Paul R. Halmos - Lester R. Ford Awards

Andrew Granville

"Using Dynamical Systems to Construct Infinitely Many Primes," *The American Mathematical Monthly*, 125:6, 483–496, 10.1080/00029890.2018.1447732.

One approach for showing that there are infinitely many primes is to construct a sequence of pairwise relatively prime integers, whose prime factors then provide an infinite collection of primes, provided the terms of the sequence are eventually greater in magnitude than 1. But how can we get our hands on such a sequence of pairwise relatively prime integers? This remarkable paper, assuming little more than a familiarity with elementary number theory, illustrates a dynamical approach, obtaining sequences by iterating maps given by polynomials with integer coefficients. The number-theoretic question (of showing that the resulting sequences are pairwise relatively prime, or at least have enough prime factors) leads directly to questions about dynamics, including the periodic orbits of polynomial maps and the rate of growth of terms in non-periodic orbits. The dynamic questions are in turn answered by number-theoretic methods, mostly elementary. By the end, we see how the *abc*-conjecture plays a natural role. This welcoming invitation to arithmetic dynamics will appeal to every reader's curiosity.

Response

I love the *Monthly*, both reading and learning from it, and greatly enjoy trying to explain mathematics, that has charmed me, to a wider audience. The *Monthly* is my favorite platform to share my joy in the distribution of prime numbers: be it races, patterns and now, dynamics, and how much beautiful and diverse mathematics can be involved in some simple-sounding questions.

Biographical Sketch

Andrew Granville is a professor at both the University of Montreal, and University College, London. He is the author of many research articles as well as a forthcoming graphic novel *Prime Suspects; The Anatomy of Permutations and Integers*, which can ordered from Princeton University Press. His first textbook, *Number Theory Revealed,* will also be published in the Fall in two versions, both as *An Introduction*, and as *A Masterclass.*