Describing Cognitive Orientation of Calculus I Tasks Across Different Types of Coursework

Nina White*, Vilma Mesa

* Corresponding Author

whitenj@umich.edu; Department of Mathematics, 530 Church St., Ann Arbor MI, 48109-1043

vmesa@umich.edu; School of Education, 4041 SEB, 610 East University, Ann Arbor MI 48109-1259

Article Note: This work was partially funded by the National Science Foundation under grant No. 0910240. The opinions expressed do not necessarily reflect the views of the Foundation. The Undergraduate Research Opportunity Program at the University of Michigan also provided funding for this project. We thank Cameron Blum who assisted with coding.

Abstract

We discuss the findings of an analysis of cognitive orientation of 4,953 mathematical tasks (representing all bookwork, worksheets, and exams) used by five instructors teaching Calculus I in a two-year college in the United States over a one-semester period. This study uses data from one of 18 cases from the *Characteristics of Successful Programs in College Calculus* (CSPCC) and characterizes the tasks using and adaptation of an analytical framework developed by Tallman & Carlson (2012). We found differences in the cognitive orientation by type of course work assigned (graded vs. ungraded) and differences by the instructors who assigned the course work. We discuss implications for practice and propose some areas for further exploration

Keywords: cognitive orientation of mathematical tasks, assessment, calculus 1, student coursework, graded coursework, ungraded coursework, textbook

1 Introduction

The National Council of Teachers of Mathematics' *Principles and Standards*, the American Mathematical Association of Two-Year Colleges' *Crossroads in Mathematics*, and substantial research highlight the importance of using challenging mathematical tasks to help students learn (Blair 2006; National Council of Teachers of Mathematics 2000; Stein and Lane 1996; Watson and Shipman 2008). The problems posed to students are seen as the conceptual link between teaching and learning (Stein & Lane 1996). When the goals of mathematics instruction are for students to build their abilities to reason with novel problems, fluently use multiple representations, and communicate and justify mathematical ideas, then the tasks given to students need to display those characteristics, so students can become proficient in those practices.

The overarching question guiding this study is: what is the quality of instructors' learning goals and students' opportunities to learn at an institution that has been identified as successful in teaching Calculus I? We use the homework and assessment tasks assigned to students to access these constructs. To this end, we characterize the potential cognitive demand of 4,953 Calculus I tasks coming from homework, worksheets, quizzes, and exams. We use cognitive orientation of tasks to compare (1) this particular institution to the national sample, (2) different types of coursework (homework vs. exams, etc.), and (3) different instructors.

These items represent the complete set of tasks assigned by five instructors at a two-year college in the US identified as "successful" by the *Characteristics of Successful Programs in College Calculus* (CSPCC) project (Bressoud, Carlson, Mesa, & Rasmussen 2013) and the analyses reported here is part of the process we are using to identify analytical strategies that would allow us to identify features that makes these eighteen institutions exceptional.

The rest of this paper is organized into six sections. In Section 2 we present our research questions followed by a conceptualization of tasks and a review of how tasks have been analyzed in the literature. In Section 3 we describe the analytical framework we developed to categorize cognitive orientation, highlighting and describing the main features of this framework. In Section 4 we present the methods, followed in Section 5 by our findings, and their discussion in Section 6. We conclude (Section 7) with implications for research and practice.

2 Research Questions and Literature Review

In order to address our overarching question (to identify the quality of instructors' learning goals and students' opportunities to learn in a "successful" Calculus I program by characterizing tasks instructors give to students in homework, worksheets, and exams) we ask the following three research questions focusing on cognitive demand of tasks, cognitive demand of different types of coursework, and consistency in cognitive demand between instructors:

RQ1: What is the cognitive orientation of mathematical tasks instructors give to their students?

RQ2: What is the cognitive orientation of different types of coursework that instructors give to their students? Are there differences between types of coursework?

RQ3: Can we detect differences between instructors at the same institution by characterizing the cognitive orientation of their tasks? Further, do instructors display preferences for higher or lower cognitive orientation of tasks that persist across different types of coursework?

In the following sections, we discuss important constructs in our questions and how our study differs from related, prior work.

2.1 Tasks and their Cognitive Orientation

Doyle (1988) defined tasks as composed of (1) the product students are asked to produce, (2) the operations used to produce the product, (3) the resources available, and (4) the significance (or "weight") in the course. Doyle derived this categorization from analyses of elementary mathematics classrooms, in which it was typical for the teacher and the students to work collectively in a mathematics problem during class. A main difference with tasks as implemented in tertiary settings is that for the most part, the actual work that students do is completed outside the classroom (Mesa and Griffiths, 2012). For this reason we define tasks as:

1. A question posed by the instructor that students are expected to produce an answer for (i.e., the task as written);
2. The hypothetical operations used to produce the answer (i.e., cognitive demand);
3. The hypothetical resources available (e.g., time, study groups, internet)
4. The significance of the product in the course (i.e., grade weight)

In our analysis we use the potential cognitive demand of a task to characterize the hypothetical operations used to produce the answer. Because we look at the potential cognitive demand of a task, and not what students actually experience, we prefer to use the term *cognitive orientation* to make this distinction clear. To address the significance of the product in the course and the resources available, we categorize tasks as belonging to different types of *coursework* (described in Section 3.3).

The underlying assumptions we make in choosing to analyze tasks is that (1) tasks represent what instructors want students to learn, and (2) students learn by engaging with those tasks. That is, we argue that tasks provide us with an approximation of instructors' learning goals and student learning opportunities. These are the constructs we want to understand better and why we choose to look at these tasks.

2.2 Tasks in the Curriculum

Studies of tasks are situated in the literature on curriculum. Curriculum has been described as existing at several *levels*: intended, potentially implemented, implemented, and attained (Valverde, Bianchi, Houang, Schmidt, & Wolfe 2002). We point this out because many of the existing studies on curriculum (in particular most textbook studies, some of which we describe below) attend to the *potentially implemented* curriculum. Our study, in contrast, does not use the textbook as the unit of analysis but focuses only the tasks the instructors actually assign. Thus the tasks we analyze represent both the instructors' intentions and also the implemented instruction, or students' opportunities to learn.

A more detailed conception of curriculum (Mesa, Gómez, Chea, 2013) brings in Rico and colleagues' (1997) four *dimensions* of curriculum: conceptual, cognitive, formative, social. The conceptual dimension refers to the discipline, the cognitive dimension refers to the learner, the formative dimension refers to the teacher, and the social level refers to society's values. The idea of "curriculum" can be studied along each of these dimensions as well as how they interact. Using this definition of curriculum helps us to locate our study within the broader idea of curriculum: the cognitive dimension of the implemented curriculum (see Figure 1).

	Conceptual	Cognitive	Formative	Social
Intended				
National/Federal				
Regional/State				
District				
School				
Classroom				
Potentially Implemented				
Textbooks				
Implemented				
Classroom		X		
Homework				
Attained				
Classroom Assessments				
State Assessments				
National Assessments				
International Assessments				

Figure 1: Curricular levels (Valverde et al. 2002) and curricular dimensions (Mesa, Gomez, & Cheah 2013; Rico et al. 1997) and where our study falls in the matrix. Adapted from Mesa et al. 2013.

2.3 Prior Work Analyzing Complexity of Tasks and Curriculum

There is a long history of analyzing the cognitive demand of tasks students encounter. We discuss here three distinct types of studies that use task as a central feature of the analysis:

1. Analysis of tasks by looking at textbooks,
2. Analysis of tasks by looking at assessments, and
3. Comparison of cognitive demand of tasks at different levels of curriculum.

We describe briefly the frameworks and data used in some of these analyses, highlighting how this study complements to these efforts.

2.3.1 Tasks in textbooks

An important number of studies of tasks compare textbooks across countries (Fan, Zhu, & Miao 2013). For example, Li (2011), in his study of integer addition and subtraction problems in American and Chinese textbooks, uses four categories to characterize their cognitive demand (Procedural Practice, Conceptual Understanding, Problem solving, and Special Requirement). In a study comparing addition and subtraction of fractions in three countries, Charalambous et al (2010) attend to potential cognitive demand in one of their dimensions of analysis using Stein et al.'s (2000) framework (Memorization, Procedures without Connections, Procedures with Connections, and Doing mathematics).

In a non-comparison study, Lithner took a different approach to cognitive demand. He asked: "in what ways is it possible to solve textbook exercises without considering the intrinsic mathematical properties of the components involved?" (Lithner 2004, p. 408). To answer this question he applied a reasoning framework (Lithner 2000) to Calculus I tasks in a common Swedish textbook. The framework helped him characterize how tasks could be solved superficially following examples from the same section of the textbook.

These studies were explicitly described as attending to the potentially implemented curriculum, because they are comprehensive analyses of the textbook tasks. That is, they analyze all the tasks in the textbook or section. In our case, we believe that we are approximating the implemented curriculum because we are looking at what teachers actually assign students via homework and examinations, thus giving us a more accurate insight into students' opportunities to learn and instructors' learning goals.

2.3.2 Tasks in Assessments

Other studies attend to cognitive demand of assessment tasks. In particular, both Smith et al (1996) and Tallman and Carlson (2012) characterize tasks assigned in calculus I final exams using adaptations of Bloom's taxonomy (Anderson, Krathwohl, & Bloom 2001; Bloom & Krathwohl 1956). Our work intentionally departs from this model of task analyses that looks only at assessment tasks because we believe such tasks only give us one part of the picture—instructors' learning goals—while ignoring students' opportunities to learn. That is, because homework fulfills a different purpose, we claim that failing to analyze homework in conjunction with exams misses an important dimension of what students are actually expected to be learning (see Section 3.3).

2.3.3 Tasks as Intended, as Enacted, and as Attained

Studies that do take into account both the instructors' intentions as well as students' opportunities to learn follow the transformation of the cognitive demands of the tasks as teachers and students use them. Stein and Lane (1996) look at cognitive demand of tasks as posed in textbooks, cognitive demand of those tasks as implemented or mediated by the teachers[1], and the connections to student learning gains (attained curriculum). They use four categories: Memorization, Procedures without Connections, Procedures with Connections, and Doing Mathematics. Their study sheds light on the complexities of using challenging tasks for teaching. Our study starts an approximation to this transformation at the tertiary level by contrasting the cognitive demand of the work teachers assign students as homework and the cognitive demand of the work they assign in exams.

In summary, our analysis expands on existing literature in curriculum and task analysis by taking into account the tasks students actually engage, looking at both teachers' intentions and students' opportunities to learn, and expanding such research to the tertiary level.

Indirectly, our study may be characterized as focusing on teachers' implementations of textbooks (Fan et al. 2013) because we compare instructors' selections of tasks. We discuss some implications of this categorization in the discussion section of the paper.

3 Analytical Framework: Cognitive Orientation of Tasks

3.1 Background

We choose the expression *cognitive orientation* over *cognitive demand* (the term commonly used in the literature) to honor that (1) we attend to the goals of the task (or our perception of the instructor's goals of the task) and (2) we cannot observe students' actual interaction with the task. This construct is, nevertheless, useful for characterizing the range of tasks instructors choose for students to engage with. That is, gives us reasonable access to instructors' goals and students' cognitive demand during authentic engagement[2].

We started by adapting the framework that Tallman and Carlson (2012) designed to analyze 3,735 tasks from 150 final exams randomly selected from a sample of 253 institutions from a U.S. sample of Calculus I final exams.

The theoretical underpinnings of this dimension come from Anderson and Krathwohl's (2001) revision of Bloom's (1956) taxonomy. The revised taxonomy has two dimensions—Knowledge Types and Cognitive Processes (see Figure 2). The Knowledge dimension refers to the different types of knowledge that students are meant to learn and use. The Cognitive Processes refers to the mental resources that are needed in order to learn; in practical terms it refers to what is to be done with the knowledge. Whereas the categories of knowledge are not necessarily ordered in terms of importance, the categories for cognitive processes are ordered,

---

[1] In our setting, the majority of tasks are not done in class, so looking at tasks as written is our best approximation of cognitive demand.

[2] The cognitive demand of a task can be decreased through regular practice, memorization of specific routines, following examples superficially, or cheating.

depending on levels of cognitive engagement. That is, Remembering requires fewer cognitive resources, than Creating.

	Type of Knowledge			
Cognitive Processes	Factual	Procedural	Conceptual	Meta-cognitive
Remember				
Apply				
Understand				
Analyze				
Evaluate				
Create				

Figure 2: The revised taxonomy matrix (Anderson & Krathwohl 2001)

We used this framework, Tallman and Carlson's (2013) framework, and our data to generate a framework that would reliably and validly classify the tasks teachers assigned according to their cognitive orientation.

3.2 Dimensions of Cognitive Orientation of Tasks

The cognitive orientation dimension in our framework has eight categories: Remember, Recall and Apply Procedures, Recognize and Apply Procedures, Understand, Apply Understanding, Analyze, Evaluate, and Create. These definitions follow Tallman and Carlson's categories of *item orientation* with some differences. See Figure 3 for how the categories for cognitive orientation align with the Anderson-Krathwohl taxonomy and Table 1 for definitions and examples of these categories.

<table>
<tr><td></td><td colspan="6">Type of Knowledge</td></tr>
<tr><td>Cognitive Processes</td><td>Factual</td><td colspan="2">Procedural</td><td colspan="2">Conceptual</td><td>Meta-Cognitive</td></tr>
<tr><td>Remember</td><td>(R) Remember</td><td rowspan="2">(P) Recall and Apply Procedure</td><td colspan="3" rowspan="2">(RP) Recognize and Apply Procedure</td><td></td></tr>
<tr><td rowspan="2">Apply</td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td colspan="3" rowspan="2">(AU) Apply Understanding</td><td></td></tr>
<tr><td>Understand</td><td></td><td>(U) Understand</td><td></td></tr>
<tr><td>Analyze</td><td></td><td colspan="4">(A) Analyze</td><td></td></tr>
<tr><td>Evaluate</td><td></td><td colspan="4">(E) Evaluate</td><td></td></tr>
<tr><td>Create</td><td></td><td colspan="4">(C) Create</td><td></td></tr>
</table>

Figure 3: Mapping of cognitive orientation categories (White, Blum, Mesa, 2013) relative to Anderson-Krathwohl's taxonomy. Empty cells did not appear in our data.

3.2.1 Remember

In Anderson & Krathwohl's taxonomy, the least demanding cognitive process is *Remember*. Remember tasks ask students to recall factual information. We coded the task:

> Write the limit definition of the derivative of $f(x)$ at $x = a$

as a remember task in the context of Calculus I.

3.2.2 Recall Apply Procedure

In Tallman and Carlson's framework, the next level of cognitive orientation is Apply Procedure. This level corresponds roughly to the Apply/Procedural cell of the matrix in the Anderson & Krathwohl taxonomy. Tasks of this type are characterized by prompts to carry out procedures, usually involving symbolic manipulation. The tasks make no connections between the procedure and broader concepts or meanings. Examples of these tasks include "find the derivative of $f(x)$," where $f(x)$ is defined symbolically; they are very common in the textbooks.

We coded tasks according to what we considered was their plausible potential for eliciting certain kinds of cognitive processes, given the content in the textbook and our experience as teachers and students of calculus. When confronted with tasks that experience told us were frequently proceduralized, but in which conceptual understanding could potentially, fruitfully be engaged we had a difficult time making a classification. For these tasks we created a new code, *Recognize and Apply Procedures*.

3.2.3 Recognize and Apply Procedure

We added this code to the Tallman and Carlson framework. Tasks of this type are characterized by the *potential* for the task to elicit students making connections and applying conceptual understanding, while acknowledging that, depending on instruction and on student, the tasks also had the potential to be proceduralized and worked without conceptual understanding or connections. One example of a task of this type is the following:

> Let $f(x) = \sin(x)\cos^2(x)$. What is the equation of the tangent line at to the graph of $f(x)$ at $x = 5$?

A student could plausibly memorize a formula for the local linearization—in this case $y = f'(5)(x - 5) + f(5)$—making the problem procedural (the result of applying a known formula). On the other hand, conceptual understanding can also play an important role. If a student understands the meaning of the derivative as the slope of the tangent line, this task could engage that knowledge and reinforce connections between symbolic and graphical representations of the derivative.

Routine optimization tasks such as:

> Find the global maximum and minimum of $f(x) = (x^2)\sin(x - 2)$ on the interval $0 \leq x \leq 5$.

were also coded as Recognize and Apply Procedure because, like in the local linearization task, students could approach this task procedurally by memorizing a set of sets (e.g. "take the derivative, set it equal to zero, etc."). However, if the student understands the graphical meaning of the derivative and the conceptual importance of critical points, the problem can be solved using reasoning instead of a memorized set of steps.

By adding this code we were able to more reliably capture tasks that were difficult to categorize as either Recall and Apply Procedure or as Apply Understanding.

3.2.4 Understand

Tallman and Carlson described Understand tasks in terms of Piaget's (1968) notion of assimilating a concept into a scheme and coded a task as Understand if the task "demonstrate[s] a student has assimilated a concept into an appropriate scheme (Tallman & Carlson 2012, p. 10).” We used Anderson and Krathwohl’s definition of Understand: “determining the meaning of instructional messages, including oral, written, and graphic communication (Krathwohl 2002, p. 215).” In particular, the main cognitive tasks of Understand include: interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining (Anderson et al. 2001). A typical example is:

> Let $c(x)$ be the pressure in mmHg felt by a scuba diver at x meters. Interpret the meaning of $c'(10)$ using everyday language.

Our data included many tasks asking students to extract features from graphs (such as limit values, intervals of concavity, or intervals of increasing/decreasing behavior). These tasks were difficult to code because they were routine without being procedural. However, we coded such tasks as Understand because they connect a definition (for example, of “increasing” or “concave up") to a graph of a function. Heuristically, we differentiated Understand Tasks from Procedure tasks by noticing that in Understand tasks information is *extracted* from a situation, as opposed to a Procedure, which is *enacted.*

3.2.5 Apply Understanding

Tasks of this type require students to make inferences and interpretations and to apply a procedure. Tasks in this category include optimization[3] and related rates problems in which students generate equations based on verbal or graphical information and then work with those equations (usually by taking derivatives, solving for variables, etc.) An example is:

> A lighthouse is located on a small island 3 km away from the nearest point P on a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1km from P? (Stewart 2010, p. 250)

---

[3]Section 3.2.3 describes optimization problems that are not Apply Understanding.

Table 1: Categories of Task Orientation, Definitions, and Examples.

Orientation	Definition	Example
Remember	Students are prompted to retrieve knowledge from long-term memory	Write the definition of the derivative of f at $x=a$.
Recall and Apply Procedure	Students must recall the algorithms for applying certain procedures and carry them out.	Find the derivative of f at $x=a$.
Recognize and apply procedure[a]	Students must recognize what knowledge or procedures to recall without being directly prompted. Conceptual knowledge plays a plausible role in this venture, but some students may be able to answer the question with memorized procedures or formulas. Students may have to string together several procedures.	At what value of x does f attain a minimum?
Understand	Students are prompted to make interpretations, provide explanations, make comparisons or make inferences that require an understanding of a mathematics concept	Is this graph a plausible graph for f given the table for values of f'? Explain why or why not.
Apply Understanding	Students must recognize when to use (or apply) a concept when responding to a question or when working a problem. To recognize the need to apply, execute or implement a concept in the context of working a problem requires an understanding of the concept.	Given a flask shaped like an inverted cone, write a function rule that expresses the relationship between the height of the water in the flask and number of ounces of water in the flask.
Analyze	Students are prompted to break material into constituent parts and determine how parts relate to one another and to an overall structure or purpose. Differentiating, organizing, and attributing are characteristic cognitive processes at this level.	Graph the given functions on a common screen. How are these graphs related? $y = 3^x$, $y= 10^x$, $y = (1/3)^x$, $y=(1/10)^x$. What general conclusions do you draw?
Evaluate	Students are prompted to make judgments based on criteria and standards. Checking and critiquing are characteristic cognitive processes at this level.	Is it reasonable to use this model to predict the winning height [of the high jump] at the 2100 Olympics? (Stewart, 2012, p. 35)
Create	Students are prompted to put elements together to form a coherent or functional whole; reorganize elements into a new pattern or structure. Generating, planning, and producing are characteristic cognitive processes at this level.	A student in our class wants to estimate the integral of a function using a Riemann sum with 5 partitions. Give a function and domain of integration such that the right-hand Riemann sum will be off of the true integral by more than 200% but the left-hand Riemann sum will be exact.

3.3 Types of Coursework

We now characterize types of coursework in our sample drawing on two aspects of Doyle's (1988) definition of task: the grade weight and the resources available.

We attend to the full spectrum of tasks in Doyle's definition, including daily homework, which typically has low "weight" and wide access to resources, as well as exams, which carry the most weight but limited access to resources. We include homework because, although low in grade weight, these tasks constitute a significant portion of students' engagement with the material, and they represent students' primary opportunities to *learn* the material.

We identified five distinct types of coursework in our context—Bookwork, Webwork (Web-based Bookwork), Worksheets, Quizzes, and Exams—using grade weights and time/resources available (Figure 4).

To Doyle's dimensions we add that tasks can be seen as fulfilling one of two major roles: (1) to engage students in learning the content (*opportunities to learn*) and (2) to demonstrate that students have indeed learned the content (*opportunities to demonstrate that learning has occurred*). These roles stem from teachers' professional obligations (Herbst & Chazan 2011).

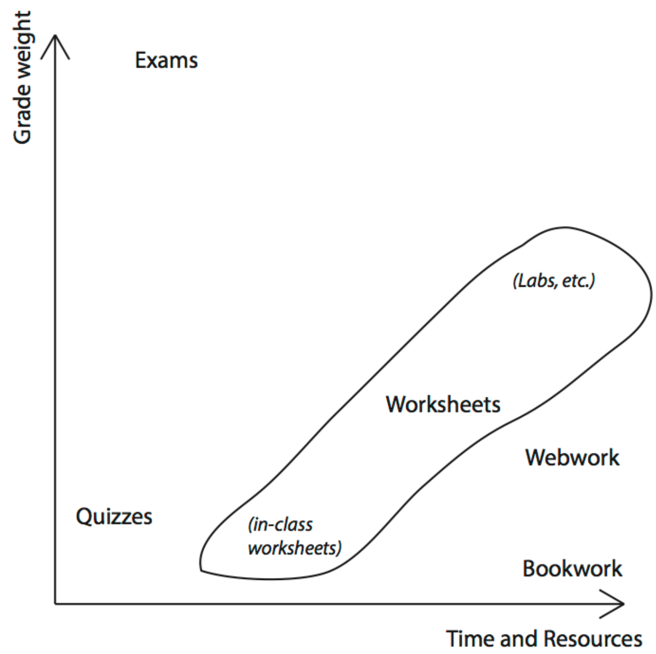

Figure 4: coursework types in our sample. The Worksheets area reflects the variation we found in terms of the time and resources needed (can be completed in or out of class, require group work or library resources) and how much they count towards the course grade.

3.3.1 Bookwork

Bookwork refers to the problems in the textbook that the instructors assigned to their students to be completed outside of class. Bookwork may be graded, but the grade has little impact on students' final grade. At our site, bookwork was ungraded. The availability of solution manuals makes the bookwork inappropriate as a tool for assessing students' learning and de-incentivizes instructors' grading of this work, In our site, students completed these tasks outside of class, over a few days, with unlimited access to resources. We characterize bookwork tasks as opportunities to learn.

3.3.2 Webwork (Web-based Bookwork)

Webwork refers to the problems in the textbook that the instructors assigned to their students to be completed outside of class over a web-based platform. The time and resources available for completing webwork are similar to those for completing bookwork. However, webwork is more likely to contribute to a student's grade. In our case only one instructor assigned webwork and it counted as 15% of the final grade (~1.5% per assignment). We characterize webwork tasks as opportunities to learn.

3.3.3 Worksheets

Worksheets refer to instructor-created (as opposed to textbook publisher-created) tasks for students to work on in class, in the lab, or at home. They may be ungraded or graded. This is our most amorphous category of coursework. In our site, worksheets completed in class tended to contribute minimally towards the final grade (<1%), although their weight increased when there was more time and resources available. We characterize worksheet tasks as opportunities to learn.

3.3.4 Quizzes

Quizzes refer to the tasks solved with limited time/resources (e.g., 10 minutes, no textbook). Quizzes are usually graded, but tend to contribute little to the overall grade. Quizzes seem to be opportunities to attest that learning is happening—students have to demonstrate some progress in their learning in a limited amount of time. However, they can provide metacognitive dissonance,—a wake-up call that students need to prepare for the upcoming exam. Quizzes also afford opportunities for practice—similar items might appear on a higher-stakes exam. We characterize quiz tasks as opportunities to learn and as opportunities to demonstrate that learning has occurred.

3.3.5 Exams

Exams refer to high-stakes situations in which a set amount of tasks need to be completed with limited access to time/resources. At our site, a single exam counted for between 10% and 25% of the final course grade. All exams were completed individually, in one 50-75-minute class period. We characterize exam tasks as opportunities to demonstrate that learning has occurred.

4 Method

4.1 Data

In the fall term of 2010, the Mathematical Association of America (MAA) conducted a national survey of Calculus I instruction across a stratified random sample of two- and four-year colleges and universities. The survey was restricted to what is known in the United States as "mainstream" calculus, the calculus course that is designed to prepare students for the study of engineering or the mathematical or physical sciences. In contrast to other countries, Calculus I (limits, derivatives, and integration) is usually a first-year university course in the United States. Some questions in the survey data were aligned with six student "success" variables: (1) confidence in own ability to do mathematics, (2) enjoyment of mathematics, (3) inclination to choose to take more math, (4) course increased interest in math, (5) intention to take Calculus 2, and (6) expected grade. With the goal of creating explanatory case studies of successful programs, colleges showing the best gains in those success variables over one semester were selected for further, in-depth, site visits in order to identify why and how their calculus programs had good performance on those success variables. Suburban College was chosen as a case study. We had access to all of the Calculus I coursework that five instructors teaching Calculus I assigned during the fall term of 2012 at Suburban College.

Suburban College is a large, two-year college located in southern United States. Its full-time enrollment in the 2010-2011academic year was 12,492 students. In Fall 2012 the math department had 35 full-time instructors and 30 part-time instructors. The Calculus I classes enroll 30 students per class. At the time of data collection, there were two honors sections capped at 15 students each, and one section taught at a satellite campus (these courses were not included in this analysis). Counting only non-honors, main campus sections, there were 8 sections of Calculus I taught by six full-time instructors. We include almost all the coursework[4] assigned by five of those instructors (representing six of the eight sections). We use pseudonyms for the five instructors: Albert, Bob, Charles, David, and Ethan.

Besides collecting instructors' coursework, we interviewed them, observed their classes, and conducted focus groups with their students. These additional sources were used to help in our understanding of the site, its Calculus I program, and its instructors and students, but are not the focus of the analysis presented here.

4.2 Analysis

We performed two types of analysis on the tasks collected, qualitative—to identify the cognitive orientation of all the tasks assigned to students—and quantitative—to identify trends in terms of cognitive orientation, and its distribution by type of coursework and by instructor.

4.2.1 Qualitative Analysis

This analysis involved three phases: development of a coding system, testing its reliability, and final coding.

The first author and a research assistant worked together closely to develop the coding system for the corpus of tasks. The units of analysis were task *parts* (i.e., when a problem or exercise has multiple questions, each one was identified as a separate task). In phase one, using data from textbooks and exams that were not part of the case data, the two coders engaged in coding the cognitive orientation of the tasks using Tallman and colleagues' framework. This was an iterative process by which each coder worked on 5-10 tasks, discussed agreement and disagreements, stating heuristics to define the categories. After several cycles of this coding—which used a wide variety of tasks that tested the limits of the coding

[4] We do not have labs and quizzes from one instructor.

process—they started phase two: coding a larger number of tasks at a time (20-50 tasks per session, not coming from the corpus). This process was used to test the reliability between the coders. Their goal was to reach a 0.7 level of agreement using the Cohen's Kappa coefficient.[5] Reaching this level of agreement for this dimension proved harder than anticipated.[6] In particular, the coders identified difficulties in distinguishing between Recall and Apply Procedure and Apply Understanding for certain kinds of tasks (White et al. 2013). The coders resolved the difficulty by creating the category, Recognize and Apply Procedure (see Section 3.2.3). The coding session before this change yielded $\kappa = 0.18$; the coding session after this change yielded $\kappa = 0.70$, which was deemed sufficient for moving on to phase three: coding the site data. In phase three, the research assistant coded all the tasks and the first author randomly selected 10% of the tasks to code. In less than 0.1% of the tasks coded, the research assistant required consultation before deciding on a code; in those cases the coders discussed the options and assigned the best code. The final agreement for cognitive orientation on the 10% of the corpus data was $\kappa = 0.475$.

4.2.2 Quantitative Analyses

Each of the coded tasks was entered into a spreadsheet that included fields for instructor, its coursework type, and its cognitive orientation. To simplify the presentation of the findings we grouped cognitive orientation into three categories: Simple Procedures, Complex Procedures, and Rich Tasks. Simple Procedures included all tasks coded as Remember and Recall and Apply Procedure, Complex Procedures included all tasks coded as Recognize and Apply Procedures, and Rich Tasks included all tasks coded as Understand, Apply Understanding, Analyze, Evaluate, and Create (see Table 1).

In order to answer RQ1 (*What is the cognitive orientation of the tasks that instructors give to their students?*), we produced frequencies for the three categories of cognitive orientation. In order to answer RQ2 (*What is the cognitive orientation of different types of coursework that instructors give to their students? Are there differences between types of coursework?*), we tested the significance of the association between type of coursework and item orientation using a chi-squared test. This test was selected because we were interested in looking at the variability in the distribution of the tasks across coursework; other approaches could have included the creation of a variable that would give a score for cognitive orientation (0 for simple procedures, 1 for complex procedures, and 2 for rich tasks) and allow mean comparisons. We decided against this approach, because it is not clear that there is a linear ordering of the categories in such a

---

[5] Cohen's κ allows assessing inter-rater reliability when there are two coders and the variables have several categories. This coefficient calculates agreement taking into account chance agreement. For that reason it is more stringent than calculating the rate of agreements to the total of agreements and disagreements. According to Landis and Koch (1977), $\kappa = 0.40$ to 0.59 reflects moderate inter-rater reliability, 0.60 to 0.79 substantial reliability, and 0.80 or greater outstanding reliability.

[6] This process included the development of a system that included five other dimensions besides cognitive orientation. For details on the development of this framework see White et al. (2013).

way, or what their relative numerical values should be. In other words, converting these categorical frequencies into a continuous variable led to a loss of the nuanced distinctions that we wanted to capture. The chi-squared test is appropriate for the data we have. In order to answer RQ3 (*Can we detect differences between instructors at the same institution by characterizing the cognitive orientation of their tasks? Further, do instructors display preferences for higher or lower cognitive orientation of tasks that persist across different types of coursework?*), we restricted our sample to one type of coursework at a time then used a chi-squared test to test the significance of the association between instructor and proportions of tasks of each level of cognitive demand. We also looked at ternary plots to graphically see where instructors' proportions of different proportions of cognitive orientation stood relative to one another.

5 Results

We organize the results into three sections. First we present basic descriptive information on the variable of interest: task counts and percentages of the different cognitive orientations. We follow this by looking at the association between cognitive orientation and types of coursework. Finally, we present analyses disaggregated by instructor according to type of coursework, looking at differences in proportions of levels of cognitive orientation.

5.1 Descriptive Information

Our original intention was to describe the nature of tasks students engage with in a semester of Calculus. Looking across the whole corpus ($N = 4{,}953$) we found that nearly half the tasks were Rich Tasks and Complex Procedures (see Table 2).

Table 2: Frequency and percent of tasks assigned by task orientation.

Task Orientation	*Frequency*	*Percent*
Simple Procedure	2,583	53%
Complex Procedures	916	19%
Rich Task	1,406	29%
Total	4,905[b]	100% [a]

[a]: Note that, due to rounding, the individual percentages do not add up to 100%.

[b] Excludes quizzes.

However, this corpus represents different instructors and different types of coursework. In the remainder of our analysis we look at how the proportions of tasks by cognitive orientation vary as we look at different slices of the task corpus.

5.2 Cognitive Orientation Across Coursework Types

Table 3 shows the frequency and percent of the different types of coursework assigned by teachers at this college. Tasks on quizzes were only a small portion of the tasks assigned to the students (1%, by only two instructors). For this reason, we did not include them in the remainder of the analysis. Note that 78% of tasks fall into either Bookwork or Webwork, while only 10% of tasks analyzed were in Exams, and 11% in Worksheets. These relative proportions are unsurprising; teachers may want to give nine tasks to students so they can practice and learn a new procedure/concept, but only one task to verify that it has been learned.

Table 3: Frequency and percent of tasks assigned by coursework.

Type of coursework	*Frequency*	*Percent*
Bookwork	3191	64%
Webwork[a]	701[a]	14%
Worksheets	538	11%
Quizzes	48	1%
Exams	475	10%
Total	4,953	100%

[a]: These tasks were assigned by only one instructor, Charles. He used the on-line platform that came along with the textbook, which were automatically graded by the system.

In our sample, only one instructor assigned problems through the web-platform (Charles, 701 tasks); he assigned Webwork in place of Bookwork. Because Webwork fulfills a similar role to that of Bookwork tasks (high-volume opportunities to learn and practice at low- or no-stakes, comparable grading stakes, unlimited access to time and resources), we compared Charles' Webwork tasks with the other instructors' Bookwork tasks (see Table 4). We found that the proportions of cognitive orientations of the tasks in Charles' Webwork assignments were similar to the proportions found in the full sample; further, a chi-squared test found that they were not significantly different ($\chi^2(2, N = 3,892) = 1.36, p > .50$). For this reason we included Charles's Webwork tasks in our further analysis of Bookwork tasks.

Table 4: Frequency and Percent of webwork and bookwork by cognitive orientation.

	Simple Procedure		*Complex Procedure*		*Rich Task*		*Total*	
	N	*%*	*N*	*%*	*N*	*%*	*N*	*%*
Webwork (1 instructor)	393	56%	135	19%	177	25%	701	100%
Bookwork (4 instructors)	1715	54%	676	21%	800	25%	3,191	100%

Table 5 presents the frequency and percent of tasks of each orientation across different types of coursework, Bookwork/Webwork, Worksheet, and Exam. There is a significant association between coursework and proportions of tasks of different cognitive orientations ($\chi^2(4, N = 4,905) = 158.73, p < .001$). That is, the observed proportions of cognitive orientations of tasks vary across coursework in ways that are not due to chance.

Table 5: Frequency and percent of tasks assigned by coursework and by task orientation.

	Simple Procedure		Complex Procedure		Rich Task		Total	
	N	*%*	*N*	*%*	*N*	*%*	*N*	*%*
Bookwork[a]	2,104	54%	811	21%	977	25%	3,892	100%
Worksheet	288	54%	52	10%	198	37%	538	100%
Exam	191	40%	53	11%	231	49%	475	100%
Total[b]	2,583	53%	916	19%	1406	29%	4905	100%

[a] Includes Webwork. [b] Excludes Quizzes.

As can be seen in Table 5 Bookwork and Worksheets comprised over half simple procedures (54%). Bookwork comprised only 25% rich tasks. Exams, on the other hand, tended to include a much higher proportion of rich tasks, nearly 50%, yet still a sizable proportion of simple procedures (40%).

5.3 Cognitive orientation across different instructors

As faculty and students in math departments know, students taking the same course at the same institution can have different experiences depending on the instructors they have. Our analysis confirms that this is the case at Suburban College when we look at the level of tasks instructors assign. There is significant variation ($\chi^2(8, N = 4{,}905) = 70.94, p < .001$) in the proportions of tasks of different cognitive orientations by instructor (see Table 6). That is, the observed frequencies of task orientation vary across instructors in ways that are not due to chance.

Table 6: Frequency and percent of tasks assigned by instructor and by task orientation aggregated across all types of coursework. The sample includes Webwork and excludes Quizzes.

	Simple procedure		Complex procedure		Rich task		Total	
	N	*%*	*N*	*%*	*N*	*%*	*N*	*%*
Albert	642	47%	257	19%	459	34%	1358	100%
Bob	282	61%	105	23%	76	16%	463	100%
Charles	514	57%	158	17%	233	26%	905	100%
David	422	48%	171	20%	278	32%	871	100%
Ethan	723	55%	225	17%	360	28%	1308	100%
Total	2583	53%	916	19%	1406	29%	4905	100%

For example, although the proportion of complex procedures is very similar across instructors, Bob assigns fewer rich tasks (16%) than what could be expected by chance in the sample (29%) and Albert assigns more (34%). Aggregating across all coursework, however, can mask distinctions between how instructors design different kinds coursework. We present those analyses next: bookwork, worksheets, and then exams.

5.3.1 Bookwork

There were 3,892 tasks assigned as Bookwork/Webwork. Overall, about 54% of tasks assigned from the textbook were Simple Procedures. Bob tended to assign the highest percentage Simple Procedure tasks (62%), whereas Albert assigned the least, 48%. The other three instructors assigned between 52% and 58% of Bookwork tasks that were Simple Procedures (See Table 7). Note how few Rich

Tasks Bob assigned in Bookwork (15%) relative to the other instructors (from 23% to 31%). We found that the association between instructor and cognitive orientation in Bookwork was statistically significant ($\chi^2(8, N = 3{,}892) = 67.35$, $p < .001$).

Table 7: Frequency and percent of tasks assigned in bookwork by instructor. The sample includes Webwork and excludes Quizzes.

Bookwork	*Simple procedures*		*Complex procedures*		*Rich tasks*		*Total*	
	N	*%*	*N*	*%*	*N*	*%*	*N*	*%*
Albert	536	48%	240	21%	342	31%	1118	100%
Bob	249	62%	93	23%	60	15%	402	100%
Charles	393	56%	133	19%	175	25%	701	100%
David	335	52%	149	23%	158	25%	642	100%
Ethan	595	58%	194	19%	240	23%	1029	100%
Total	2108	54%	809	21%	975	25%	3892	100%

Because we are working with three categories of cognitive orientation, these figures can be illustrated in a ternary plot (see Figure 5). A ternary plot allows us to graph proportions of three quantities that sum to 100%. In our case, the three quantities are the categories of cognitive orientation and the category names can be seen at the vertices of the triangle. Rich Tasks is located in the upper corner of the triangle. The light horizontal lines represent percentage levels associated to Rich Tasks. Thus Albert, Charles, David, and Ethan have over 20% Rich Tasks (because their markers are above the 20% Rich Tasks line), but Bob (the open triangle) is below it. Bob's tasks (the open triangle) are over 60% Simple Procedures (looking at the level lines going from lower right to upper left) and just over 20% Complex Procedures (looking at the level lines going from lower left to upper right). The precise percentages can be seen in Table 7. This plot provides a graphical view of how the instructors stand relative to one another along the three categories of cognitive orientation of their tasks. Notice how overall the markers are clustered towards the corner of Simple Procedures, suggesting that, in general, one intention of homework is to give students opportunities to practice the procedures they need to learn (in contrast to challenging their conceptions). The plot also shows how, in spite of this, there are differences by instructors, with some being a bit closer to either the complex procedures vertex (David), or the rich task vertex (Albert).

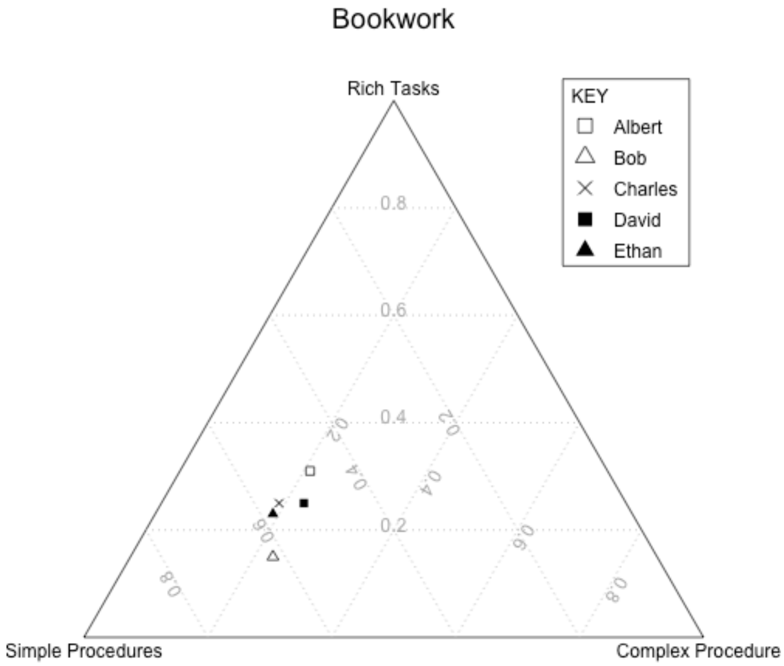

Figure 5: The ternary plot illustrates the proportions of three quantities that add up to 100%, in this case, the categories of cognitive orientation for the set of tasks assigned by each instructor. A point in the Rich Tasks vertex (upper corner of the triangle) represents 100% of Rich Tasks and 0% of the other two categories. Each horizontal lines (parallel to edge opposite to the Rich Tasks vertex) represents the percentage levels associated to Rich Tasks. Albert, Charles, David, and Ethan have over 20% Rich Tasks (because their markers are above the 20% Rich Tasks line); Bob's marker (the clear triangle) is below it. Bob's clear triangle is over the 60% line for Simple Procedures and just over the 20% line for Complex Procedures. The precise percentages are given in Table 7.

These instructor differences in Bookwork tasks are interesting because the instructors used the same textbook (Stewart 2012). This suggests that choice of textbook does not completely dictate the characteristics of the tasks instructors will choose to assign, and, consequently, it will not dictate the characteristics of the tasks students engage with.

5.3.2 Worksheets

When the context is worksheets (N = 538) instructors tend to assign slightly fewer Simple Procedure and slightly more Rich Tasks (see Table 8) than in Bookwork. Charles assigns the greatest proportion of Simple Procedures (65%) and the smallest proportion of Rich Tasks (23%) whereas the other three all assign around 50% Simple Procedures and 40% Rich Tasks. The association between instructor and item orientation in Worksheets is significant ($\chi^2(6, N = 538) = 12.91, p < .05$). That is, the proportions of cognitive orientations on Worksheet tasks vary by instructors in ways that do not depend on chance.

Worksheets	*Simple Procedures*		*Complex Procedures*		*Rich Tasks*		*Total*	
	N	*%*	*N*	*%*	*N*	*%*	*N*	*%*
Albert	56	52%	9	8%	42	39%	107	100%

Charles	80	65%	15	12%	29	23%	124	100%
David	68	49%	12	9%	59	42%	139	100%
Ethan	84	50%	16	10%	68	40%	168	100%
Total	288	54%	52	10%	198	37%	538	100%

Note: Bob did not submit worksheets.

These findings are illustrated in the ternary plot in Figure 6. Notice how the markers for Albert, David, and Ethan are clustered together, whereas Charles's marker is separate. The overall trend in worksheets is also apparent: few Complex Procedures, about 40% Rich Tasks (for the cluster of three instructors), and over 50% Simple Procedures.

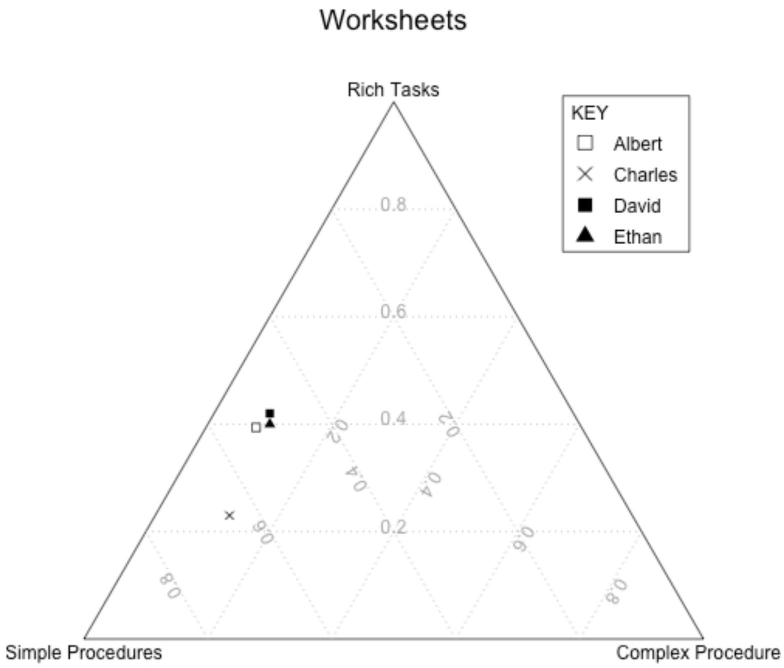

Figure 6: Ternary plot showing the proportion of the three dimensions of cognitive orientation of tasks instructors assigned in worksheets. See Figure 5 for an explanation of how to read the plot.

5.3.3 Exams

When the context is Exams ($N = 475$), differences between instructors are again observable (Table 8), but instructors in general tend to assign fewer Simple Procedures in the Exams relative to Complex Procedures and Rich Tasks: Bob and Charles assign the greatest number of Simple Procedures (54% and 56% respectively); Albert and Ethan around 40% (38% and 40% respectively), and David assigns the fewest (21%). When we look at Rich Tasks, we see percentages on Exam tasks ranging from 26% (Bob) to 68% (David). The association between instructor and item orientation in Exams is significant ($\chi^2(8, N = 475) = 42.65$, $p < .001$).

Table 8: Frequency and percent of tasks assigned in all exams, by instructor.

Exams	*Simple*	*Complex*	*Rich*	*Total*

	Procedures		*Procedures*		*Tasks*			
	N	*%*	*N*	*%*	*N*	*%*	*N*	*%*
Albert	50	38%	8	6%	75	56%	133	100%
Bob	33	54%	12	20%	16	26%	61	100%
Charles	45	56%	8	10%	27	34%	80	100%
David	19	21%	10	11%	61	68%	90	100%
Ethan	44	40%	15	14%	52	47%	111	100%
Total	191	40%	53	11%	231	49%	475	100%

These results are displayed in Figure 7. Unlike what we saw for Bookwork and Worksheets, the instructors are not clustered. Although their exams can altogether be characterized by having few complex procedures tasks, there is definitive difference in the proportions of Rich Tasks and Simple Procedures.

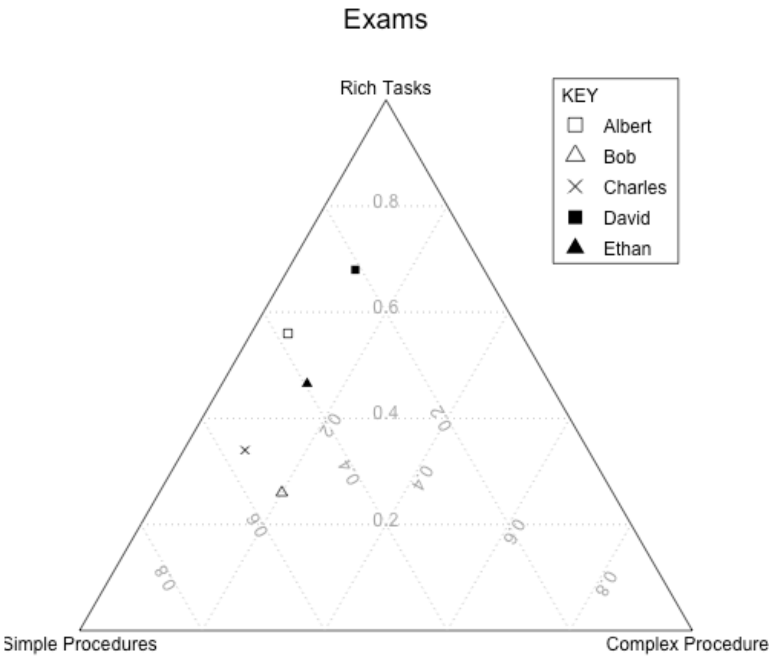

Figure 7: Ternary plot showing the proportion of the three dimensions of cognitive orientation of tasks instructors assigned on Exams. See Figure 5 for an explanation of how to read the plot.

6 Discussion

We organize the discussion by the three research questions we posed.

RQ1: What is the cognitive orientation of the tasks that instructors give to their students?

We found that nearly half of tasks in the corpus were Complex Procedures or Rich Tasks. This result suggests that, looking across all instructors and types of coursework, the Calculus I program at Suburban College puts emphasis on tasks covering the spectrum of cognitive orientations. This is unsurprising when we take into account what we know about Suburban College: it was identified by the CSPCC as being successful and it has a long history of Calculus I in their department, including a large intentional overhaul in the 1990s, a change that still influences their department attitude towards Calculus I today.

RQ2: What is the cognitive orientation of different types of coursework that instructors give to their students? Are there differences between types of coursework?

First, looking at homework-type tasks, we did not find differences in the cognitive orientations of tasks in Webwork versus Bookwork. This suggests to us, that the choice to use a Webwork platform might not necessarily change how instructors select tasks, that is, if they tend to assign a majority of Simple Procedures tasks, they will do so in either platform. Although students' access to resources while completing homework online can alter the cognitive engagement of the student from that intended by the task design (e.g., because they use the "see an example" feature), it seems that the availability of the homework in various platforms does not result in faculty changing the complexity of the homework assigned.

Overall, comparing cognitive demand across coursework types (Bookwork/Webwork, Worksheets, Exams), we found statistically significant differences.

Bookwork/Webwork had a higher proportion of Simple Procedures and low proportion of Rich Tasks relative to other coursework types. We believe this may point to instructors' understanding of how students build procedural skills—through high-volume practice—and to the central role they give the Bookwork/Webwork in providing that practice. Indeed, instructors openly recognized this role of homework during interviews:

> Interviewer: What about the homework problems from the book? What are you hoping they get mathematically out of those?
>
> Albert: Practice. You know? A lot of math is repetition. If they work enough problems they… You do the homework you're trying to learn.

When we asked Bob where his student learned skills like taking a derivative, he replied:

> Bob: Typically from the, from the online homework. So the homework to just drill 'em with things. So on the online homework we're drilling them, anywhere between 30 to 40 questions a week. Typically. And they just have to just and they're very repetitive…

David also expressed something similar, but further acknowledged bookwork as a place for students to engage with applications and modeling:

> Interviewer: What is it mathematically that you want students to get out of the book problems?
>
> David: Uh the problems? I want them to understand the basic manipulations of algebraically… algebraic manipulations basically. That's for a homework, then the, uh, I choose basic ones. And also there are lots of application problems, so how to apply it, that stuff each student learned, to realistic situations.

However, about one fourth of the tasks assigned in Bookwork/Webwork were indeed Rich Tasks. This corresponds roughly to 249 to 595 tasks in a semester, depending on the instructor. Thus, as instructors design their homework, they seem to be aware that they need to give students opportunities to learn more cognitively demanding skills. As students at this site indicated that they believe that doing the homework is essential to their success in the course, this suggests that these students are indeed being exposed to a substantial number of conceptual tasks. Doyle points to two consequences of completing tasks: "first, a person will acquire information—facts, concepts, principles, solutions…Second, a person will practice operations...(Doyle, 1983, p. 162)" It appears instructors at our site see both of these purposes in Bookwork/Webwork tasks in looking at the cognitive demand of the homework tasks they assign.

The proportions were different when we analyzed Exams; they included relatively more Rich Tasks than Simple Procedures across all instructors. We propose several possible reasons for this contrast to Bookwork/Webwork. First, although instructors will need some evidence that students have mastered basic procedures, they may choose to use limited exam time to assess more complex work. Second, complex work might in itself require the use of simpler procedures, thus fulfilling other needs in the assignment. Third, instructors may need fewer examples to assess students' basic skills. That is, an instructor might assign 100 tasks for students to learn to differentiate, but use only two tasks of this type on an exam to assess proficiency. In contrast (due to quality of engagement and constraints on time), and instructor might assign only 10 related rates problems as learning opportunities, but use just as many assessment tasks on this content as for the more procedural content in the courses. Fourth, we do not take into account student exposure to specific Rich Tasks; routine exposure to specific tasks could decrease their cognitive demand when seen on Exams. Further investigation on the nature of assessment, on how instructors conceptualize them and use them is needed to understand this phenomenon at our site.

Although these reasons may explain why we saw such a high proportion of Rich Tasks on Exams (49%) compared to homework, such a high proportion of Rich Tasks on Exams should not be taken for granted. Indeed, in the Tallman et al. (2012) analysis only 15% of the 3,735 exam tasks "required students to demonstrate an understanding of an idea or procedure (p. 15)." That is, at most 15% were rich tasks.[7] When they restricted their analysis to the two-year colleges in their sample (10% of their sample institutions), this proportion of Rich Tasks drops to 11%. Even taking into account inherent differences between the coding, this difference is striking.

RQ3: Can we detect differences between instructors at the same institution by characterizing the cognitive orientation of their tasks? Further, do instructors display preferences for higher or lower cognitive orientation of tasks that persist across different types of coursework?

---

[7] Recall that our categorization includes an extra category not accounted for in the Tallman et al.'s analysis. Our Rich Task category is a subset of their category "required students to demonstrate an understanding of an idea or procedure." This means that our proportions of Rich Tasks are conservative estimates of what their framework would produce.

Research in K-12 settings has documented that there is less variance in students' achievement on standardized tests across schools than within schools (Lee, Croninger and Smith 1997). That is, schools across a single district, for example, are very similar to each other, but within schools there is substantial variation that is solely dependent on the instructors in the school. Such variation has also been documented in trigonometry teaching at community colleges (Mesa and Lande in press). In our study we found significant differences in the relative proportions of tasks' cognitive orientation by instructor.

Though not large, the Bookwork discrepancies are most surprising. These discrepancies reveal instructors' different orientations towards assigning cognitively demanding work even when using the same department-mandated textbook. This might be connected to instructors' own perceptions of calculus as a subject or about their perception of its role in preparing students for more advanced work. It might also be connected to teachers' prior experiences with mathematics (either applied or research), or their assumptions about the students that are taking the course (e.g., their cognitive abilities and future careers). It might be that some instructors do not see calculus as a place in which conceptual work is needed, rather that it is a place for developing proficiency with procedures. We do not have means to tease out the driving forces behind these different results, but we do see consistency within instructors' own selection of tasks.

In practical terms, the marked differences we see between exam tasks of different instructors mean that students, all taking Calculus I at the same institution, will be demonstrating competency in their exams by different standards. A high score in an exam by David will be taking into account a larger share of rich tasks and complex procedures, whereas a high score in an exam by Bob or Charles will represent students who might be more proficient with simple procedures. Thus, students are being assessed by different requirements in these classes.

This particular finding raises also interesting questions about the role of the textbook in mathematics courses at the tertiary level. In our interviews of the selected institutions, instructors and chairs indicated that the selection of the textbook was a process that was taken very seriously, as it not only benefitted the students, but also allowed for internal consistency in the course. That is, the instructors and administrators indicated that the single textbook policy was an effective mechanism for controlling curriculum, especially when there were many instructors teaching the same course. However, this particular finding suggests that choosing the same textbook may not be as a strong influence in the curriculum (see Lattuca and Stark 2009 for similar acknolwedged influence of the textbook in higher education). It might be the case too, that even with a more reform oriented textbook (e.g., Hughes-Hallett, Gleason, McCallum and Others 2005; Ostebee and Zorn 2002) little change might actually occur.

7 Conclusion

In this article we have illustrated that a framework for analyzing the cognitive orientation of mathematical tasks suggests that including coursework beyond exams and comparing multiple instructors can provide a richer picture of students' opportunities to learn within a single Calculus program.

This analysis provides us with baseline information to contrast with the rest of the institutions (n = 17) in the CSPCC project; in addition to simply analyzing cognitive demand across types of institutions, we will also look at patterns of cognitive demand across coursework and patterns of cognitive demand across instructors as ways to compare Calculus I programs.

In terms of broader research, this analysis points to four areas of further study outside of the CSPCC context. First, to more fully understand instructors' beliefs behind their tasks selection and design, it would be important to explore further the relationship between instructors' task orientation "signatures" and other characteristics, such as their training in teaching, their ideas about teaching and learning, or their years of experience. Second, to more fully capture opportunities to learn, we would be interested in seeing how tasks, even those worked at home, are supported by instructors in the classroom. Third, we could look in more detail about how students use resources out of class and how that affects the cognitive demand of problems worked at home. Lastly, we believe that this framework is a low-cost instrument that could be used in other subjects/contexts beyond Calculus to explore students' opportunities to learn as well as instructors' learning goals and orientations towards task selection/design.

We point to two implications for practice. First, we find it notable that more tasks of high cognitive demand were assigned when students had access to fewer resources and that such tasks counted with greater weight in the course grade, namely on Exams. This may mean that students may not learn how to use resources for solving more complex tasks or may not realize the importance of such tasks in the subject until it is "too late." We see a greater role for Worksheet/Lab activities to manage these two issues: they can be a context for instructors to assign cognitively demanding tasks to be completed with more resources (time, peers, teacher's assistance) and with a higher grade weight, relative to homework. The higher grade weight would signal the importance of these tasks for learning the material. Second our findings serve as a caution to curriculum committees about the assumption that choosing a common textbook necessarily will serve as a mechanism for aligning instructors' enactment of learning goals within a department. There needs to be other mechanisms for ensuring such alignment.

8 References

Anderson, L. W., Krathwohl, D. R., & Bloom, B. S. (2001). *A Taxonomy for Learning, Teaching, and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives*. Boston, MA: Ally & Bacon.

Bloom, B. S., & Krathwohl, D. R. (1956). *Taxonomy of educational objectives: The classification of educational goals. Handbook I: Cognitive domain*. NY, NY: Longmans, Green.

Bressoud, D. M., Carlson, M. P., Mesa, V., & Rasmussen, C. (2013). The calculus student: insights from the Mathematical Association of America national study, (0910240), 685–698.

Charalambous, C. Y., Delaney, S., Hsu, H.-Y., & Mesa, V. (2010). A Comparative Analysis of the Addition and Subtraction of Fractions in Textbooks from Three Countries. *Mathematical Thinking and Learning*, *12*(2), 117–151. doi:10.1080/10986060903460070

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: development status and directions. *Zdm*, *45*(5), 633–646. doi:10.1007/s11858-013-0539-x

Herbst, P., & Chazan, D. (2011). Research on Practical Rationality: Studying the Justification of Actions in Mathematics Teaching. *Montana Mathematics Enthusiast*, *8*(3).

Krathwohl, D. R. (2002). A Revision of Bloom ' s Taxonomy : *Theory into Practice*, *41*(4), 212–218. doi:10.1207/s15430421tip4104

Li, Y. (2011). A Comparison of Problems That Follow Selected Content Presentations in American and Chinese Mathematics Textbooks. *Journal for Research in Mathematics Education*, *31*(2), 234–241.

Lithner, J. (2000). Mathematical Reasoning in Task Solving. *Educational Studies in Mathematics*, *41*(2), 165–190.

Lithner, J. (2004). Mathematical reasoning in calculus textbook exercises. *The Journal of Mathematical Behavior*, *23*(4), 405–427. doi:10.1016/j.jmathb.2004.09.003

Mesa, V., Gomez, P., & Cheah, U. (2013). Influence of international studies of student achievement on mathematics teaching and learning. In *Third International handbook of mathematics education* (pp. 861–900). Springer New York.

Piaget, J. (1968). *Six Psychological Studies*. (Trans. A. Tenzer., Ed.). New York, NY: Vintage Books.

Rico, L., Castro, E., Castro, E., Coriat, M., & Segovia, I. (1997). Bases Teóricas del Currículo de Matemáticas en Educación Secundaria [Theoretical basis for mathematics curriculum in secondary education]. *Madrid: Editorial Síntesis*, 1–11.

Smith, G., Wood, L., Coupland, M., Stephenson, B., Crawford, K., & Ball, G. (1996). Constructing mathematical examinations to assess a range of knowledge and skills. *International Journal of Mathematical Education in Science and Technology*, *27*(1), 65–77.

Stein, M. K., & Lane, S. (1996). Instructional Tasks and the Development of Student Capacity to Think and Reason: An Analysis of the Relationship between Teaching and Learning in a Reform Mathematics Project. *Educational Research and Evaluation*, *2*(1), 50–80.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (Eds.). (2000). *Implementing Standards-Based Mathematics Instruction: A Casebook For Professional Development*. New York: Teachers College Press.

Stewart, J. (2010). *Calculus: Early Transcendentals* (7th ed.). Cengage Learning.

Tallman, M., & Carlson, M. P. (2012). A Characterization of Calculus I Final Exams in U.S. colleges and Universities. In *Proceedings of the 15th Annual Conference on Research in Undergraduate Mathematics Education* (pp. 217–226). Portland, OR: Portland State University.

Valverde, G. A., Bianchi, L. J., Houang, R. T., Schmidt, W. H., & Wolfe, R. G. (2002). *According to the Book: Using TIMSS to investigate the translation from policy to pedagogy in the world of textbooks*.