Preface

Initial Comments

This volume has its origins in a contributed papers session, "Innovative Methods in Courses Beyond Calculus," held at Mathfest 2001. This session was organized to discuss the following question:

> What can be done to generate and then maintain student interest in the mathematics courses that follow calculus?

Presentations were made by faculty who are addressing this question by doing "something different" in these courses and having a good deal of success with what they are doing. At that time, it was suggested to me that some of the papers presented at that session, when combined with a number of solicited outside papers, might make a useful contribution to the MAA Notes series. The result is this volume, *Innovative Approaches to Undergraduate Mathematics Courses Beyond Calculus,* which contains a wide range of papers that encourage students to take an active role in the learning process and to stretch their learning to ideas and concepts not presented in the classroom.

There is a real need for material of the type contained in this volume, a need that is reinforced by the **CBMS 2000 Survey**. This survey indicates that the number of mathematics majors continues to decline, even though enrollments in Calculus I and II and in the standard second year courses are increasing. Furthermore, a growing number of students in the life and social sciences are seeking more mathematical training in connection with their own disciplines. We should be teaching these students; they should not end up getting this material in courses offered within their own departments with titles like "Mathematical Methods in XX." A similar comment applies to students in the physical sciences and engineering. We have to attract students in all these areas but we are not doing so, even though we are in the midst of an era of increasing college enrollments.

If we are to obtain the results that we desire, we have to rethink what we are doing. We have to make our courses more interesting and more attractive to students. But making courses more interesting is not synonymous with hand waving or watering down content. The authors of the articles in this volume show how we can do all this and still teach **mathematics** courses. These articles introduce new material, offer a variety of approaches to a broad range of courses, and even bring students into contact with ongoing research both in mathematics and in other disciplines. Furthermore, this exposure can take place as early as the second semester of the sophomore year. The articles point out how students can work both individually and in collaboration in order to stretch their mathematical boundaries. This stretching can be accomplished in a variety of ways but the end result is always the same: students leave the course having done something beyond the material covered in the classroom. This volume does have one ongoing theme: it is possible to excite our students about mathematics. But to do so, it is not enough for students to sit and listen to lectures about mathematics. They must actually DO mathematics.

Some Specifics

The multifaceted nature of *Innovative Approaches to Undergraduate Mathematics Courses Beyond Calculus* allows it to address a variety of needs. Faculty trying to revitalize their major offerings will find a host of helpful ideas. Instructors seeking new ways to approach the courses they teach will find a number of models that they can either adopt or adapt. Individuals looking for ideas to incorporate into a specific course will find a wealth of suggestions both in the articles themselves and in their associated references. And teachers who want to expose their students to current mathematical activity will find several avenues to follow. The net result is a volume that offers a variety of options to meet a broad range of needs.

Each of the papers in this volume indicates how the approach discussed can be incorporated into different courses. Specific references are made to twenty-four different upper division mathematics courses, with double references to six of them. These courses include abstract algebra, applied mathematics, biostatistics(two different courses), combinatorics, differential equations, discrete mathematics, game theory, geometry, graph theory, group theory, history of mathematics, linear algebra, mathematical biology, module theory, multivariable calculus, number theory, probability, real analysis, statistics, and topology, along with three capstone courses. In addition, the interdisciplinary applications that are cited involve biology, computer science, economics, engineering, physics, and the social sciences. The reference section of each paper includes additional material. Each article discusses assessment and mentoring. Several of the articles, in addition to the references, also include extensive bibliographies. In general, every effort has been made to make the contents as transparent as possible.

The First Chapter

The first chapter of this volume contains five papers with approaches that are applicable in a variety of courses:

1) *Using Writing and Speaking to Enhance Mathematics Courses*: In this paper, Nadine Myers of Hamline University discusses both writing intensive and oral intensive methods that have been effective in Linear Algebra, Modern Algebra, and Modern Geometry, as well as in structuring a capstone course (Topics in Advanced Mathematics). This approach requires students to write and present proofs frequently and often involves student research on an advanced topic coupled with written and oral presentations. An extensive set of guidelines is included to assist in implementing such an approach.

2) *Enhancing the Curriculum using Readings, Writing, and Creative Projects*: A variety of student-oriented activities can be used to enhance student learning in advanced courses. In this paper, Agnes Rash of St. Joseph's University indicates a number of methods that have been used successfully in Discrete Structures, Group Theory, Number Theory, and Probability and Statistics, as well as in a capstone course involving student research. This article includes a host of specific examples and several extensive reading lists. There is a strong emphasis on student presentations.

3) *How to Develop an ILAP*: The applications of mathematics, while powerful motivators in and of themselves, can have even more impact if they are developed in conjunction with departments that use mathematics within their own disciplines. In this article, Michael Huber and Joseph Myers, both of the United States Military Academy, provide a detailed description of how to work with colleagues in other departments to construct ILAP's(Interdisciplinary Lively Application Projects). The article includes details on how to select a suitable topic, how to work in conjunction with a partner department, what material to include in the student handout, mentoring students during the project, and how to organize student presentations of the results. It also provides several examples of ILAP's and contains references to many others.

4) *The Role of the History of Mathematics in Courses beyond Calculus*: The history of mathematics can provide a good deal of content motivation if it is integrated seamlessly into the course; it cannot appear to be just thrown in. This article, by Herbert Kasube of Bradley University, discusses how such an integration can be accomplished in a variety of courses beyond Calculus, including Abstract Algebra, Combinatorics, Graph Theory, and Number Theory. This paper also provides an extensive bibliography of source material.

5) *A Proofs Course that Addresses Student Transition to Advanced Applied Mathematics Courses*: This paper, by Michael Jones and Arup Mukherjee of Montclair State University, is unique in that it describes a proofs course directed toward a specific curriculum. While emphasizing the construction of proofs, the approach described also encourages students to go through a process that moves from exploration to conjecture to proof in a specific curricular area. It often uses technology to motivate or consolidate ideas.

The Second Chapter

The second chapter of this volume contains five articles that, while more course specific, also contain approaches that are adaptable in other courses.

6) *Wrestling with Finite Groups: Abstract Algebra need not be Passive Sport*: Abstract Algebra is, by its nature, abstract. But it does not have to be approached as a list of definitions and theorems that need to be verified. In this paper, Jason Douma of the University of Sioux Falls discusses how an abstract algebra course can be structured around an open-ended research project. The project is not an application of material covered in class but rather a basis for motivating the actual course content. The paper provides all the details needed to implement such an approach, including information on organization, classroom activity, and assessment.

7) *Making the Epsilons Matter*: Students all too often view an introductory real analysis course as a mechanism that provides the theoretical foundation for calculus results that they already have accepted intuitively. In this paper, Stephen Abbott of Middlebury College describes how an introductory real analysis course can be used to challenge and sharpen intuition as opposed to merely verifying it. He also shows how these outcomes can be reached by a shift of emphasis and not necessarily content, leaving the students with a thorough grounding in the basic concepts of continuity, differentiability, integrability, and convergence.

8) *Innovative Possibilities for Undergraduate Topology*: In this paper, Samuel Smith of St. Joseph's University approaches the undergraduate course in topology as one intended for a broad range of majors and not just those planning on graduate study. To achieve this outcome, the author describes in detail how to structure a course in which an initial geometric approach can be used to motivate the axiomatic structure that characterizes topology. A major goal is to maximize the number of ideas that the students discover for themselves. The possibility of using topology as a capstone course also is explored.

9) *A Project-Based Geometry Course*: Geometry is an axiomatic subject but these axioms need not always be presented in lecture mode. In this paper, Jeff Connor and Barbara Grover, both of Ohio University, discuss a geometry course in which the students develop their own axiom systems, using technology when appropriate. The students receive early experience with both Euclidean and non-Euclidean geometries and also obtain the intellectual tools that they will need to learn any new and unfamiliar mathematics.

10) *Discovering Abstract Algebra: A Constructivist Approach to Module Theory*: Students in upper division mathematics courses can profit from guided discovery, an approach that encourages students to construct their own knowledge and choose their own course of study, while retaining subtle guidance on the part of the teacher to generate definitions, examples, and eventually theorems. In this article, Jill Dietz of St. Olaf's College discusses how to use such an approach in a course on Module Theory that is taught as a follow-up to a first course in Abstract Algebra. It begins with the initial question: "What happens if we replace the vector space axiom that requires an action of a field on an additive abelian group with the

new requirement that there is instead a ring action on an additive abelian group?" The course then uses guided discovery to generate results in what the students eventually find out is module theory.

The Third Chapter

The third chapter of this volume also contains five papers. The first two describe courses relating mathematics and biology, while the third and fourth papers discuss voting theory. The last paper in this chapter discusses a course in Classical Applied Mathematics in which technology is an integral component.

11) *The Importance of Projects in Applied Statistics Courses*: Advanced Statistics courses, particularly those directed toward biology students, no longer are places where formulas are emphasized and data summarized. The focus now is, or should be, on the importance of statistics in providing legitimate answers to the questions posed by researchers. In this paper, Timothy O'Brien of Loyola University Chicago discusses how projects that not only study the assumptions and limitations inherent in research studies but also require student to address statistical topics that are not a part of the standard curriculum greatly expand student learning. These projects may require the analysis of a previously unstudied data set or a critique of original research articles from refereed journals. They also must involve or require the use of techniques that are beyond those covered in the formal classroom presentation.

12) *Mathematical Biology Taught to a Mixed Audience at the Sophomore Level*: Most Mathematical Biology courses are either modeling courses designed for upper division mathematics majors or lower level courses, often with minimal mathematical prerequisites, for biology majors. In this article, Janet Andersen of Hope College describes a team taught course in Mathematical Biology that serves both mathematics and biology majors. The prerequisite for the mathematics majors is a course in Linear Algebra and Differential Equations while the biology majors are required to have one semester in Calculus plus a sophomore level course on Ecology and Evolutionary Biology. The course is based on biology research papers that use matrix analysis or differential equations in their development. Class requirements include both collaborative work and classroom presentations.

13) *A Geometric Approach to Voting Theory for Mathematics Majors*: Voting theory can be incorporated into a variety of upper division mathematics courses, which can allow students to obtain some insight into ongoing mathematical research and its outcomes. In this article, Tommy Ratliff of Wheaton College discusses how this can be done in a course that also covers game theory. With discrete mathematics as a prerequisite, this course delves into the geometric framework that underlies some of the recent results obtained in voting theory. One outcome is to help students become better judges of the choice procedures available to them in their everyday lives.

14) *Integrating Combinatorics, Geometry, and Probability through the Shapley-Shubik Power Index*: Voting theory is a rapidly developing area of mathematics with a broad range of applications both inside and outside of mathematics. This paper, by Matthew Haines of Augsburg College and Michael Jones of Montclair State University, serves as a primer for instructors who wish to introduce the elements of voting theory into their courses. This article also discusses how its contents can be applied in several upper division courses.

15) *An Innovative Approach to Post-Calculus Classical Applied Math*: Classical Applied Mathematics typically is considered to be a formal development of the theorems and problem-solving techniques of applied analysis. In this article, Robert Lopez, formerly of Rose-Hulman Institute of Technology and now with Maplesoft, indicates why a computer algebra system should be the working tool for teaching, learning, and doing classical applied mathematics. The result of such an approach is a richer, more efficient, and more effective learning system. One key point is that technology must be an integral part of the course that is available for all parts of the course, including examinations. Two in-depth examples are provided to illustrate the effectiveness of this approach.

Conclusion

Innovative Approaches to Undergraduate Mathematics Courses Beyond Calculus is intended to serve as a starting point both for those who plan to adopt or adapt the approaches it discusses and for those who plan to develop their own ideas. This volume contains fifteen papers that provide useful information on alternative methods that are being used with great success in the courses following calculus. These papers not only present new concepts and related applications that can be introduced into these courses but in several cases also bring students into contact with ongoing research. In all cases, the material is presented in detail. When the presentation is course specific, techniques for using the approach in other courses also are discussed. Much of the heavy lifting already has been done and all that remains is for instructors to adapt the suggestions to their own individual settings.

Hopefully, this volume will provide a body of information that will prove helpful to instructors teaching the courses that follow calculus. At this time, there does not appear to be anything in print that discusses how to generate and maintain student interest in the courses beyond calculus. While *Innovative Approaches to Undergraduate Mathematics Courses Beyond Calculus* may well be the first of its kind, it most certainly will not be the last. The need to make our major programs more attractive and to draw students outside our discipline into our courses will see to that.