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PRELIMINARY RESULTS OF THE STUDY ON CHARACTERISTICS OF SUCCESSFUL PROGRAMS IN COLLEGE CALCULUS

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This is a preliminary report of results from a large-scale survey of Calculus I students in the United States. The analysis highlights students' mathematical background as well as aspects of instruction that contribute to successful programs. Success of a university's program of Calculus I instruction is defined both in terms of the percentage of students who complete the course with a grade of C or higher and the percentage of students who maintain or increase their interest in continuing the study of mathematics beyond Calculus I.

Keywords: Calculus, Survey, Hierarchical Linear Modeling, Case Studies

INTRODUCTION

The Mathematical Association of America (MAA) is currently engaged in a five-year study of Calculus I instruction at universities in the United States.[1] The first phase of this study was a large-scale survey of Calculus I instruction that was conducted across a stratified random sample of two- and four-year undergraduate colleges and universities during the fall term of 2010. The survey was restricted to what is known in the United States as "mainstream" calculus, the calculus course that is designed to prepare students for the study of engineering or the mathematical or physical sciences. Within mathematics, it serves as the initial introduction to calculus, preceding a first course of analysis.

The second phase of the study, which will be conducted during the fall term of 2012, will consist of case studies examining Calculus I instruction at sixteen colleges and universities identified as having a notable measure of success with their Calculus I program. Success was defined in terms of both the percentage of students who had successfully completed the course and the percentage of students who maintained or increased their interest in

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continuing the study of mathematics beyond Calculus I, controlling for the varying academic strengths and interests of the entering students at different universities.

BACKGROUND AND GOALS

In the United States, Calculus I instruction is a 12- to 15-week course covering basic differential and some integral calculus. It is viewed as a university-level course. Each fall, approximately 325,000 college or university students (Lutzer *et al.*, 2007), most of them in their first post-secondary year, take this course. This number has been essentially constant over the past quarter century, as has the number of students graduating with degrees in engineering, the physical sciences, or the mathematical sciences.

In contrast to calculus enrollments at university, the number of high school students who take a course that ostensibly covers the same material as Calculus I grew from around 200,000 in 1986–87 to approximately 620,000 during 2011–12.[2] Roughly two-thirds of the secondary school students studying calculus do so under the auspices of the College Board's Advanced Placement (AP) Calculus Program. This program is designed to enable students to begin university-level work in mathematics while in secondary school, thus encouraging more students to pursue more advanced mathematics. In practice, however, at least 85% of the students who study calculus in secondary school either take no calculus at university or retake Calculus I when they get to university.

There has been very little large-scale data collected on who elects to study Calculus I at university or the effect of this course on student intention to pursue a career in mathematics, science, or engineering. Even information as basic as the national success rate, the percentage of students in university Calculus I who successfully complete the course, has not been known.

Five goals were identified for this project:

1. To improve our understanding of the demographics of students who enroll in calculus,
2. To measure the impact of the various characteristics of calculus classes that are believed to influence student success,
3. To conduct explanatory case study analysis of exemplary programs in order to identify why and how these programs succeed,
4. To develop a theoretical framework that articulates the factors under which students are likely to succeed in calculus, and
5. To use the results of these studies and the influence of the MAA to leverage improvements in calculus instruction across the United States.

SURVEY PREPARATION AND IMPLEMENTATION

A total of five online surveys were constructed, one for the calculus coordinator, two for the calculus instructors of which one was administered immediately before the start of the

[2] In spring 2011, 341,000 students took the AP Calculus exam. Both (NCES, 2007) and this survey found that about 55% of the students who complete a course of calculus in high school take the AP Calculus exam.

course and the other immediately after it ended, and two for the students in the course (one at the start of the term and one at the end). In addition, instructors reported on the distribution of final grades and submitted a copy of the final exam. One year after the surveys were administered, a short follow up survey was sent to those students who had volunteered their email addresses. No incentives were given for completing the surveys.

The sampling frame

For the purposes of surveying post-secondary mathematics programs in the United States, the Conference Board of the Mathematical Sciences separates colleges and universities into four types, characterized by the highest mathematics degree that is offered: Associate's degree (hereafter referred to as two-year colleges), Bachelor's degree (referred to as undergraduate colleges), Master's degree (referred to as regional universities), and Doctorate (referred to as national universities). We sampled according to this stratification. Within each type of institution, we further divided the strata by the number of enrolled full time equivalent undergraduate students, creating from four to eight substrata. We sampled most heavily at the institutions with the largest enrollments. In all, we selected 521 colleges and universities: 18% of the 2-year colleges, 13% of the undergraduate colleges, 33% of the regional universities, and 61% of the national universities. Of these, 222 participated: 64 two-year colleges (31% of those asked to participate), 59 undergraduate colleges (44%), 26 regional universities (43%), and 73 national universities (61%).

The survey variables

There were six dependent variables, four of which were measured by student responses on a Likert scale at the end of the term: student confidence in their mathematical abilities, student enjoyment of mathematics, student desire to study more mathematics, and agreement with the statement that this course had increased their interest in studying more mathematics. The first three of these had also been assessed in the start of term survey, so that we were able to measure changes in these variables. The fifth dependent variable was the intention (yes or no) to take the next course in Calculus. This also was measured both at the start and end of the term so that we could look at the change in this variable. The final dependent variable was whether or not the student passed the course.

There were a number of control variables: gender, parental education, race/ethnicity, secondary school experience in mathematics including which courses were taken and what grades were received (including score on the AP Calculus exam, if relevant), SAT/ACT (college admission exam) scores, year at university, prior mathematics courses at university, and career intention.

The independent variables were collected at the student, the classroom, and the institutional level. At the student level, variables included student beliefs and attitudes about learning mathematics, study habits, level of intellectual engagement with the course, and experience with technology (graphing calculators and/or computer software). At the classroom level, instructor-supplied input included instructor experience and background; instructor beliefs, attitudes, and interests; class size; instructional practices; assessment practices; out of class interactions with students; use of technology including use of web resources; and the textbook as well as additional instructional resources provided for students. The student-supplied input at the classroom level included student perceptions of instructional practices, use of technology, assessment practices, and the intellectual community outside of class. At the institutional level, the variables were placement procedures, technological support, institutional support for students, and institutional support for instructors. The actual survey instruments are available at www.maa.org/cspcc.

There were 660 instructors and over 14,000 students who responded to at least one of the surveys. There is complete data (all five surveys completed and linked with each other) for 3103 students enrolled with 309 instructors at 125 colleges or universities.

SUMMATIVE DATA

While there is a great deal of variation by type of institution, the summative data are still interesting because this is our first glimpse at who studies calculus at university and why. Of the students who responded, 55% are men, 97% were full time students, 75% were in their first year of university, and 75% intended to major in science or engineering. The racial/ethnic breakdown was 76% White, 14% Asian, 5% Black, 10% Hispanic, as opposed to the general undergraduate population which is 73% White, 9% Asian, 12% Black, and 12% Hispanic.

Of those taking Calculus I at university, 61% had completed a course in Calculus in high school, and 61% of them had earned an A in this high school course. Just over 20% of all university Calculus I students had earned a 3 or higher on an AP Calculus exam which should mean that they enter this university course with at least as much knowledge of calculus as any passing student has at the end of the course.

The students who entered Calculus I were confident in their mathematical abilities and enjoyed mathematics: 95% believed that they had the knowledge and abilities to succeed in this course, 89% said that using reasoning to solve mathematics problems is a satisfying experience, 83% said that they enjoy mathematics, and 65% would want to take this course even if it were not required. These students were looking to understand Calculus and not simply complete the course: 74% preferred to make sense of mathematics rather than simply memorizing it and 72% saw the role of the instructor as helping students to reason through problems on their own rather than showing students how to work the problem. Finally, 58% believed that they would earn an A in this course.

In fact, 22% received an A, 28% received a B, 23% a C, and the remaining 27% received a D, failed the course, or withdrew. From the start to the end of the course, there was a sharp drop in the dependent variables that were measured in both student surveys. On a 1 to 6 scale (6 being most confident), student confidence dropped from 4.89 with standard deviation 1.01 to 4.42 with standard deviation 1.18, an effect size of –0.46. Enjoyment of mathematics dropped with an effect size of –0.27, and intention to continue to Calculus II dropped with an effect size of –0.20. Additional summative results can be found at (Bressoud, 2011a, 2011b).

STATISTICAL MODEL OF DATA

Phillip Sadler and Gerhard Sonnert of the Science Education Department at the Harvard-Smithsonian Center for Astrophysics are undertaking the statistical modeling of the set of complete data. The primary tool is a multiple linear regression with hierarchical linear modeling, recognizing that student variables are nested within classes and classroom variables are nested within institutions. They are also using propensity weighting (Rosenbaum & Rubin, 1983), recognizing, for example, that students who studied calculus in high school probably differ on a host of characteristics. Propensity matching or propensity weighting can be used statistically to even out those differences between those who studied calculus in high school and those who did not, providing a clearer picture of the difference attributable to studying calculus in high school. Sadler and Sonnert also have been working on variable reduction, recognizing that many variables are collinear. They have applied methods of variable reduction, such as factor analysis or multidimensional scaling, to create robust composite variables. The construction of the statistical model is still in its early stages, and there are as yet no results to report. The relatively small $N = 3103$ of students for whom we have complete data is complicating this analysis.

STUDENTS WHO OPT OUT OF TAKING MORE CALCULUS

In an analysis of US post secondary institutions, the Higher Education Research Institute (2010) found that over 50% of students who start their studies in a Science, Technology, Engineering and Mathematics (STEM) major do not complete their degree in five years. In Seymour's (2006) testimony to the US Congress, she noted that students leave STEM majors primarily because of poor instruction in their mathematics and science courses, with calculus often cited as a primary reason. In light of these findings, the data from this study provides a unique opportunity to better understand who is and who is not choosing not to continue taking more calculus and why. To gain insight into this issue, Chris Rasmussen, Jess Ellis and Kristin Duncan analyzed survey data from 7,260 students, representing 491 instructors and 61 institutions (Rasmussen, 2012). By triangulating multiple survey questions, they were able to determine which students planned to take more Calculus (that is, they were on a STEM trajectory) and which students switched off this trajectory. Of the 7,260 students analysed, 5,381 were classified as being on a STEM trajectory, with 12.5% of these choosing to opt out of taking more calculus. In the summary that follows, students who opted out of taking more calculus are referred to as "switchers" and students who followed through with their plan to take more calculus are referred to as "persisters."

Three research questions were identified for the analysis of switchers and persisters:

1. What is the profile of students who choose not to continue with calculus?
2. What characterizes the behavior of switchers and the behavior of their instructors?
3. What are the reasons that students give for switching out of calculus?

To answer the first research question Rasmussen, Ellis, and Duncan examined a number of attributes, including type of institution attended, gender and ethnicity, career path, academic preparation, beliefs and attitudes, and Calculus I grade. The following is a summary of their findings for each of these attributes:

Institution – In comparison to the overall percentage of students attending the different types of institutions, switchers represented a significantly greater percentage at large national universities (those with over 20,000 undergraduate students). Specifically, 32.6% of the STEM intending students attended a large national university, whereas 45.6% of switchers were from a large national university ($p < 0.001$). In light of this finding, what we learn from the case studies about more successful Calculus programs will be of particular interest to large national universities.

Gender and Ethnicity – Findings about the gender makeup of switchers reveal that switchers are disproportionally female. Of the STEM intending students, only 41.5% were female. However, 56.1% of the switchers were female. This finding raises a serious concern as to why women are disproportionately choosing to switch out of the STEM trajectory. Unlike the case with gender, there were no significant differences in the ethnic background of switchers and persisters.

Career Path – Recall that overall, 12.5% of STEM intending students were switchers. For students pursuing a career in the medical profession, 23.7% were switchers. The situation for engineers, however, is just the opposite. Only 5.9% of the engineering students were switchers. While this is good news considering the worldwide need for more engineers, it raises questions about why engineers are more likely to persist in Calculus, in comparison to other majors. Further study of this question is needed.

Academic Preparation – Contrary to what was conjectured, there was little difference in the academic preparation between switchers and persisters. In particular, switchers and persisters did not significantly differ in their SAT mathematics score and in whether or not they took calculus in secondary school (either a non-AP version or an AP version). However, on the end of the term survey, a significantly greater percentage of switchers compared to persisters reported that their previous mathematics courses did not prepare them for Calculus (32.4% of switchers felt unprepared whereas only 18.8% of persisters felt unprepared). One interpretation of these findings is that SAT score in mathematics and secondary school course taking pattern may not be sensitive enough measures to pick up on the kind of academic preparation that is important for success in Calculus I.

Beliefs and Attitudes – A greater percentage of switchers (48% compared to 39%, $p < 0.001$) believe that their success in Calculus relies on their ability to solve specific kinds of problems, as opposed to making connections and forming logical arguments. This and other related findings suggest that there may be important epistemological differences between switchers and persisters. Regarding enjoyment and confidence in mathematics, all STEM intending students lost joy and confidence in mathematics, however this loss was significantly greater for switchers than persisters.

Grade in Calculus I – 81% of the switchers received a grade of C or higher (the minimum needed to continue on in Calculus) compared to 98% of persisters. This finding indicates that other factors besides grade are contributing to the majority of switchers' decision to opt out of taking more Calculus.

Because of space limitations only an overview of the findings on the second and third research questions are provided. In terms of behavior, switchers report that they are less likely to contribute to class discussion and more frequently find themselves lost in class. Consistent with this, the majority of switchers were more likely to seek help outside of class with either their instructor or a tutor. Similar to persisters, about 70% of switchers worked only 0-5 hours per week. The vast majority of switches spent about the same amount of time studying calculus as did persisters. Similar to persisters, about 60% of switchers met with other students to study calculus. One of the most interesting findings, and perhaps one with the greatest implication, is that switchers report having quite different classroom experiences than persisters (the differences on multiple measure of this are significant). For example, switchers are more likely to report that their instructors did not actively engage them in doing and thinking about mathematics. In addition, most STEM intending students felt encouraged by their instructors, but significantly more switchers felt less supported and encouraged compared to persisters. All of this indicates that switchers are putting as much effort into Calculus I, if not more, than persisters. However, switchers report having less intellectual connection with Calculus and their instructor. This may account, in part, for why switchers are to a greater extent losing joy and confidence in mathematics.

Data to answer the third research question came primarily from an open-ended question that prompted students to reflect on their experience in Calculus I. This question was on the follow up survey that was taken by 1230 students a year after completion of the main surveys. A preliminary qualitative analysis indicates that students found the teaching of Calculus I to be ineffective and uninspiring, the course was “over stuffed” with content and delivered at too fast a pace, the assessments were poorly aligned with what was taught, and their instructor lacked connection to students and the course.

FINAL EXAMINATION ANALYSIS

As part of the end of term instructor survey, instructors were asked to submit a copy of the final examination. Marilyn Carlson and Michael Tallman analysed these exams (Carlson *et al.*, preprint). There were 253 submitted exams of which 150 were randomly selected for analysis. Of the 150 exams, 61.9% were administered at national universities, 22.7% at regional universities, 9.4% at two-year colleges, and 6.0% at undergraduate colleges.

Data analysis in this study consisted of three phases. In the first phase, an exam characterization framework was developed and used to code the exams. In the second phase, Carlson and Tallman compared the coded exam data with data obtained from a post-term instructor survey with the intention of determining the extent to which their characterization of the exams corresponds with instructors’ perceptions of their exams relative to their conceptual orientation. In the third phase, they conducted a comparison by coding a national selection of 13 Calculus I final exams that had been administered in 1986–87 and collected by MAA. For comparison purposes, they also coded the free response portion of the AP Calculus AB and BC exams administered during 2009-2011.

The Exam Characterization Framework characterizes exam items according to three distinct item attributes: (a) *item orientation*, (b) *item representation*, and (c) *item format*. The development of the framework constructs were initially informed by other frameworks (*e.g.*, Li, 2000; Lithner, 2004: Mesa, 2004; Mesa *et al*., in press; Smith, 1996) that had been used to characterize mathematics tasks relative to their cognitive demand, representation, format and solution type. The item orientation that was chosen was an adaptation of Anderson & Krathwohl's (2001) modification of Bloom's taxonomy (Table 1).

Table 1: Adaptation of intellectual behaviors from Anderson & Krathwohl (2001).

Cognitive Behavior	Description
Remember	Students are prompted to retrieve knowledge from long-term memory (e.g., write the definition of the derivative).
Recall and apply procedure	Students must recognize what knowledge or procedures to recall when directly prompted to do so in the context of a problem (e.g., find the derivative/limit/integral of *f*).
Understand	Students are prompted to make interpretations, provide explanations, make comparisons or make inferences that require an understanding of a mathematics concept.
Apply understanding	Students must recognize when to use (or apply) a concept when responding to a question or when working a problem. To recognize the need to apply, execute or implement a concept in the context of working a problem requires an understanding of the concept.
Analyze	Students are prompted to break material into constituent parts and determine how parts relate to one another and to an overall structure or purpose. Differentiating, organizing, and attributing are characteristic cognitive processes at this level.
Evaluate	Students are prompted to make judgments based on criteria and standards. Checking and critiquing are characteristic cognitive processes at this level.
Create	Students are prompted to put elements together to form a coherent or functional whole; reorganize elements into a new pattern or structure. Generating, planning, and producing are characteristic cognitive processes at this level.

Most items were coded as "recall and apply procedure" (78.7%) or "apply understanding" (10.3%). No items were coded as "create." This analysis is not in agreement with instructors' beliefs about their examinations. At the end of the term, instructors were asked "How frequently did you require students to explain their thinking on exams?" They responded on a scale from 1 (not at all) to 6 (very often). "Very often" was chosen by 38% of the instructors, and over two-thirds of them rated the frequency as 4 or higher.

When asked, "On a typical exam, what percentage of the points focused on skills and methods for carrying out computations?", the median of the instructor responses was 50% with an interquartile range of [40%,70%]. In fact, the median percentage of "recall and apply procedure" questions on their final exams was 82.8% with an interquartile range of [70.5%, 88.2%] (Figure 2). In 90% of the exams, over 70% of the items were coded either as "remember" or "recall and apply procedure." Additionally, 89.4% of all exam items required students to perform symbolic computation. Carlson and Tallman (Carlson *et al.* preprint) have speculated on the reasons for this disconnect between instructor perception and reality.

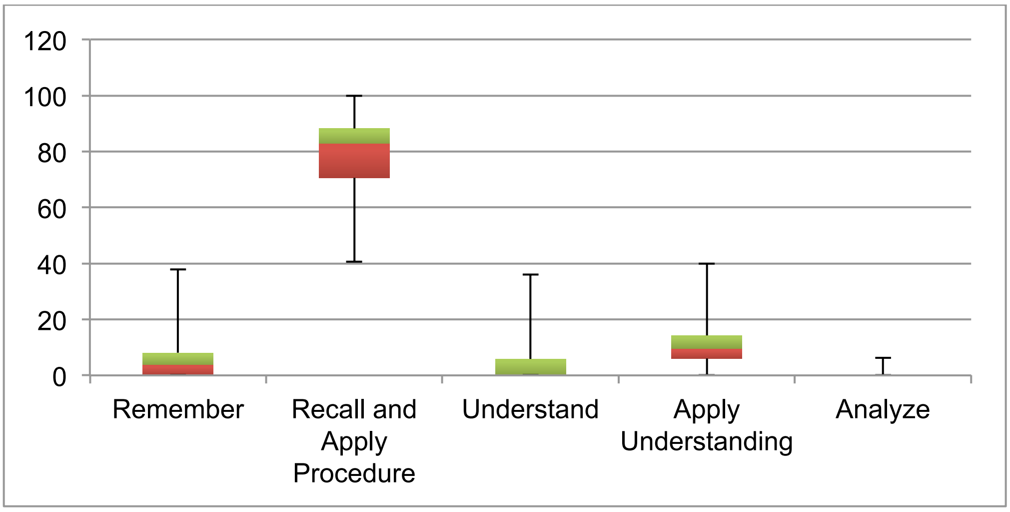

Figure 2: Box plots representing the percentage of items per exam within each category of the item orientation taxonomy.

Comparison with the final examinations from 1986–87 shows that the distribution of item orientation has changed very little. For the thirteen exams collected in that year, 3.7% of the items were classified as "remember," 87% as "recall and apply procedure," and 8.2% as "apply understanding." The AP Calculus exams were different. None of the questions classified as "remember," 60.3% were "recall and apply procedure," and 39.7% were "apply understanding."

CONCLUSION

These are still early days in the analysis of the massive quantity of data collected in the fall 2010 survey. While the relatively small $N = 3103$ of complete student data is disappointing, the richness of these data promise much insightful mining for the future.

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