MathFest 2010

Prizes and Awards

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Program

Opening and Closing Remarks

David Bressoud, President
Mathematical Association of America

Carl B. Allendoerfer Awards

The Carl B. Allendoerfer Awards, established in 1976, are made to authors of expository articles published in *Mathematics Magazine*. The Awards are named for Carl B. Allendoerfer, a distinguished mathematician at the University of Washington and President of the Mathematical Association of America, 1959-60.

Ezra Brown and Keith Mellinger

"Kirkman's Schoolgirls Wearing Hats and Walking Through Fields of Numbers," ***Mathematics Magazine*****, 82:1 (2009), p. 3-15.**

The historical basis for this interesting article is a problem in recreational mathematics posed by T. P. Kirkman in 1850. Kirkman's problem states:

> "Fifteen young ladies of a school walk out three abreast for seven days in succession: it is required to arrange them daily so that no two shall walk abreast more than once."

Using this problem as a springboard, the authors treat the reader to a captivating exploration of the theory and applications of block designs. In the process, solutions to the schoolgirls problem are uncovered in such seemingly unrelated areas as the subfield structure of algebraic number fields and the configuration of "spreads" and "packings" in finite projective geometry.

Additional connections to the schoolgirls problem are revealed by the authors' extension of Todd Ebert's "Three Hats" problem to an analogous problem involving fifteen hats. Their solution to this extension is based upon an interesting relationship between Kirkman's problem and the theory of error correcting codes. In particular, a solution to the schoolgirls problem leads to a Hamming code, which then can be used to solve the fifteen hats problem.

This well-written and accessible article invites the reader to join the authors on a fascinating journey into the modern theory of block designs and the surprising connections of these designs to diverse areas of mathematics. Readers who accept the invitation will be left with both a deeper understanding of Kirkman's problem and an appreciation of the ubiquitous nature of its solution.

Biographical Notes

Ezra (Bud) Brown grew up in New Orleans, has degrees from Rice and Louisiana State University, and has been at Virginia Tech since 1969, where he is currently Alumni Distinguished Professor of Mathematics. His research interests include number theory and combinatorics. He particularly enjoys discovering connections between apparently unrelated areas of mathematics and working with students who are engaged in research. He has been a frequent contributor to the Mathematical Association of America journals, and he just finished a term as the Maryland, District of Columbia, and Virginia Section Governor. He and Art Benjamin edited *Biscuits of Number Theory*, a collection of number theory articles published in 2009 by the Mathematical Association.

In his spare time, Bud enjoys singing (from opera to rock and roll), playing jazz piano, and talking about his granddaughter Phoebe Rose. Under the direction of his wife Jo, he has become a fairly tolerable gardener, and the two of them enjoy kayaking. He occasionally bakes biscuits for his students, and he once won a karaoke contest.

Originally from Lancaster County, Pennsylvania, **Keith E. Mellinger** graduated with his Ph.D. in mathematics from the University of Delaware and was a post-doc at the University of Illinois at Chicago before moving to the University of Mary Washington in 2003. He is currently an Associate Professor and Chair of the Department of Mathematics at the University of Mary Washington. His research interests are in discrete mathematics, usually connected to finite geometry, and he regularly mentors undergraduate researchers. In 2005 he earned a research grant from the National Security Agency, and in 2008 he won the Outstanding Young Faculty Award presented by the University of Mary Washington.

Outside of mathematics, Keith is a decent cook, an average tennis player, and an accomplished musician. He has performed in different bands over the years, usually on acoustic guitar. However, most of his time outside the office these days is spent with his two beautiful children, Gabriel and Cecilia.

Response from Ezra Brown and Keith Mellinger

It was in graduate school that each of us first learned Thomas P. Kirkman's elegant solution to his Fifteen Schoolgirls problem, Bud in Dave Roselle and Brooks Reid's graduate course in Combinatorics, and Keith in Gary Ebert's similar course. In both of us, there began a fascination with combinatorial designs, and both of us have written on the topic for *Mathematics Magazine* in the past. It happens that Kirkman's solution turns up in a variety of different settings, including algebraic number fields, finite geometries, coding theory, and the so-called fifteen hats problem in recreational mathematics. We had this wild idea of presenting our discoveries to a general audience, and so we wrote this paper together.

Actually, truth be told, we wrote the paper for each other. We will be forever grateful to Frank Farris who pointed out that we need to stop writing for each other and start writing for others. Frank, they just don't pay you enough. We want to thank the paper's two referees, who suggested numerous improvements that greatly improved the paper's readability—and how! We also thank our families for their support and encouragement over the years. It is a great honor to receive the Allendoerfer Award: we are truly grateful for all that the Mathematical Association of America does for our community!

David Speyer and Bernd Sturmfels

"Tropical Mathematics," ***Mathematics Magazine*****, 92:3 (2009), p. 163-173.**

Contrary to popular opinion, mathematics is not static. New branches of mathematics are born and develop in response to impulses from within the subject and from areas of application. But it is hard to find good expository articles which capture the excitement of a new field. This article by David Speyer and Bernd Sturmfels succeeds brilliantly in making the tropical approach to mathematics attractive and accessible to a wide readership.

The adjective "tropical" was chosen by French mathematicians to honor Imre Simon, the Brazilian originator of min-plus algebra, which grew into this field. The basic object of study is the tropical semiring consisting of the real numbers $\boldsymbol{R}$ with a point at infinity under the operations of $\oplus$ equals minimum and $\odot$equals plus. The arithmetic and algebra of this simple number system yield surprising connections with well-studied branches of classical mathematics. For example, tropical polynomials in n variables are precisely the piecewise-linear concave functions on $\boldsymbol{R}^n$ with integer coefficients.

Further developments and generalizations connect with combinatorics, algebraic geometry, and computational biology. This article offers an introduction to diverse aspects of this new subject. Each section begins with very elementary material and ends with research problems. In a short amount of space the authors manage to convey the depth of this attractive new field and its broad reach.

Biographical Notes

David Speyer has just begun serving as an associate professor of mathematics at the University of Michigan. Before that, he was a research fellow funded by the Clay Mathematics Institute. He has an A.B. and a Ph.D. in Mathematics, from Harvard University and the University of California Berkeley respectively. Speyer's research focuses on problems which combine questions of algebraic geometry and combinatorics; both the many such questions which arise in tropical geometry and those which occur in the study of classical algebraic varieties and representation theory. He blogs at http://sbseminar.wordpress.com.

Bernd Sturmfels received doctoral degrees in Mathematics in 1987 from the University of Washington, Seattle, and the Technical University Darmstadt, Germany. After two postdoctoral years in Minneapolis and Linz, Austria, he taught at Cornell University before joining the University of California Berkeley in 1995, where he is Professor of Mathematics, Statistics and Computer Science. His honors include a National Young Investigator Fellowship, a Sloan Fellowship, a David and Lucile Packard Fellowship, a Clay Senior Scholarship, and an Alexander von Humboldt Senior Research Prize. Presently, he serves as Vice President of the American Mathematical Society. A leading experimentalist among mathematicians, Sturmfels has authored or edited fifteen books and 180 research articles, in the areas of combinatorics, algebraic geometry, polyhedral geometry, symbolic computation, and their applications. His current research focuses on algebraic methods in optimization, statistics, and computational biology.

Response from David Speyer

I am highly honored to receive the Carl B. Allendoerfer Award. Tropical mathematics is an exciting but frustrating field to introduce people to: the motivating computations are extremely explicit and elegant, but learning what questions to ask often requires absorbing a great deal of background and sophisticated technology. We hope that our article has helped explain what this field is about and excited our readers to consider entering it.

If it is odd to say so, it is also true that my first thanks should go to my co-author and advisor Bernd, who has driven me both to discover and to expound tropical mathematics. I also want to thank the Clay Institute for their support of my research. Finally, I must thank my fellow graduate students at the University of California Berkeley where I began writing this article and my colleagues at the Massachusetts Institute of Technology where I finished it, for listening to my many attempts to explain the tropical perspective.

Response from Bernd Sturmfels

It is a great honor for me receive the Carl B. Allendoerfer Award for the article with David Speyer on tropical mathematics. This prize means a lot to me, especially since I find myself following in the footsteps my late advisor, Victor Klee, who received this award in 1999. Vic's passion for the combinatorics of convex polyhedra has been a great inspiration for me over the years.

Tropical mathematics is a delightful subject that challenges our basic assumptions about arithmetic and geometry and thus leads us to a deeper understanding of familiar structures we are so accustomed to. It has been a great pleasure for me to embark on this tropical journey with numerous students and postdocs. I wish to thank them for being so patient with their impatient mentor.

Please allow me to also use this opportunity to share my view that the Mathematical Association of America is doing a marvelous job in its various programs. They have always reminded me of the unity of mathematics, and the fact that the benefits of integrating research with teaching at all levels cannot be overestimated.

Trevor Evans Awards

The Trevor Evans Awards, established by the Board of Governors in 1992 and first awarded in 1996, are made to authors of expository articles that are accessible to undergraduates and are published in *Math Horizons*. The Awards are named for Trevor Evans, a distinguished mathematician, teacher, and writer at Emory University.

Pamela Pierce, John Ramsay, Hannah Roberts, Nancy Tinoza, Jeffrey Willert, and Wenyuan Wu

"The Circle-Square Problem Decomposed," *Math Horizons*, November 2009, p. 19-21, 33.

This outstanding article examines Miklós Laczkovich's 1990 shocking and famous result that a circle can be decomposed into a square of the same area, which is a task not attainable with paper and scissors as proven by Dubins, Hirsch and Karush in 1964. Enabling readers to gain visual understanding of Laczkovich's theoretical result, this article studies how closely the circle-squaring process can be approximated using polygons that can be moved only with translation. By the end of the article, a reader will be able to dissect an n-gon to form a circle and understand why a method of dissection answers the challenge of decomposition. The captivating figures of this article include notably sized polygons with 12 and 20 sides. The article also offers insightful conjectures regarding the number of pieces necessary for such constructions. Readers of this article will enjoy an entertaining and enlightening hands-on introduction to a remarkable proof that traded in "scissors for the Axiom of Choice."

Biographical Notes

Pamela Pierce is a professor of mathematics at The College of Wooster, in Wooster, Ohio. She holds a B.A. from Amherst College, an M.Ed. from the University of Massachusetts, and a Ph.D. in Mathematics from Syracuse University. Her primary area of research is real analysis, focusing on functions of generalized bounded variation. This year she received an NSF grant to help fund the 34th Summer Symposium in Real Analysis, which she hosted at Wooster last month. In recent years she has been active in leading undergraduate research projects as part of the summer

AMRE program at Wooster. In 2008 she hosted the Midstates Conference for Undergraduate Research in Computer Science and Mathematics. In 2009 her team of student researchers was selected to participate in the CUR-sponsored "Posters on the Hill" session in Washington, D.C. Pam enjoys teaching the talented students at Wooster, in courses such as calculus and real analysis. Recently she served a two-year term as an associate dean at Wooster. She is currently serving as the department chair. Pam was a fellow in Project NExT (a blue dot) and is a consultant for the current class of NExTers.

John Ramsay is a professor of mathematics at The College of Wooster. He holds a B.A. from Berea College and an M.S. and Ph.D. from the University of Wisconsin, Madison. His doctoral work was in algebraic topology and he completed a Ph.D. minor in operations research. He is just completing a sabbatical leave and is looking forward to getting back into the classroom where he has enjoyed teaching Wooster students for the past 23 years. John is founder and director of Wooster's 17-year old AMRE program. AMRE is a summer consulting and research program that employs Wooster students and faculty in consulting projects for business and industry in northeast Ohio as well as in mathematics and computer science research projects. Most recently John has worked with a team on a consulting project for Goodyear Tire and Rubber and on a research team working in knot theory.

Hannah Roberts is a junior mathematics major at The College of Wooster, and is from Youngstown, Ohio. Hannah and Nancy Tinoza worked on the circle squaring problem in the summer of 2009. They made improvements to the dissection algorithm that reduced the number of pieces needed in the dissections.

Nancy Tinoza is a junior mathematics and economics double major at The College of Wooster. She is from Harare, Zimbabwe.

Jeff Willert is a 2009 graduate of The College of Wooster, and is from Chagrin Falls, Ohio. He is currently in his second year of a Ph.D. program in mathematics at North Carolina State University. Jeff worked on the circle squaring problem in the summers of 2007 and 2008. He and Wenyuan Wu developed the initial dissection algorithm.

Wenyuan Wu is a senior mathematics and economics major at The College of Wooster. She is from Chengdu, China. Wenyuan worked on the circle squaring problem in the summer of 2008.

Response from Pamela Pierce
It is truly an honor to receive the Trevor Evans award, and I am very grateful to the people that helped to make this piece what it is. John Lindner suggested that I have undergraduate researchers investigate Tarski's Circle Squaring Problem; that turned out to be a great springboard for the work that followed. Thanks to some funding from the College's grant from the Howard Hughes Medical Institute, I was able to hire excellent students to work on this project. The hard work and dedication of our student co-authors—Jeff Willert, Wenyuan Wu, Hannah Roberts, and Nancy Tinoza— were crucial to the success of this paper. I was fortunate to have John Ramsay join the research team, and he provided additional support for the research through the AMRE program. Several people have helped me develop as a writer; these include my husband Jim Foley and colleagues Dan Waterman, Dan Velleman, and Ivars Peterson. The MAA PREP workshop entitled "Expository Writing to Communicate Mathematics" helped immensely with the drafting process. And lastly, I am grateful to the editors of *Math Horizons*, Steve Abbott and Bruce Torrence, who provided encouragement and helpful suggestions along the way.

Response from John Ramsay
The Trevor Evans award is a very unexpected honor and I can only express humble appreciation to be included with the others that were responsible for the circle squaring research and article. It is a recognition of three things: a research area that has turned out to be very interesting and fruitful, some bright and creative students, and Pam Pierce's hard work and excellent writing in presenting the work in such an interesting and understandable form. I am very appreciative of The College of Wooster and the Howard Hughes Medical Institute whose funding allowed us to begin a mathematics research component within the AMRE consulting program.

Adrian Rice

"Gaussian Guesswork (or Why 1.19814023473559220744... is Such a Beautiful Number)," *Math Horizons*, November 2009, p. 12-15.

Mathematics can be viewed as being static and lifeless, like the ink that printed the theorems, formulas and graphics on a page. In this delightful exposition, Adrian Rice draws on stories from the life of Carl Friedrich Gauss to demonstrate the experimentation, observation, invention, and imagination that nurture the living and growing subject of mathematics. The article discusses entries in Gauss' mathematical diary. He began with numerical experimentation, which led to proving an amazing relationship between a sophisticated form of average, a particular value of an elliptic integral and the well-known ratio of the circumference of a circle to its diameter. Readers will walk through critical stages of Gauss' intellectual journey and see how one of the giants of mathematics, on whose shoulders we often stand today, used intuition as part of his approach to discovering new mathematics. As the article notes, "...even geniuses make guesses sometimes!" Such guesses can open a door of discovery through which we may eventually pass with a rigorous proof in hand.

Biographical Note

Adrian Rice received a B.Sc. in mathematics from University College London in 1992 and a Ph.D. in the history of mathematics from Middlesex University in 1997 for a dissertation on Augustus De Morgan. He is currently a Professor of Mathematics at Randolph-Macon College in Ashland, Virginia, where his research focuses on 19th-century and early 20th-century mathematics. His publications include *Mathematics Unbound: The Evolution of an International Mathematical Research Community, 1800–1945* (edited with Karen Hunger Parshall) and *The London Mathematical Society Book of Presidents, 1865–1965* (written with Susan Oakes and Alan Pears.) He has recently finished co-editing a book on mathematics in Victorian Britain with Robin Wilson and Raymond Flood.

Response from Adrian Rice

I am thrilled and very honored to have been selected as a recipient of the Trevor Evans Award for my article on "Gaussian Guesswork." That article will always be special to me, not only because it was a great pleasure to write, but also because I completed it just two days before the birth of my son! It is therefore particularly gratifying to know, from the comments and e-mails I have received over the past few months, that many people seem to be finding it an interesting and enjoyable read. My thanks go to Bruce Torrence and Steve Abbott, for their editorial help and attention to detail when preparing the piece for publication, and of course to the Mathematical Association of America for this great honor.

Lester R. Ford Awards

The Lester R. Ford Awards, established in 1964, are made to authors of expository articles published in *The American Mathematical Monthly*. The Awards are named for Lester R. Ford, Sr., a distinguished mathematician, editor of *The American Mathematical Monthly*, 1942-46, and President of the Mathematical Association of America, 1947-48.

Tom M. Apostol and Mamikon A. Mnatsakanian

"New Insight into Cycloidal Areas," *American Mathematical Monthly*, 116:7 (2009), p. 598-611.

We all know that the area under a cycloidal arch is three times the area of the generating circle. You probably don't know, unless you have read this wonderful article, the marvelous fact that this area relationship holds throughout the generation of the cycloid. That is, the area of the cycloidal sector at each instant of its generation is three times the area of the circular segment determined by the portion of the perimeter through which the circle has rolled.

This surprising theorem opens this beautiful paper and it is proven by a simple application of Mamikon's sweeping-tangent theorem: the area swept out by a collection of tangent vectors to a curve is preserved if the tangent vectors are parallelly translated to share a basepoint (see Apostol-Mnatsakanian, *Monthly*, 109 (2002) 900-908.) Various generalizations to epi- and hypo-cycloids are derived as are other stunning cycloidal quadratures. Throughout the method of reasoning is a continuation of the remarkable dynamic-geometric logic that these authors have been presenting over the last decade in a series of articles in Mathematical Association journals. The mathematical results surprise and delight, the methods of proof thrill and charm, and the exposition leads one clearly and simply through the beautiful geometry.

Biographical Notes

Tom M. Apostol joined the Caltech faculty in 1950 and is now Professor Emeritus. He is internationally known for his textbooks on calculus, analysis, and analytic number theory, (translated into six languages), and for creating *Project MATHEMATICS!*, a video series that brings mathematics to life with computer animation, live action, music, and special effects. The videos won first-place honors at a dozen international festivals, and were translated into Hebrew, Portuguese, French, and Spanish. He has published 102 research papers, has written two chapters for the Digital Library of Mathematical Functions (2010), and is coauthor of three texts for the physics telecourse: *The Mechanical Universe…and Beyond*.

He has received several awards for research and teaching. In 1978 he was a visiting professor at the University of Patras, Greece, and in 2001 was elected a Corresponding Member of the Academy of Athens, where he delivered his inaugural lecture in Greek.

Mamikon Mnatsakanian is a former Soviet doctor of sciences in theoretical and mathematical physics and astrophysics, and professor at Yerevan State University in Armenia. He is the author of a hundred published scientific and popular articles. Mnatsakanian developed 'Generalized General Theory of Relativity with Variable Gravitational Constant' and 'New Apparatus of Radiation Transfer Theory', and has created hundreds of educational games and puzzles (www.mamikon.com).

As a student he invented 'CaKuLuS' - a simple, visual and dynamic approach for solving advanced calculus problems without formulas or equations. He has taught it successfully to students of various levels in the United States. After the devastating earthquake in Armenia in 1988, he began seismic safety investigations in cooperation with Californian specialists. As the Soviet Union collapsed, he stayed in the USA where he found a true appreciation of his new friends and colleagues. In 1996 he met his dream colleague, Professor Tom Apostol, and works with him at Caltech's *Project MATHEMATICS!*

Response from Tom M. Apostol

Lester R. Ford impacted my career in many ways. His 1929 book on automorphic functions and his earlier 1915 monograph strongly influenced my choice of analytic number theory as a field of research. His superb 1929 text on differential equations helped me greatly in my early teaching at Caltech. And his 1938 *Monthly* paper on circles associated with Farey fractions (which I later called *Ford Circles*) made it possible to simplify the derivation of Rademacher's historic convergent series for the partition function. I consider it a special honor to share another Ford Prize with my colleague Mamikon Mnatsakanian, whose remarkable geometric insights have led to joint publication of 28 papers during the past twelve years.

Response from Mamikon Mnatsakanian

I believe every mathematician's dream is to meet Archimedes or other giant thinkers who struggled intellectually and suffered emotionally while paving the mathematical routes for modern sciences. A huge army of geniuses - Galileo, Newton, Leibniz, Wren, Huygens, Descartes, Fermat, Roberval, Torricelli, Bernoulli, and Mersenne- worked so successfully on understanding the nature of the cycloid, that even climbing on their shoulders doesn't enable one to see further. Rather it helps to look back in time and to slightly polish some of their grandiose insights into this Helen of geometry.

No appreciation from these ancestors themselves is expected, so this Lester Ford Award of the Mathematical Association of America gets us possibly the closest we can come to the extraordinary company of these great men; and it feels great!

Judith Grabiner

"Why Did Lagrange 'Prove' the Parallel Postulate?," *American Mathematical Monthly*, 116:1 (2009), p. 3-18.

"I've got to think about this some more." According to Augustus de Morgan, this is what Lagrange said while reading his paper "proving" the parallel postulate at the Institut de France in 1806. And "I've got to think about this some more" is what readers are likely to say when reading this excellent article, but for different reasons than Lagrange's.

Grabiner's intriguing exploration of understandings and beliefs concerning Euclidean geometry and the parallel postulate focuses on the questions of what Lagrange's proof was and why he took this approach, and why the problem was so important to him. In her discussion of these questions, the author explains that Lagrange was following Leibniz' "principle of sufficient reason." One part of it says that something is true unless there is ample reason for it not being so. This insight leads to an exploration of the understanding of the nature of geometry and space in the seventeenth and eighteenth centuries, with mathematics, physics, philosophy, and art all being important to the story. It begins with Descartes, Leibniz, and Newton, who, although disagreeing on some aspects of space, agreed that Euclidean geometry was essential to physics. Then Euler comes onto the scene, followed by the philosophers Kant and Voltaire, and others. Thus, Euclidean geometry was very much part of the thinking of the intellectual establishment of the day, and it is proposed that this may account for the fact that non-Euclidean geometry originated outside the mainstream, and was not universally accepted at the time. It all makes absorbing reading, and readers will indeed be stimulated into "thinking about this some more."

Biographical Note

Judith V. Grabiner is the Flora Sanborn Pitzer Professor of Mathematics at Pitzer College, one of the Claremont Colleges in California. She is the author of *The Origins of Cauchy's Rigorous Calculus* (MIT Press,) *The Calculus as Algebra: J. -L. Lagrange, 1736-1813* (Garland Press,) and a forthcoming book from the Mathematical Association of America, *A Historian Looks Back: The Calculus as Algebra and Selected Writings*. She also is the author of a Teaching Company DVD course called "Mathematics, Philosophy, and the 'Real World.'" Besides writing many articles about the history of mathematics and history of science, and receiving several Lester R. Ford and Carl B. Allendoerfer awards from the Mathematical Association of America, she won the Deborah and Franklin Tepper Haimo Award for Distinguished College or University Teaching in 2003.

Response from Judith Grabiner

I would like, first and foremost, to thank the Mathematical Association of America – for so many things besides this prize: for its support of excellence in teaching and writing mathematics, for its outstanding publications, and for its sponsorship of so many activities for students, for junior faculty, and for the mathematical community at large. I also thank my students, who have taught me most of what I know about being clear and presenting material in ways that interest others than just me. I thank the Institut de France for letting me work with Lagrange's manuscript "proof" of the parallel postulate, the Department of History and Philosophy of Science at the University of Leeds, England, for vigorous discussions as I worked on "Why Did Lagrange 'Prove' the Parallel Postulate?", the Pitzer family for the generous research support that comes with my Flora Sanborn Pitzer professorship at Pitzer College, and three wonderful geometry teachers: Professor Dana Scott from his time at the University of Chicago, and Miss Kranz and Mr. Roche from my days at Fairfax High School in Los Angeles.

Jerzy Kocik and Andrzej Solecki

"Disentangling a Triangle," *American Mathematical Monthly*, 116:3 (2009), p. 228-237.

One would think that in the 2200 years since the Greek astronomer Hipparchus published the first table of chords, or in the 1800 years since this work was extended, improved, and highly popularized by Claudius Ptolemy in his *Almagest*, or at least in the 550 years since Regiomontanus modernized trigonometry for study by Western mathematicians, that we would have arrived at a unified approach to the subject, one that would bring together all the standard results, relating the familiar trigonometric functions with their geometric representations as elements of triangles: no doubt there are many expositions of the subject that can claim this. Well, here we find a candidate for the world's most elegant presentation of trigonometry's central theorems: in a few brief pages, decorated with repeated versions of the same simple figure consisting of an acute triangle set within its inscribed circle - each of the figures only slightly modified to highlight some particular relation among its pieces, we are presented with a wealth of relationships illustrating how the ideas behind these theorems can be compellingly visualized and simply derived.

Ptolemy's theorem, the law of sines, the nine-point circle, the standard sum and difference formulas for the trigonometric functions, these and a host of other well-known (and less well-known) results spring forth geometrically in these diagrams and in a few simple equations. The authors tease each new result out of the diagram by turning our heads first this way, then that, across the pictures. It is remarkable how this perspective on trigonometry has gone unnoticed for so long.

Biographical Notes

Jerzy Kocik studied theoretical physics in Wroclaw, Poland, but he strayed into mathematics when he realized that it explores many realities, while physics is confined to one… He is currently an associate professor at Southern Illinois University at Carbondale. His main research interests lie in Lie algebras and differential geometry. Recently, he has given in to the charms of Apollonian circle packing. Nothing thrills him more than encountering bridges between the simple and the deep, between the obvious and the concealed.

When **Andrzej Solecki** decided to study mathematics at Wrocław University, he heard that the place was okay but why mathematics? The answer was that maths would not depend on geopolitics, it would not demand much time, it could be fun and one could play with it all alone. It took him a long time to discover that the answer was only 50% right.

Response from Jerzy Kocik and Andrzej Solecki

We are truly honored by the Lester R. Ford Award and are very happy to receive it! Who would have imagined that a slight variation of a two thousand year-old figure would lead to rewriting the basic trigonometry of triangles. We appreciate the recognition even more because it comes from people who have invested a lot of effort into bringing mathematical culture to a wider audience. We thank the Mathematical Association of America for their generosity.

Obviously we share this award with all involved: with the editors who worked hard at improving the quality of the original manuscript, with readers, and with our supporting friends and families. And with triangles themselves – thank *you* for being so elusively attractive! After so many years with us, you are still hiding simple truths about yourselves.

Bob Palais, Richard Palais, and Stephen Rodi

"A Disorienting Look at Euler's Theorem on the Axis of a Rotation," ***American Mathematical Monthly*, 116:10 (2009), p. 892-909.**

"In whatever way a sphere be rotated around its own center, a diameter can always be chosen whose direction in the rotated configuration would coincide with the original configuration." This remarkable 300-year-old theorem by Leonhard Euler is so famous and so important in both pure and applied mathematics that the general concept of rotation is often confused with rotation about an axis. As the authors point out, it is true, but not immediately obvious, that the composition of rotations about two different axes of the two dimensional sphere is again a rotation about a single axis. The authors lead us on a journey through the mathematics of spherical rotations during which (1) they present a new and constructive proof of Euler's theorem using only notions available from a muscular undergraduate first course in linear algebra, (2) they provide a translation (possibly the first) and explication of Euler's original proof. (The first line of this citation is from their translation of Euler's original paper), (3) they give another proof "using only the kind of classical spherical geometry arguments that Euler himself used, translated into a modern idiom," but which includes the orientation reversing case (cf. their title), (4) finally, they review some short modern (non-constructive) proofs using, respectively, linear algebra, topology, differential geometry and Lie theory. We come away from this journey admiring the ability of the authors to move smoothly between modern linear algebra and classical spherical geometry and for the power of Euler's mathematics.

Biographical Notes

Bob Palais received his B.A. from Harvard College in 1980 and his Ph.D. from the University of California Berkeley in 1986. He is currently a Research Professor at the University of Utah in Salt Lake City. He splits his time between the Math department (teaching and doing research on scientific computation and mathematical visualization) and the Pathology Department, where he works on DNA melting analysis and bioinformatics. His article "Pi is Wrong!" (http://www.math.utah.edu/~palais/pi.html) still generates the most spirited discussions. In his spare time, he enjoys mountaineering, often with other mathematicians. He and his father Richard recently co-authored *Differential Equations, Mechanics, and Computation* (http://odemath.org) His mother Eleanor is a founding member of AWM, and had an exceptional streak with 100% of her BC calculus students receiving 5 on the AP exam for 9 years! His grandmother, Madeleine Galland, also taught mathematics, with Ronnie (`Christopher') Walken among her favorite students.

Richard S. Palais received his B.A. from Harvard College in 1952 and his Ph.D., also from Harvard, in 1956. After a long career in teaching and theoretical research at Brandeis, he partially retired in 1997 to work on developing a mathematical visualization program (see http://3D-XplorMath.org). He moved to the University of California Irvine in 2004, where he teaches part time while continuing to program and write about novel algorithms for visualizing complex mathematics. He is proud of his many mathematical descendants (72 according to the Mathematical Genealogy Project) and also for entries (together with Luc Benard) in the 2006 and 2009 National Science Foundation/*Science* Magazine Scientific Visualization Challenge that both won First Prizes. The 2006 entry was the September 22, 2006 cover of *Science*:
(http://www.sciencemag.org/content/vol313/issue5794/cover.dtl)

Stephen Rodi is a professor of mathematics at Austin Community College (Texas). He joined the infant college in 1976; it now boasts a booming 40,000 students. He has held various administrative posts, but returned exclusively to teaching in 1998. Rodi received a classical secondary education in New Orleans. He then spent seven years in Jesuit seminary training, including a philosophy curriculum taught in Latin. This explains his B.A in 1965 from Spring Hill College with a triple major in mathematics, philosophy, and Latin. He holds a Master's degree from Marquette University (1967) and a Ph.D. from The University of Texas at Austin (1974), both in mathematics.

Rodi has participated in many professional activities: president of the American Mathematical Association of Two-Year Colleges; chair of the Mathematical Association of America Committee on Two-Year Colleges; member of the Conference Board of the Mathematical Sciences (CBMS) and the Mathematical Sciences Education Board; chair of the advisory board for the calculus project at Harvard; co-editor in 2000 and 2005 of the CBMS statistical survey of undergraduate mathematics programs in the United States.

Response from Bob Palais

It is a special treat to share the Lester R. Ford Award with my father and Stephen, and on behalf many unnamed others who contributed directly and indirectly. Every stage of this mathematical adventure has been a team effort. The question arose in discussions related to mathematical visualization; could you compute the axis of a rotation without any multiplications. I knew how when the matrix was not symmetric, and my father knew how when it was. As usual when we collaborate, we each thought the other did the more challenging case. To make it a proof we could not use the Euler-Rodrigues formula whose starting point is the existence of an axis. My dad's suggestion that we survey the variety of other proofs led to our enlightening and entertaining correspondence with Stephen Rodi on topics just as diverse as proofs of this theorem. The idea of a joint paper in the *Monthly* became more realistic and especially appropriate, since in 1964 my father had published there on a related topic an oft cited characterization of quaternions. Even more serendipitous, the input of the reviewers and especially the editor and editorial assistant were essential to reaching a good conclusion. Since the fixed axis theorem is false for orthogonal transformations that are orientation reversing, though neither Euler nor Whittaker's early proofs exclude this condition explicitly. we saw an opportunity to characterize proper vs. improper maps in terms of this difference. The publication led us to come in contact with Johan Sten, who has translated an algebraic proof by Lexell that appeared alongside Euler's, but again, where is the orientation hypothesis? The story is not over it seems.

Response from Richard S. Palais

It may sound like dull work to reprove a 300 year old theorem, even one by the great Euler, but I have never had as much fun collaborating on a paper as I did on this one. When Bob first sent me his remarkably elementary, short, and constructive proof of the existence of an axis for a rotation I felt it must have a mistake—it just couldn't be that easy I said to myself. When I saw that it really was valid, I became excited and was very pleased when I was able to add something to the proof and Bob suggested that we write it up together. When we started searching the literature, we soon discovered that there were many purported proofs that subtly begged the question. We also discovered that Euler's original proof had never been translated into English, so we were very pleased when Steve Rodi agreed to join our team and provided us with a verbatim translation. On studying Steve's translation, we found that even Euler's original proof, while certainly correct in its basic outline, was less than airtight by modern standards. Since the theorem is so central and so often cited, we decided to write an article on Euler's beautiful result that did a serious job of "covering all the bases". We are of course very grateful that the Lester R. Ford Award Committee found the result of our efforts worthy, and we would also like to thank the Editor of the *Monthly*, Dan Velleman, for all of his incisive comments, criticisms, and suggestions. Dan put in so much effort to help us improve our paper that he could reasonably be listed as a fourth recipient of the award!

Response from Stephen Rodi

Serendipitously, I have been on a lovely journey over a few sharp peaks and deep valleys of Euler's mathematics, a ride to rival a rail journey through Euler's native Switzerland, ending now at the high terminus of the Lester R. Ford Award. I mostly have been a passenger, but I have the satisfaction of knowing the train may never have left the station without my push. I could not be happier, even giddy, and am very grateful.

Thanks to Dennis Allison, an old friend and lecturer at the University of Utah, who passed my name to Bob Palais when Palais asked if anyone knew "a mathematician who could read Latin." Thanks to my co-authors for their trust. Kudos to those secondary school Latin teachers (Condry, Schilling, Cazanavette, Bienvenue, Coco: Jesuits all) who 55 years ago laid the foundation. Lastly, a hat tip to Thomas Aquinas, whose philosophical texts were the perfect preparation for someday translating Euler.

Mike Paterson and Uri Zwick

“Overhang,” *American Mathematical Monthly*, 116:1 (2009), p. 19-44.

The problem of how far off the edge of a table one can reach by stacking n identical, friction free, blocks of unit length first appeared in the *Monthly* in 1923 and has been considered elsewhere many times. Conventional wisdom says that the optimal solution is asymptotic to $\log(n)/2$ as n increases, and is obtained by the harmonic stacking. This result is correct under the implicit added hypothesis that each block can rest on only one block below it. The authors investigate this problem without the one-on-one restriction. By clarifying the mechanics of the physical model and using numerical investigations, they lead readers to deepen their intuition of the problem on the way to their main result: for large n and without the one-on-one hypothesis, the overhang of a balanced n-block stack is asymptotic to at least $cn^{1/3}$ for some positive constant c.

The exposition is direct and illustrated with helpful diagrams and clear definitions. The critical notion is that of a "balanced stack"—one in which each block is in equilibrium under vertical forces and moments. The paper teases out clearly what assumptions—implicit as well as explicit—have been used by others investigating this problem. The paper concludes with a number of attractive problems. (Solutions to some of these by Paterson, Zwick, and others have since appeared in the *Monthly*.)

Biographical Notes

Mike Paterson is emeritus professor of Computer Science at Warwick University in the UK and is a Fellow of the Royal Society. His BSc and PhD in mathematics are from Cambridge University. Paterson started his teaching career at Massachusetts Institute of Technology and continued at Warwick where he has remained for nearly forty years. He was president of the Trinity Mathematical Society as a student, and of the European Association for Theoretical Computer Science somewhat later. He co-invented Sprouts with John Conway and received the Dijkstra Award in Distributed Computing. His interests have been mainly in algorithmic complexity, beginning from his early years at MIT, but have extended broadly within that area to include distributed algorithms, algorithmic game theory and, of course, the work recognised by this award.

Uri Zwick is a professor of Computer Science at Tel Aviv University, Israel. He received his BSc in Computer Science from the Technion, Israel Institute of Technology, and his MSc and PhD in Computer Science from Tel Aviv University. His main research interests are: algorithms and complexity, combinatorial optimization, mathematical games, and recreational mathematics. Zwick spent two years as a PostDoc at Warwick University after completing his PhD and has been collaborating with Mike Paterson ever since.

Response from Mike Paterson and Uri Zwick

We are delighted and honored by selection for the Lester R. Ford Award, joining the list of distinguished past winners. Unfortunately, neither of us is able to be present on Friday for the Prize Session. We had hard work and great fun collaborating on our *Monthly* paper over several years and are pleased that others have enjoyed it too.

George Pólya Awards

The George Pólya Awards, established in 1976, are made to authors of expository articles published in the *College Mathematics Journal.* The Awards are named for George Pólya, a distinguished mathematician, well-known author, and professor at Stanford University.

Andrew Barker

"Evolutionary Stability in the Traveler's Dilemma,"
***College Mathematics Journal*, 40:1 (2009), p. 33–38.**

Two travelers return home with identical souvenirs destroyed by baggage handlers. Each must secretly write a value, between two and one-hundred dollars, for the lost souvenir. The traveler who claims the lower value x receives $x + 2$, while the traveler who claims the higher value receives only $x - 2$. If they tie, then each receives x. How much should each traveler claim? This is the "Traveler's Dilemma," and Andrew Barker's paper begins with a brief survey of three game-theoretic tools—dominated and undominated strategies, Nash equilibrium, and the more familiar Prisoner's Dilemma—using each of them to explain why the counterintuitive strategy of claiming only two dollars is actually the rational option.

As the reader might expect, human subjects normally do not take this rational option, but their "irrationality" is usually rewarded with higher payoffs. The rest of this paper uses evolutionary game theory to help reconcile the dichotomy between how people should play this game and how they actually do play it. Barker describes using a (stochastic) strategy vector to accommodate more general mixed strategies, where probability distributions govern the travelers' claims, and then interprets such a vector as a population distribution, with payoff as a sort of evolutionary fitness. In particular, the author recalls the definition of evolutionarily stable strategy (ESS), noting that the pure strategy of claiming two dollars all the time is the unique ESS, and then uses this framework to investigate a more subtle issue: finding the "degree of stability" of that pure strategy. It is in terms of this stability measure that Barker is able to meaningfully contrast the Traveler's

Dilemma with the Prisoner's Dilemma. The article concludes with an analysis of why strategies other than the ESS might still be considered desirable from an evolutionary perspective.

The Traveler's Dilemma, already an interesting problem, is given a precise, nuanced, novel, and eminently readable treatment by Barker. The article teaches game theory in an accessible way, and provides a compelling example of the ability of mathematics to model real decision-making.

Biographical Note

Andrew T. Barker is a postdoctoral researcher at Louisiana State University, where he works under Susanne Brenner. He received his Ph.D. in Applied Mathematics from the University of Colorado at Boulder in 2009, doing his dissertation under Xiao-Chuan Cai. He has research interests in the areas of numerical analysis, finite element methods, domain decomposition methods, pre-conditioning, high-performance computing, and game theory. Outside of work he enjoys playing the piano, hiking, cycling, and reading. He lives in Baton Rouge with his wife Katie, where they drink lots of coffee and eat too much unhealthy Cajun food.

Response from Andrew Barker

It is a great honor to receive this award. The work that led to this paper was a lot of fun, and my first thanks has to go to Pat Yannul, who encouraged me to think about interesting ideas and in particular the ideas behind this project. An important part of sanity (and, I think, success) in mathematics is to pursue questions because they are enjoyable and intriguing, even if they never lead to anything prestigious or publishable. I began to learn the lesson that math should be fun from Paul Isihara, and I learned the related lesson that it can be rewarding and fulfilling from Bob Brabenec, both of whom have my thanks for starting my career. Thanks also to James Adler, Greg Norgard, and David Simpson for their support, and to Xiao-Chuan Cai for teaching and modeling the importance of writing and communicating ideas simply and clearly. Finally, thanks to my wife Katie for making all the non-mathematical areas of my life more fun.

Curtis Feist and Ramin Naimi

“Topology Explains Why Automobile Sunshades Fold Oddly,” *College Mathematics Journal*, 40:2 (2009), p. 93-98.

Mathematics can be right in front of your nose . . . or right above your dashboard. In “Topology Explains Why Automobile Sunshades Fold Oddly,” Curtis Feist and Ramin Naimi examine “automatic folding” sunshades that coil up when not in use. From the authors’ experience, it seems impossible simply to fold such a sunshade in half (i.e. coil it into exactly two loops.) The object here is to figure out how many loops can appear in the coil and to understand why.

The result is an engaging expedition into braid theory, a branch of knot theory. Specifically, the paper uses the concepts of braid position and linking number to investigate which numbers can be, and which cannot be, the number of coils in a closed sunshade. Braid position and linking number give students a nice demonstration that “abstract” mathematics can actually be quite visual and intuitive. In the end, the authors pull these topics together to show that this kind of sunshade cannot be folded into any even number of loops; hence the word “oddly” in the title is no accident. It is not often that students can actually purchase an inexpensive object and use it both to follow a mathematical argument and to test the accuracy of the stated result.

This article is a clear and well-illustrated introduction to an interesting and accessible branch of topology, and it is also a reminder that we should be on the lookout for mathematics everywhere we go. Finally, to those of us who have wrestled with those sunshades in an innocent but futile attempt to fold them in half, the article provides reassurance. It’s not you, Feist and Naimi tell us: it’s the topology.

Biographical Notes

Curtis Feist received his B.S. and M.S. from California Polytechnic State University, San Luis Obispo, and his Ph.D. (in Topology) from the University of California, Davis, under the supervision of Abigail Thompson. He is currently an Associate Professor of Mathematics at Southern Oregon University.

Ramin Naimi obtained his Ph.D. in Topology from Caltech in 1992. He has been enjoying teaching at Occidental College since 1998. He also enjoys collaborating in research, currently working on projects involving symmetry groups of graphs embedded in 3-space, and knots and links in spatial graphs.

Response from Curtis Feist and Ramin Naimi

We are very grateful to the selection committee for this honor. The award announcement came as a complete surprise to us, but a very pleasant one, of course. We consider this award as our second reward: the first was the process itself of working together and figuring out why automobile shades fold as they do. It all started, not with the intention of writing a mathematics article, but with the goal of helping one of us with his immeasurable frustration in trying to fold (with an extremely low rate of success) a similarly designed but larger beach shade. So it came as a very pleasant surprise when we realized mathematics was at work behind the scenes; and the fact that we ended up using knot theory, a field we both work in, was like the frosting on the cake. We again thank the committee for this honor, and we hope this article will inspire students, educators, and researchers to continue to look for and study the mathematics that occurs all around us.

Annie and John Selden Prize for Research in Undergraduate Mathematics Education

In January 2005, the Annie and John Selden Prize for Research in Undergraduate Mathematics Education was established by the MAA Board of Governors to honor a researcher who has established a significant record of published research in undergraduate mathematics education and who has been in the field at most ten years.

Keith Weber

Dr. Weber is Associate Professor of Mathematics Education at Rutgers University. He received his Ph.D. in 2001 from Carnegie Mellon University. His first published paper in collegiate mathematics education was in 2001: "Student difficulties in constructing proofs: The need for strategic knowledge," published in *Educational Studies in Mathematics*, a highly regarded international journal.

Dr. Weber has 25 publications in refereed journals and 5 articles "in press," most of which are in the leading journals in mathematics education. A recent article (2008) in the *Journal for Research in Mathematics Education*, ranked by many as the most prestigious journal in the field, is particularly impressive. This article, "How mathematicians determine if an argument is a valid proof," offers a novel perspective on the work of mathematicians and important implications for teachers of undergraduate mathematics.

Dr. Weber is the recipient of a National Science Foundation CAREER award and has another National Science Foundation grant shared with colleagues at Rutgers. He received the 2006 Early Career Publication Award, awarded by the Special Interest Group in Mathematics Education of the American Educational Research Association. In 2009, he received the Rutgers Board of Trustees Research Fellowship for Scholarly Excellence. Dr. Weber has made impressive contributions to the Research in Undergraduate Mathematics Education Special Interest Group of the Mathematical Association of America; he has served as an Executive Board Member and Program Chair for this SIGMAA.

His selection for this award was based on his strong research program and his work in implementing this program. His work is theoretically based, product oriented, and pedagogically sound. He has a deep understanding of mathematical content that is evident in all his writing. He demonstrates how research in mathematics education can be cumulative: based on previous works and aimed at bridging existing gaps.

Biographical Note

Keith Weber is an associate professor of mathematics education in the Graduate School of Education at Rutgers University. He earned a B.S. and M.S. in mathematics and an M.A. and Ph.D. in instructional science from Carnegie Mellon University. Dr. Weber has won several prestigious awards, including the 2006 Early Career Publication Award from the Research in Mathematics Education Special Interest Group of the American Education Research Association, the 2007 Early Career Award from the National Science Foundation, and the Best Paper Award at the 12th Conference on Research in Undergraduate Mathematics Education in 2009. Dr. Weber's current research is on students' reading and comprehension of proofs.

Response from Keith Weber

I am honored to receive this prestigious award. I am grateful to my dissertation committee for preparing me to be a successful researcher in mathematics education. My committee—John R. Hayes (a cognitive psychologist), Fred Reif (a physics educator), and James Cummings (a mathematician)—were not researchers in mathematics education. Each generously agreed to mentor me even though they would gain no professional benefit for doing so. I would especially like to thank Annie and John Selden, who have made it their professional mission to improve the quality of research in collegiate mathematics education. Finally, I am indebted to all of those who have mentored me (including Lynne Reder, Annie and John Selden, and Carolyn Maher) and all of my collaborators who have made my professional experience so enjoyable.

Henry L. Alder Awards for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member

The award was established in January 2003 to honor beginning college or university faculty whose teaching has been extraordinarily successful and whose effectiveness in teaching undergraduate mathematics is shown to have influence beyond their own classrooms. An awardee must have taught full time in a mathematical science in the United States or Canada for at least two, but not more than seven, years since receiving the Ph.D. Henry Alder was MAA President in 1977 and 1978 and served as MAA Secretary from 1960 to 1974.

Nathan Carter

Nathan Carter of Bentley University is cited as an extraordinary teacher who elicits high levels of student participation and shares his pedagogical innovations with his colleagues and the wider mathematical community. He is consistently ranked by students as one of the best teachers in the department.

His Group Explorer software is used worldwide. It has been downloaded over 9,000 times and regularly receives "laudatory comments from grateful users." He recently published the book *Visual Group Theory*. He is a member of the editorial board of *Math Horizons* and reviews for the National Science Foundation. He received a National Science Foundation grant to develop a computer program to help students develop their proof-writing abilities. He has directed three students in senior theses and has received a university-wide teaching award.

Biographical Note

Nathan Carter uses computers to advance mathematics. He earned a bachelor's degree in both mathematics and computer science at the University of Scranton in 1999, masters degrees in mathematics and computer science at Indiana University, and a Ph.D. in mathematical logic at Indiana University in 2004. Besides research in mathematical logic he has also worked on group theory visualization, resulting in the free software Group Explorer and the expository book *Visual Group Theory*, which was published in 2009 by the MAA. He enjoys being the computational mathematician on research teams and performing simulations in areas such as social network analysis and population dynamics. He is currently working with Kenneth Monks on the Lurch Project, a general validation environment for mathematical reasoning. It aims to give students of mathematics frequent and immediate feedback on each step of their work, from computations to proofs. All of Nathan's educational software projects are available free and open source at: http://www.platosheaven.com.

Response from Nathan Carter

I am very grateful to the MAA for selecting me to receive this honor, but more grateful still to the educators from whom I benefited as I sought to become one. I have been so blessed with good teachers from elementary school through college and graduate school that I have not enough space to list all the good ones, but I know that much of what I know and can do comes from their love of mentoring students. I hope to show my gratitude by doing the same work. Thank you also to my excellent colleagues and administrators at Bentley University who have given me generous guidance and support as my career begins, from advice on how to run a classroom discussion to encouragement to try big projects and the resources to support them. Particular thanks to Charlie Hadlock, who does not restrict his mentoring to students, but includes young faculty as well.

Kathleen Fowler

Kathleen Fowler of Clarkson University is cited as an exceptional teacher both inside and outside the classroom. Her students view her as "awesome" and appreciate her encouragement. Her peers affectionately refer to her as "Queen of Calculus" and recognize her curriculum innovation and outreach activities.

She has directed ten students in research projects and this work has resulted in five publications. She works with or directs an impressive number of programs, including Mathcounts, Pi Day, the McNair Scholars program, and numerous grants, including a summer mathematical camp for middle and high school students. She develops and runs workshops for teachers and she mentors junior faculty. She created a "flourishing" student chapter of the Association for Women in Mathematics with over fifty members, and she advises the University's Mathematical Contest in Modeling teams. She has received two University-wide teaching awards.

Biographical Note

Katie Fowler joined Clarkson University in 2003 after obtaining her Ph.D. from North Carolina State University in computational applied mathematics. Her efforts as a mathematician are split between the scientific and campus/local communities. Her research area is in optimization with an emphasis on developing hybrid derivative-free algorithms for simulation-based engineering problems. Her interdisciplinary projects span hydrology, polymer processing, psychology, and physiology. She is an active member in the scientific community—giving over 25 talks and organizing 7 mini-symposiums in the last 6 years at national and international conferences. In 2008, she hosted the American Institute of Mathematics Workshop on Derivative-Free Hybrid Optimization Methods for Simulation-Based Problems in Hydrology. She is committed to educational outreach, bringing in approximately $3 million in funding to her community. She co-developed nine science, technology, engineering, and mathematics (STEM) summer institutes for teachers, in addition to the IMPETUS for Career Success program, which includes a mathematics-physics roller coaster summer camp for students and teachers.

Response from Kathleen Fowler

It is an honor to receive this award for doing something that I genuinely love. Supportive colleagues, motivated students, and an understanding family make it much easier. Thank you!

Index of Award Recipients