than students who do not. Those who have leveled up but did not score well on the in-class exam also have an opportunity to re-take it.

6 Benefits after college

Employers want graduates to have experience with technology, be it programming or using software applications. Using an appropriate tool to solve a problem is a universally-valued skill. Effective communication of ideas often requires technology for images, data representations, and notation. Most important, for professional and personal needs, students need the ability to learn to use emerging technologies. We therefore have a responsibility to encourage and enable our students to learn technologies alongside their mathematics.

Graduate schools often expect knowledge of TEXto properly and professionally prepare material for communication, be it homework assignments, technical materials for posting, or articles for publication. Sometimes knowledge of a programming language might be appropriate for preliminary or even deep analyses, exploration and analysis of large data sets, or to modify established routines. Indeed, in a 2001 paper [25] a well-known algebraist, Cheryl Praeger, says:

> It is impossible to over-emphasize the impact of computer systems such as MAGMA and GAP on the professional lives of algebraists ... both on their teaching and on their research. All of my research students use these computer systems as essential learning tools to explore new concepts. They demand illustrative examples to examine by computer to aid their understanding.

7 Department and institutional issues

CUPM has long recommended a department culture of "continuous improvement". Departments and institutions have a responsibility to explore how their programs can or should change in light of technology. The place of technology in the curriculum should always be in service of general cognitive and content-specific learning goals, as outlined elsewhere in this Guide. We have highlighted ways this can happen by fostering exploration, computation, communication, and motivation, and attending to assessment.

Departments can work on technology and the curriculum in many ways, e.g., study by department committees, external reviews, seeking and obtaining grant support, or as a consequence of institutional commitments. To have positive outcomes, we argue that this programmatic work requires input from all department constituents and education of all on what technology is available and what the possibilities are in the department's setting. It is not healthy if individuals feel disenfranchised because they lack experience or interest in using technology, and we should not assume that these categories necessarily align with age. We recommend collegial processes and shared learning, including demonstrations by faculty who have experience using technology successfully, but not coercion. It is important to pick easy entry points for novice users: key places where technology saves time, enhancements of things they already do (such as illustrating simple Riemann sums), and simple ways to engage students more actively during class.

Technology often comes into our classrooms through student use of calculators and online CAS, very sophisticated CAS. The implications of this and other technology for what is important and/or necessary for our students to know is a large and sometimes contentious issue. It is a truism that a CAS should not be able to pass a calculus exam. But what does this mean? Is it an argument to forbid the use of technology on exams, or is it an argument to re-think our testing and, more broadly, our syllabi? We argue for the latter. If technology is used on an exam, students should be asked to describe the process and goal of any calculation, interpret its result, and explain its significance. An exam question can give the result of a computation and ask students for explanation and interpretation. Students should also be asked higher-order, conceptual questions that go beyond techniques and cannot be answered merely with technology. Thus technology should enable us all to elevate our "game" and our expectations of our students. At the same time, we have to carefully bring our students up to the level to which we aspire.

To illustrate, consider an example from calculus: integration by parts. Rather than merely a specific technique (which a CAS can carry out), this is a process of transformation (a new integral can be *s*et up by hand), and the idea of transformation should be emphasized more than facility with the details of the technique. The current interest in building concept inventories for calculus [8] to test more than computational skills is a move in the direction of raising the bar in student understanding and de-emphasizing the ability to do calculations for which technology is well suited.

Not everything need happen department-wide. Departments should also foster the work of individual teachers in finding or developing new and effective uses of technology in the classroom. This can occur through encouraging exploration and supporting reasonable risk-taking in the use of technology by individuals, particularly younger faculty. Varied mechanisms exist for providing incentives and rewards for individuals doing substantial work on curriculum and teaching in new ways. Annual evaluations of teaching should certainly recognize work of this kind. Some institutions offer their teachers internal grants for course development. For example, Georgia Southern University has for over 25 years offered supporting summer grants of $3-4000. Much of the work that was supported involved the use of technology, sometimes using software and courseware already available on campus. There is a premium on innovation, and a campus-wide committee judges proposals. Recently development of online courses has been particularly encouraged. At other schools, institution-wide committees select faculty for teaching awards to recognize their work in the classroom, including technology innovations. The recognition itself sends a signal, to the recipients and to others, that the institution values the work. Such efforts can be featured externally (for alumni, media, and prospective students) and internally through invited presentations for faculty gatherings and retreats.

Reasoned and reasonable experimentation in using technology should be encouraged and supported. Acknowledging the risks in the first use of any innovation is important, and institutional encouragement to learn from mistakes and improve should be in place.

In departments where a large fraction of the teaching is done by part-time adjuncts, incorporating the use of technology poses special challenges. These teachers may need help using the technology themselves. More important, they may need help using the technology to enhance the mathematical ideas, not to replace them.

Finally, we note that the use of technology has implications for institutional infrastructure. While some use of technology imposes new costs, there can also be some savings. For example, some departments and institutions are changing their "computer labs" to make use of student-owned devices.

While we cannot ignore the presence of problems, including some that can only be dealt with at the institutional level (such as difficulties with course management software), we believe that moving forward in using technology in teaching mathematics is essential for our discipline. To ignore technology is to ignore the future. We do this at our professional peril.

Conclusion

There is no conclusion, only on-going conversation and inquiry into the uses of technology in the undergraduate mathematics curriculum. We believe this is an evolutionary as well as revolutionary process, with new species and approaches being introduced (and some dying out). We encourage individual teachers, departments, and institutions to foster and support reasonable experimentation in the uses of technology to teach undergraduate mathematics. We believe wider discussion and sharing of implementations through MAA activities (meetings, speakers, newsletters, etc.), journal publications, [4, 9, 12, 18, 21, 27], campus visits, etc. can help support innovators in their local environments. We see this report as part of that process, and we believe that everyone—teachers and students—can benefit from open, objective, and energized discussions about the uses of technology in teaching undergraduate mathematics.

References

[1] *American Institute of Mathematics.* `http://aimath.org/textbooks/`.

The American Institute of Mathematics (AIM) seeks to encourage the adoption of open source and open access mathematics textbooks. The AIM Editorial Board has developed evaluation criteria to identify the books that are suitable for use in traditional university courses. The Editorial Board maintains a list of Approved Textbooks which have been judged to meet these criteria.

[2] Boelkins, M. 2014. *Active Calculus*.

This is a full-text free PDF calculus text with activities workbook made available by the author at `http://faculty.gvsu.edu/boelkinm/Home/Download.html`.

[3] Cline, K. 2014. Resources for Classroom Voting in Mathematics. `http://mathquest.carroll.edu/resources.html`.

This website contains question libraries (by course) and useful web pages with regard to uses of classroom voting clicker devices.

[4] *The College Mathematics Journal*. `http://www.maa.org/publications/periodicals/college-mathematics-journal/the-college-mathematics-journal`.

This journal is designed to improve classroom learning and stimulate thinking regarding undergraduate mathematics.

[5] *MAA Course Communities site*. `http://www.maa.org/programs/faculty-and-departments/course-communities`.

Links for Developmental Mathematics, Linear Algebra, One-Variable Calculus, Multivariable Calculus, Ordinary Differential Equations and Probability courses are provided. A "Featured Items" is *Calculus Applets using GeoGebra*. The resources on this site are very extensive with 17 pages of listings for Linear Algebra alone.

[6] *Demos with Positive Impact: A Project to Connect Mathematics Instructors with Effective Teaching Tools*. http://www.mathdemos.org/mathdemos/index.html.

There are links here to physical, calculator, databased, browser-based, and software demonstrations with a useful key word index.

[7] *EduCreation*. https://www.educreations.com/.

Software to permit creation of educational dynamic video lessons that students and colleagues can access any time, as needed.

[8] Epstein, J. 2013. Concept Inventory - Measurement of the Effect of Teaching Methodology in Mathematics.. *Notices of the American Mathematical Society*. 60(8): 1018-1026.

A description of efforts in concept-inventory development with a thorough bibliography.

[9] *FOCUS*. http://www.maa.org/publications/periodicals/maa-focus.

This news magazine of the MAA contains information about MAA activities, news about mathematics and the mathematical community, and lively articles about interesting new (or sometimes not so new) ideas in mathematics, mathematics education, and related areas

[10] *GAP - Groups, Algorithms, Programming - a System for Computational Discrete Algebra*. http://www.gap-system.org/.

GAP is a system (programming language, library of functions, and data sets) for computational discrete algebra, with particular emphasis on Computational Group Theory. GAP is now included in the Sage software.

[11] *GeoGebra*, www.geogebra.org.

This software is free and can also be used on multiple platforms (Chrome app, ipad, etc.). It is a useful free "substitute" for Geometer's Sketchpad or Cabri Geometry (but with much more in its current version).

[12] *International Journal of Mathematical Education in Science and Technology*. http://www.tandfonline.com/loi/tmes20.

This is a medium by which a wide range of experience in mathematical education can be presented, assimilated and eventually adapted to everyday needs in schools, colleges, polytechnics, universities, industry and commerce.

[13] Katz B. P. and E. Thoren. 2014. WikiTextbooks: Designing Your Course Around a Collaborative Writing Project. *PRIMUS*. 24(7): 574-593.

This article describes how students build a reference text for a mathematics course as the course progresses.

[14] *LiveScribe*. http://store.livescribe.com/.

Corporate source of live pens which enable users to write on a table screen and save in real time the session for future read and analysis.

[15] MAA Committee on the Teaching of Undergraduate Mathematics. 2015. *Pedagogy Guide*. Washington DC: MAA.

[16] *Magma*. magma.maths.usyd.edu.au.

From that site: Magma is a large, well-supported software package designed for computations in algebra, number theory, algebraic geometry and algebraic combinatorics. The software is free and is subsidized by the Simons Foundation.

[17] *Maple*. http://maplesoft.com

Maple is one of the successful computer algebra systems which is widely used in undergraduate education and has many features designed for teaching and discovery. *Maple* features Maple T.A. for online assessment, assignments, and placement testing.

[18] *Math Horizons*. http://www.maa.org/publications/periodicals/math-horizons.

A vibrant and accessible forum for practitioners, students, educators, and enthusiasts of mathematics, dedicated to exploring the folklore, characters, and current happenings in mathematical culture.

[19] *Mathematica*. http://wolfram.com.

Mathematica is one of the successful computer algebra systems which is widely used in undergraduate education and has many features designed for teaching and discovery.

[20] Mathematics Leadership Program at Mount Holyoke College. http://mathleadership.org/wp-content/uploads/2014/08/Gazette-Zoom-Article.pdf

A story about the hybrid course offered using Zoom.

[21] *Mathematics Magazine*. http://www.maa.org/publications/periodicals/mathematics-magazine.

Offers lively, readable, and appealing exposition on a wide range of mathematical topics.

[22] *MATLAB*. http://www.mathworks.com/products/matlab/.

From the web site, "MATLAB is a high-level language and interactive environment for numerical computation, visualization, and programming. Using MATLAB, you can analyze data, develop algorithms, and create models and applications." This software is used by many engineers.

[23] Merow, Katharine. 2014. Make the MOOC Work For You. *FOCUS*. 38(5): 15-16.

A description of Robert Ghrist's (University of Pennsylvania) Calculus Single Variable (Coursera) lectures freely available to the public with no registration or account in what is called "preview" mode. See coursera.org/course/calcsing and click on the "Preview Lectures" button on the top left, and you get Ghrist's 58 lecture titles to browse.

[24] *OpenIntro*. http://www.openintro.org/.

Supports educational materials that are open and innovative to manage a course or use selected materials such as free downloadable pdf texts like, *OpenIntro Statistics: Second Edition*.

[25] Praeger, Cheryl. 2001. Computers in Algebra: new answers, new questions. *J. Korean Mathematical Society*. 38: 763-780.

[26] *PREZI*. https://prezi.com.

From that site: Prezi is a cloud-based presentation software, meaning you can present from your browser, desktop, iPad, or iPhone and always have the latest version of your work at your fingertips. Create or edit on the go, then auto-sync across all your devices with ease.

[27] *PRIMUS - Problems, Resources, and Issues in Mathematics Undergraduate Studies*. http://www.tandfonline.com/loi/upri20.

A peer-reviewed academic journal covering the teaching of undergraduate mathematics in ten issues each year.

[28] *R*. www.r-project.org

R is a free software environment for statistical computing and graphics. It compiles and runs on a wide variety of UNIX platforms, Windows and MacOS.

[29] *The Right Stuff: Appropriate Mathematics for All Students*. http://www.amatyc.org/?page=therightstuff.

The project encourages the use of materials that engage students in meaningful activities that promote the effective use of technology to support mathematics, further provide students with stronger problem solving and critical thinking skills, and enhance numeracy. There are links to modules.

[30] *Sage*. http://sagemath.org/.

A free open-source mathematics software system licensed under the GPL. This is a growing package which has a community of supporters and developers to incorporate desirable features. It includes GAP.

[31] Salter, A. 2014. Games in the Classroom Reading List. *The Chronicle of Higher Education*. ProfHacker Column of September 15, 2014. http://chronicle.com/blogs/profhacker/games-in-the-classroom-reading-list/58101.

[32] *Screencast-o-matic* http://www.screencast-o-matic.com.

This website enables screen capture recording on Windows or Mac computers with no installation required and at no charge. It can also be downloaded on the Mac.

[33] *Special Interest Group on Statistics Education*. http://sigmaa.maa.org/stat-ed/.

This Special Interest Group has compiled a list of technology resources for the teaching and learning statistics. Technology for teaching (especially online and supplementary) includes Jing, Camtasia, the iPad app Explain Everything, and YouTube videos.

For visualization there are applets, including

http://www.rossmanchance.com/applets/index.html,

http://onlinestatbook.com/2/index.html, http://www.math.hope.edu/isi, and

http://bcs.whfreeman.com/sris/.

For dynamic data exploration and simulation in introductory statistics they list *Tinkerplots* at www.srri.umass.edu/tinkerplots and *Fathom* at http://concord.org/fathom-dynamic-data-software.

They note that most publishers have technology-packed sites that include eBooks and other resources. They also list statistical packages and calculators for doing statistics, such as: TI8x calculators, Minitab, SPSS, JMP, DataDesk, StatKey, StatCrunch, CrunchIt!, Stata and R.

[34] *SIMIODE – Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations.* `https://www.simiode.org/`.

SIMIODE is about teaching differential equations using modeling and technology up front and throughout the learning process in a community of teachers and students in which colleagues can communicate, collaborate, publish, teach, explore, contribute, etc.

[35] *Statistics Online Computational Resource (SOCR).* `http://www.socr.ucla.edu/`.

Provides portable online aids for probability and statistics education, technology based instruction and statistical computing. SOCR tools and resources include a repository of interactive applets, computational and graphing tools, instructional and course materials.

[36] Trench, W. F. 2001. *Elementary Differential Equations with Boundary Value Problems*. Brooks Cole.

The author has made this text available without charge online at `http://www.e-booksdirectory.com/listing.php?category=93`.

[37] WeBWorK. `webwork.maa.org`.

WeBWorK is an open-source online homework system for mathematics and science courses. It is supported by the National Science Foundation and the MAA and comes with a National Problem Library (NPL) of over 20,000 homework problems. Problems in the NPL target most lower division undergraduate math courses and some advanced courses.

[38] *Wolfram Research*. `demonstrations.wolfram.com`.

Many demonstrations are offered from topics including Algebra, Applied Mathematics, Calculus & Analysis, Discrete Mathematics, Experimental Mathematics, Geometry, Historical Mathematics, Number Theory, and Statistics.

[39] *WolframAlpha*. `http://www.wolframalpha.com/`

This is a free computational knowledge engine in which complex mathematics instructions can be entered for immediate execution.

[40] Woodbury, G. 2011. Using Game Mechanics to Fix the Grading System and Motivate Students. `http://georgewoodbury.wordpress.com/tag/teaching/page/3/`.

George Woodbury of the College of the Sequoias describes a grading system using elements of gaming theory.

[41] Zoom. `http://www.zoom.us`

Technology that combines cloud video conferencing, online meeting and cross-platform group chat.