

Preface

In 2000, the International Commission on Mathematical Instruction published the ICMI Study entitled *History in Mathematics Education*, edited by John Fauvel and Jan van Maanen [1]. This volume presented the state of the art in the use of the history of mathematics in mathematics education and made numerous suggestions for further research in the field. One of the major suggestions was that there should be more empirical studies of the use of history in the mathematics classroom to get more insight into its educational implications. In recent years many researchers have followed up on the recommendations of the ICMI Study and have presented their findings in numerous international meetings, including the meetings of the *International Study Group on the Relations Between History and Pedagogy of Mathematics* (the HPM Group) and the meetings of the *European Summer University on the History and Epistemology in Mathematics Education* (ESU)¹. To further the goal of making these new results available to a wider audience, including mathematics educators, mathematics faculty in secondary schools and universities, and historians of mathematics, the contributors to these meetings were invited to submit papers based on their presentations, which were peer-reviewed to international standards by two independent referees and then revised before final acceptance. We are indebted to the referees for their willingness to support this project and provide useful reports on the 50 submitted manuscripts. This volume consists of the 24 papers that were finally accepted (coming from 13 countries worldwide) and aims to constitute an all-embracing outcome of recent activities within the HPM Group. We believe these articles will move the field forward and provide faculty with many new ideas for incorporating the history of mathematics into their teaching at various levels of education. The book is organized into four parts. The first deals with theoretical ideas for integrating the history of mathematics into mathematics education. The second part contains research studies on the use of the history of mathematics in the teaching of numerous mathematics topics at several levels of education. The third part concentrates on how history can be used with prospective and current teachers of mathematics. We also include a special fourth part containing three purely historical papers based on invited talks at the HPM meeting of 2008. Two of these articles provide an overview of the development of mathematics in the Americas, while the third is a study of the astronomical origins of trigonometry.

Theoretical Ideas for Integrating History of Mathematics into Mathematics Education

Part I contains seven articles, from five countries, dealing with theoretical aspects of using the history of mathematics classroom. In the first, David Pengelley from the United States argues that virtually all courses should be taught using original sources, and gives various reasons in support of his position. Evelyne Barbin of France then introduces the notion of dialogism in considering how one can use such sources in the classroom. In other words, she suggests that students consider an original mathematical writing as a dialogue between the author and the intended audience. Next, Gustavo Martinez Sierra and Rocio Antonio Antonio from Mexico study how the production of new mathematical knowledge emerges from the necessity of new ideas agreeing with ones that are already part of our knowledge base. Luis Puig from Spain, on the other hand, takes a look at three historical sources in algebra, considering how the authors thought about cut and paste methods in solving quadratic equations. Giorgio Bagni, an Italian who was tragically killed in a bicycle accident during the preparation of this book, considers Bombelli's *Algebra* and how its content reflects an

¹These are: ESU 5, Prague, Czech Republic, 2007; the Topic Study Group 23 of ICME 11 on *The Role of History of Mathematics in Mathematics Education*, and the Affiliated Study Group-meeting of the HPM at ICME 11 in Monterrey, Mexico, 2008; the *HPM Group Satellite Meeting of ICME 11* (HPM 2008) in Mexico City, Mexico, 2008; the Working Group 15 on *The Role of History of Mathematics in Mathematics Education: Theory and Research* of the Congress of the European Society for Research in Mathematics Education (CERME 6), in Lyon, France, 2009.

emerging change in the nature of algebra. Renaud Chorlay of France considers four viewpoints in elementary function theory and shows how a historical consideration of these viewpoints can impact the education of students. Finally, again from Mexico, Gabriela Buendia Abalos and Gisela Montiel Espinosa consider trigonometry as an example of how the ambient culture affects the teaching and learning of significant mathematical ideas.

Implementing the History of Mathematics in Mathematics Education

In Part II, there are ten articles dealing with concrete uses of the history of mathematics in teaching mathematics at levels ranging from elementary school through university. In each case, the authors have conducted at least some preliminary research to test the effectiveness of their teaching methods and conclude that the use of history has a positive effect on their students. These ten papers come from seven countries on three continents, reflecting the international nature of the research. In many of these cases, teachers elsewhere should be able to modify the methods of the original authors for use in their own classrooms. We begin with an article by Man-Keung Siu from Hong Kong, China, who discusses a course for secondary students integrating physics and mathematics. Since physics and mathematics developed together to a large extent, this course is historically based. But students today are frequently not aware that mathematics has a history or even that anything new can be discovered. Next, Batya Amit, Nitsa Movshovitz-Hadar and Avi Berman in Israel show how one can introduce news from contemporary mathematics with its historical background into a secondary classroom, including such topics as Fermat's Last Theorem, Kepler's conjecture, and the mathematics of Sudoku. From Italy, we have a study by Adriano Demattè and Fulvia Furinghetti on the use of pictures of ancient documents in teaching secondary students. Then, there is a study by Luis Casas and Ricardo Luengo of Spain on the use of historical weights and measures in an elementary classroom. This activity involved the students not only in mathematics and its history, but also in the collecting of information from older members of their community. Next, Uffe Jankvist in Denmark studied how his students thought about the history of mathematics. Moving on to the university level, we have a study by Beverly Reed of the United States on how studying the history of the idea of a function positively affected her students' understanding of the concept. From Greece, we have a paper by Theodoros Paschos and Vassiliki Farmaki on how the study of the idea of motion in the later Middle Ages can help students better understand the concepts of the definite integral and the fundamental theorem of calculus. And then Tinne Kjeldsen from Denmark shows how original sources from the eighteenth century can be used in demonstrating to students the influence of physics on the development of differential equations. Returning to Greece, Michael Kourkoulos and Constantinos Tzanakis examine their statistics students' difficulties in understanding the concept of variance, i.e., why one uses the square root of the sum of the squares, and show how this difficulty is related to the historical development of the idea of variance and standard deviation. Finally, back in the United States, we have an article by Janet Barnett and others discussing the use of original sources in teaching courses in discrete mathematics.

Using the History of Mathematics in Teacher Education

Although teacher education was relevant to some of the earlier papers, in part III we present four articles dealing directly with the education of prospective teachers of mathematics. Each article shows the importance of the use of history in educating future teachers so they can meet the challenges of their own classrooms. In the first paper in this part, Bjørn Smestad of Norway discusses how to use history of mathematics in pre-service courses for elementary and lower secondary school teachers. Kathleen Clark of the United States discusses her history of mathematics course specifically designed to show prospective secondary teachers how to use history in their own teaching. Leo Rogers of the UK reflects on how recent changes in the English mathematics curriculum are now forcing university professors teaching pre-service teachers to figure out new ways to introduce relevant historical material. And finally, in a second article, Bjørn Smestad discusses a study of some Norwegian secondary teachers to understand how they actually use the history of mathematics in their classrooms.

Invited Papers on the History of Mathematics

The final part contains three important articles based on invited presentations on the history of mathematics at the HPM meeting in Mexico City in July, 2008. In the first, Karen Parshall of the United States discusses how the mathematical

research community in North America evolved from its beginnings in colonial times up to the middle of the twentieth century. Ubiratan D'Ambrosio from Brazil complements this article with a study of the evolution of the mathematics community in Latin America from the time of independence again up to the middle of the last century. Finally, we conclude with an article by Glen van Brummelen of Canada on the history of trigonometry, showing that its primary motivation was the study of the heavens.

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Bibliography

- [1] J. Fauvel & J. van Maanen, 2000, *History in Mathematics Education: The ICMI Study*, Kluwer Academic Publishers, Dordrecht.