

**FOCUS** is published by the Mathematical Association of America in January, February, March, April, May/June, August/September, October, November, and December.

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Periodicals postage paid at Washington, DC and additional mailing offices. **Postmaster:** Send address changes to FOCUS, Mathematical Association of America, P.O. Box 90973, Washington, DC 20090-0973.

ISSN: 0731-2040; Printed in the United States of America.



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## William McCallum and Ken Ono Named Distinguished Teaching Scholars

On June 21, the National Science Foundation named seven Distinguished Teaching Scholars, honoring scholars who have achieved success in both research and teaching, and who have successfully integrated the two. Among the recipients are two mathematicians: William McCallum of the University of Arizona (and a member of the MAA) and Ken Ono of the University of Wisconsin Madison. The awards, which are worth up to \$300,000 over four years, are “NSF’s recognition of accomplishments by scientists and engineers whose roles as educators and mentors are considered as important as their ground-breaking results in research,” said NSF Director Arden L. Bennet, Jr. The Distinguished Teaching Scholars program has existed since 2001, and has so far honored 34 people.

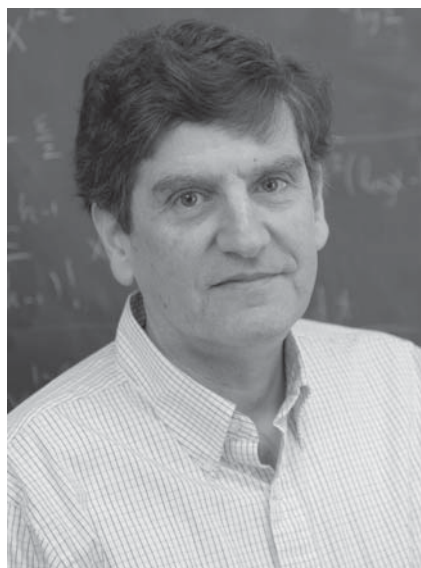
William McCallum’s research work is in number theory and arithmetic algebraic geometry, but he is best known as a leader in the calculus reform movement and one of the main authors of the “Harvard” calculus textbook. He has also been involved with the Arizona Winter School on Arithmetic Algebraic Geometry, which has made this area accessible to countless graduate students and scholars wishing to learn more about the subject. NSF reports that “His new work will focus on better communication among mathematicians, teachers and math education researchers in a systematic content analysis of problems in algebraic thinking that should lead to new instructional materials for a broad range of students.”

Ken Ono’s work has focused on modular forms and their relations with elliptic curves. He has obtained remarkable results on congruence properties of the partition function that are closely related to congruences discovered by Ramanujan many years ago. His work with undergraduates has successfully involved them in research, resulting in papers published by students and in collaboration with students. He has also worked with K-12 students on projects related to number theory. NSF reports that “His award will help fund summer institutes that will provide high-school and undergraduate students with a structured research environment. It will also allow for Ono to travel to conduct lectures and hands-on activities with middle- and high-school students alongside Nobel Prize and National Medal of Science winners.”

### MAA Election Results



*Joseph A. Gallian*  
President-Elect (2006)



*Carl Pomerance*  
First Vice-President (2006-07)



*Deanna B. Haunsperger*  
Second Vice-President (2006-07)

The MAA’s national election concluded at the end of May; 3759 votes were cast, about 35% of them electronically. Joseph Gallian was chosen as President-Elect for 2006, which means that he will be President of the Association in 2007–2008. Carl Pomerance and Deanna Haunsperger will be vice-presidents during 2006–2007.

## Saunders Mac Lane, 1909-2005

By John MacDonald

Saunders Mac Lane was one of the most influential mathematicians of the 20th century and was, together with Samuel Eilenberg, a creator of Category Theory. Details of this achievement together with much other information about his career can be found in his new book *Saunders Mac Lane, A Mathematical Autobiography*, published by A.K. Peters, Ltd. in 2005. However, in this article, I will present my personal point of view, since Saunders Mac Lane was my thesis advisor, mentor, and lifetime friend.

I first met Saunders as a graduate student in Chicago in 1961 when I took a course with him in category theory. I probably first came to his attention around that time, when I pointed out a slight error in one of the exercises in his book with Birkhoff on *Modern Algebra*.

Saunders' influence on me did, in fact, extend much further back in time, to September 1956, when I took a course in algebra at Harvard from Andrew Gleason using the book just mentioned. This course resulted in my changing majors from physics to mathematics, since it convinced me that mathematics had the richness and mystery that I wished to explore further.

In his later years when he had passed from his role of advisor and mentor to friend I was constantly amazed by his tenacity and independence. To give an example, here is a story from when he was in Coimbra, Portugal, at the Category Theory meeting in 1999. After the lectures one day, everyone was urging him to take a cab back to the hotel because of the steep walk from the university to the town. But, no, he wanted to walk back and I ended up walking back with him. The trek was difficult for him, but he was very determined.

He showed the same tenacity, and at the same time a great attention to detail, when acting as a thesis advisor. When I had written something up and thought it was fine, Saunders would read it through, make corrections and usually



Saunders Mac Lane  
MAA President 1951-52

suggest some other lemma or theorem that needed proving.

My most important interactions with Saunders, in fact, took place this way when I was a graduate student. He insisted on weekly meetings as well as on a progress report from the previous week. It was not enough to have read something — he wanted evidence of some thought applied to the research problem at hand. In this way he was very serious and not nearly as light and easygoing as he seemed with some visitors.

At this time in the early 60s, there was indeed a whirl of categorical ideas evolving with visits to Chicago from Eilenberg, Freyd, Lawvere, Beck, and Linton. Tom Hungerford was also writing a thesis with Mac Lane at the same time and John Thompson and Jonathan Alperin could be heard in the halls discussing exciting new developments in group theory. Max Kelly and John Gray came up frequently for the Midwest Category Seminar. In the meantime Saunders kept producing all these books, neatly typed chapter after chapter. *Homology* was the first one. I was

directly involved in the proofreading of each chapter as it was typed and used to delight in finding misprints because Saunders would say I had an "eagle eye". He liked to have his students read original source material and had me read early papers by Hopf, his papers with Eilenberg on  $K(\pi, n)$  spaces, Lawvere's thesis, as well as Freyd's, at that time new, book on abelian categories.

At Oberwolfach he climbed through the hills with his special walking stick. In the 1970s there were many category meetings there and many impassioned discussions amongst the participants, especially about the new developments in topos theory. There were discussions during the day and in the evening, and arguments too, often with both Eilenberg and Mac Lane present. Freyd, Lawvere, Johnstone, Kock, Tholen, Rosicky, Isbell, Barr and Tierney and many others were involved. Eilenberg's students were always part of the inner circle. Jon Beck's work, in particular, continues to command the highest respect.

I remember Saunders' first wife, Dorothy, quite vividly from the 1974 International Conference in Vancouver when I had many categorists at my house. I met Saunders' second wife Osa at MSRI in 1993. I have a much treasured photo of Osa, Saunders and my wife standing around my son Ian, then 18 months old. I saw her later at notable events like the celebration in Coimbra, Portugal, in 1999, and kept in touch with her up and through the time of Saunders' memorial service at MSRI on May 4, 2005.

Of course, these reflections are not a chronological or balanced view of Saunders' life but rather an impressionistic view of a person with a very rational and orderly view of the world. Please see the autobiography mentioned above for a more complete story.

As he used to say at the time of someone's passing — Hail and Farewell!

## Encountering Saunders Mac Lane

By David Eisenbud

*No man could so stimulate others unless, alongside an incisive intellect, he was possessed of enthusiasm and warmth, a deep interest in his fellow man, and a sympathy the more real for being unsentimental. Those who proudly call themselves his friends know these things: others will infer them in reading [his works].*

— M. Kelly, in *Saunders Mac Lane: Selected Papers*, edited by I. Kaplansky

Saunders Mac Lane was my teacher, mentor, and model almost from the beginning of my mathematical life. It is a relationship I've cherished. He was a figure of great honesty and integrity, who worked hard to advance research and to serve the mathematical community. His belief in the good, the right, and the rational, his care for the essence of mathematical ideas, his powerful enthusiasm, and his essential optimism were and are deeply attractive to me.

Nearly everything about Saunders in action was colorful, starting with the red-and-green plaid sports coat (the Mac Lane tartan, of course) and red pants that he would wear for important occasions. Perhaps a few anecdotes and reflections from my experience of him over 40 years will help the reader appreciate this color.

### First Encounter

I first met Mac Lane — in a sense I'll make precise — in 1963. He was one of the most important figures in the University of Chicago Mathematics Department, or indeed in American mathematics: His first student, Irving Kaplansky, was Chair of the department, and two other students were on the faculty — one, John Thompson, a Fields Medalist. Mac Lane was an inventor of group cohomology, a founder of homological algebra and category theory, known for the Eilenberg-Mac Lane spaces in topology. He was past President of the Mathematical Association of America, and he would soon be Vice President of the National Academy of Sciences, member of the Board governing the National Science Foundation, and President of the American Mathematical Society, as well.

I knew none of this. I was sixteen, an early entrant to the University, an uneven student with a great enthusiasm for



Saunders Mac Lane, c. 1931  
Norwalk, Connecticut

mathematics. It was the beginning of my second quarter, and I was scheduled to start a basic linear algebra class that morning. I happened to arrive a little early, settled down in the first row of the class, and sank peacefully into a daydream. Being so new, I wasn't surprised not to know the other students who settled in around me, and I didn't know the teacher that I'd have. In due course, Mac Lane walked in and began lecturing. His style was lively and colorful, and I was immediately interested — but almost at once aware that I'd made a big mistake: this was not an undergraduate linear algebra course, but an advanced graduate course on Category Theory. I'd come an hour early.

I understood nothing whatever after a few moments, but was far too embarrassed to get up and leave — instead I sank into daydreams, glassy-eyed. Mac Lane, who prided himself on

paying attention to his class, later told me he thought he could always see who was following and who was not. In a moment like a thunderclap, I looked up from my seat and found him pointing directly at me from across the room. "You!" he said peremptorily, "you don't believe this proof, do you?" Belief and disbelief were equally beyond me; I sat petrified. He advanced toward me, and I don't know what I imagined — that he would pick me up by the scruff of my neck and throw me from the room? He stopped, turned back to the board, and proceeded to explain the proof to satisfy me. Of course, I still understood nothing — but I sat in rapt attention.

Fortunately the class ended soon, and as students asking questions surrounded him, it was easy for me to slip out. I didn't tell Saunders this story until many years afterwards, when I had the privilege of re-enacting it (from the other side) in a lecture at the conference in honor of his seventieth birthday. Needless to say, the event hadn't left a trace in his memory, though it remains sharp for me to this day.

### Saunders and Tolerance

Saunders believed strongly in principles, in the rightness of right positions. I never once saw him personally intolerant, but he could sometimes be direct and candid to the point of offending. People whose judgment I respect have felt injured by what he said, and sometimes by the bluntness of his expression. In some way perhaps he didn't appreciate the magnitude of his position in mathematics, or the seriousness with which people took him. In a lesser personage some of his extreme positions might have been regarded as charming eccentricities. But given Saunders' stature, they could injure, and he might have been more cautious.

An event from late in Saunders' life may give a bit of the flavor. It was a special session run by him and Richard Askey at the Joint Mathematics Meeting in 1999, a session boldly entitled "Mathematics Education and Mistaken Philosophies of Mathematics." The audience was enormous. I found the title charming (and still find it so, even now as I become more involved with ideas in K-12 education), and I imagine that Saunders meant it to be controversial but playful. Predictably, it annoyed and needled some practitioners. Saunders began the session with introductory remarks that I found fascinating; he said that he now considered the extent of his own emphasis on category theory as a tool for learning and teaching mathematics to have been too extreme. This humbleness may have helped soften the critical tone of the session.

**Saunders and Sammy**

One of Saunders' great mathematical friendships and collaborations was with Samuel Eilenberg (widely known as Sammy, or even as S<sup>2</sup>P<sup>2</sup>: "Smart Sammy the Polish Prodigy"). I got to see them in action together only once, at the AMS Summer Research Institute on Category Theory at Bowdoin College, in 1969. They had special status at this three-week conference, not only as the senior members, but also as the very founders of the subject. So, when they began discussing its origins one evening after dinner, everyone gathered around to listen.

I dearly wish I could recall the substance of their debate, but I don't; only my sense of the contrast in the two men's styles stays with me. Sammy drew Saunders out and egged him on, always slightly evasive and mocking; Saunders, whose father

and grandfather were Congregational ministers, seemed to feel that since his view was right, his view would prevail. Once he had stated it, all he could do was

Kegel), it was finally time for me, by now a second-year graduate student, to settle on an area for a PhD thesis. I obsessed about how to make the choice. A close

mathematical friend, Joe Neisendorfer, explained to me an algorithm: forget the topic, look around the faculty for the person you like the most. It didn't take me long to choose Saunders.

I wouldn't say I ever felt personal intimacy with Saunders, but he did go out of his way to make me and other students feel welcome in more than his office. Saunders and his late wife, Dorothy, had a small but comfortable cottage in the Indiana Dunes, a beautiful area on the shore of Lake Michigan about an hour south of Chicago, and they occasionally invited students to spend an afternoon there. Saunders was an enthusiastic sailor, and I can report, from a ride in a small sailboat on rough water, that he was ready to provide

needed instruction not only in mathematics but also on how to handle the absence of a toilet — or any privacy — in that difficult situation.

If you look at the list of Saunders' 39 students, you'll see that Irving Kaplansky, who worked on valuation theory of fields, came first; I'm near the end, with a thesis on noncommutative rings. Along the way are such people as John Thompson (finite groups), Anil Nerode (logic and computation), and Robert Szczarba (algebraic topology). How did this variety come about?



*Mac Lane lecturing abroad*

bang his fist. The devious and sophisticated European versus the innocent but honest American? That's how it seemed to me at the time. Maybe I was a little innocent myself. A loyal student, I was rooting from the beginning for Saunders' point of view, but I came away feeling that he was trounced in the contest.

**Being Saunders' Student**

After flirting a while with operator theory (Paul Halmos and Felix Browder were my teachers) and group theory (learned from Jon Alperin and Otto

Perhaps the answer lies in Saunders' hospitality to these many ideas. He wanted to learn finite groups, and taught a course on them. By the end of the course he'd decided that he'd never really understand the subject, but in Thompson he found a fabulously strong student. Saunders might have tried to turn such a student toward interests close to his own, but I think he would not, on principle: he was happy to encourage his students to do what excited them.

Saunders followed an interesting, curving trajectory through mathematics, from logic and foundations to field theory and the beginnings of homological algebra, through topology to category theory, with smaller diversions along the way into Hamiltonian mechanics, finite groups, and many other subjects. Perhaps his students, or many of them, could be described as coming off on the tangents to this path, a kind of developable surface reaching broadly across mathematics.

Some other aspects of Saunders are also reflected in his students: Saunders was always active on behalf of the community, whether as Chair working to build the department at the University of Chicago or, near the end of his career, as a member of the National Science Board or as manager of the elaborate system of reports for the National Academy of Sciences. Many of his students and grand-students have followed him into this willingness for public service. When I was worrying about whether to move to my current position at MSRI, he was one of the first

people I called on for advice and blessing, and he gave both.

Returning to the more fundamental matter of being Saunders' mathematical student: I tried for a while, dutifully, to find a thesis topic in Category Theory,



*Just sailing along on his own off the coast of Maine.*

Saunders' passion in that part of his life. But I failed; somehow, the things I read and learned in that domain just didn't inspire me. When I developed an interest instead in a problem on non-commutative rings posed by a visitor of Herstein, the young Chris Robson, Saunders could easily have washed his hands of the project. He did not: though it was far from his current area of interest, he welcomed what I had done, and painstakingly read draft after draft of my thesis.

Saunders' mode of instruction in thesis writing bears mention. I had written a couple of papers, jointly with Robson, of which my thesis results were partially an extract. Robson cared a lot about exposition, and so (learning from Saunders among others) did I. We'd gone through many drafts, and I thought the writing pretty polished. Saunders did not. He began at the beginning and

worked his way through the thesis until he'd compiled a list of exactly 25 substantive suggestions. Then he stopped, and returned the document to me for an overhaul. When I had finished making the corrections he'd flagged and all their analogues, I gave it back to him, eager to be done. But... after a week or so I got a second list of exactly 25 more suggestions. The third list was a bit shorter, and Saunders allowed the process to converge before I got too frustrated.

It must be clear by now: over these forty years I learned many lessons from Saunders. I'm deeply grateful to him.

*David Eisenbud is Director of the Mathematical Sciences Research Institute in Berkeley, CA and President of the American Mathematical Society. This article is an edited version of the preface he wrote for Saunders MacLane: A Mathematical Autobiography, published by A K Peters, Ltd., Wellesley, MA. It is reprinted here with the publishers' kind permission.*

## ICM 2006 in Madrid, Spain

All information currently available about the International Congress of Mathematicians (ICM) 2006 program, organization, and registration procedure can be found on the ICM 06 website at: <http://www.icm2006.org>.

## George B. Dantzig 1914-2005

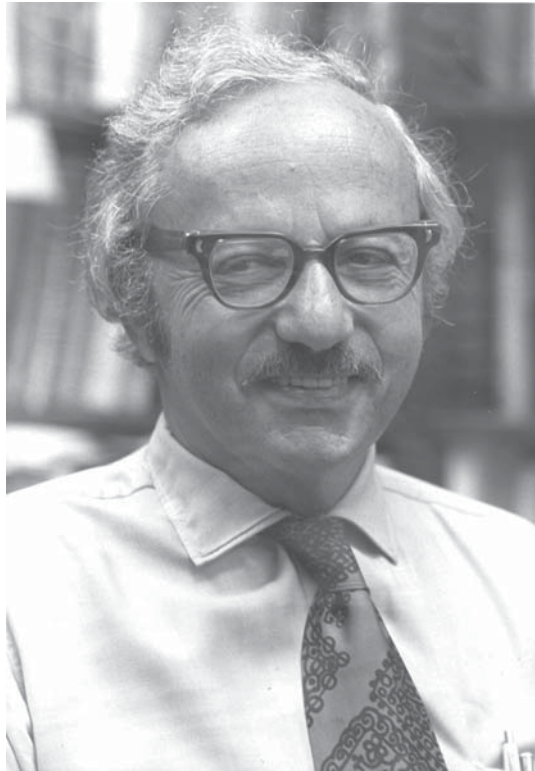
By Don Albers

George Bernard Dantzig, the “father of linear programming,” died at home in Palo Alto, CA on May 13. His father Tobias Dantzig, named him after the famous writer George Bernard Shaw in the hopes that he would become a writer. Fortunately for the world of mathematics, he became a mathematician. He was the inventor of the simplex method, which has powerful applications in several fields. Many feel that he should have received the Nobel Prize in Economics for his work.

His achievements in mathematics are particularly striking in view of the fact that he had trouble with algebra in junior high school. In a 1984 interview with the author of this obituary, he was disarmingly modest about his trouble with algebra: “To be precise, I was flunking. I remember walking home one day, furious with myself. How is it, I asked myself, that I, a son of a mathematician, do poorly while all the other kids in class do so much better? I was angry with myself. After that I sailed through algebra.” From then on, he got top marks in mathematics.

Dantzig attributed the influence of his father to the development of his analytical power: “My father taught me by giving me problems to solve. He gave me thousands of geometry problems while I was still in high school. I would say over ten thousand. But it was I who asked for problems. After he gave me one, he would say, ‘Well, I’ll give you another one.’ It seemed as if he had an infinite storehouse of them.... The mental exercise required to solve them was the great gift from my father.”

He completed his bachelor’s degree in 1936 at the University of Maryland, where he did not recall seeing a single application of mathematics in any of his mathematics courses. In 1937, he earned a master’s degree from the University of



George B. Dantzig

Michigan. He and Anne Shmuner, who he had married the previous summer, then moved to Washington, DC where he got a job as a statistical clerk in the Bureau of Labor Statistics. While there he reviewed a paper on double sampling by the famous statistician Jerzy Neyman. Dantzig was very excited by the paper and soon wrote to Neyman saying that he would like to finish his Ph.D. under him at the University of California, Berkeley.

During his first year at Berkeley, he arrived late one day to one of Neyman’s classes. On the blackboard were two problems that Dantzig assumed had been assigned as homework. A few days later he apologized to Neyman for taking so long to do the homework, saying that the problems seemed to be a little harder than usual. Neyman told him to throw the homework on his desk.

About six weeks later, on a Sunday morning, he and his wife were awakened by someone pounding on the door. It was Neyman. He rushed in with papers in hand all excited: “I’ve written an introduction to one of your papers. Read it so I can send it out right away for publication.” It turned out that what Dantzig thought were homework problems were in fact two famous unsolved problems in statistics. The story of these problems has proven to be inspirational to several ministers who have incorporated it into their sermons.

In 1941, before finishing his doctorate, Dantzig, wanting to contribute to World War II, took a position with Air Force Statistical Control. In that position he became expert at programming planning methods and set up a reporting system for combat units on the number of sorties flown, aircraft lost and damaged, bombs dropped, and targets attacked. In those days the only “computing machines” they had were people using hand-operated desk calculators.

In 1946 he returned to Washington as the civilian head of the Air Force’s Headquarters Statistical Control. Two of his colleagues challenged him to mechanize the planning process; that is, to find a more rapid way to compute a time-staged deployment, training, and logistical supply program. Mechanization in those days meant using analog devices or punch-card equipment. He set out to formulate a model, and soon became fascinated by Wassily Leontief’s Input-Output Model of the American Economy.

By late 1946, he had formulated a linear programming model, but did not yet have an algorithm. Over the next year he succeeded in producing the simplex method, an algorithm for solving linear programming problems, and his group

at the Air Force started experimenting with it. He continued to look for a better algorithm, but in June of 1948, his group asked him why he continued to look elsewhere when the simplex method was working so well on the test problems.

The rest, as they say, is history. The simplex algorithm has proven to be of great utility in a wide range of applications. Airlines use it to schedule crews and make fleet assignments, shipping companies use it to determine how many planes they need and where their delivery trucks need to be deployed. It is used in refinery planning, manufacturing, telecommunications, architecture, revenue management, circuit design, advertising, and many other areas. It has provided managers with a powerful tool for modeling problems and comparing a large

number of alternative courses of action to find one that is optimal.

In 1952, he became a research mathematician at the RAND Corporation where he began implementing linear program-

Center of the University of California, Berkeley, and in 1966 professor of operations research and computer science at Stanford University, where he remained until his retirement.

His pioneering book *Linear Programming and Extensions* was published in 1963 and revised in 1997 and 2003.

Dantzig was the recipient of many prizes and eight honorary degrees. He was a member of the National Academy of Engineering, the National Academy of Sciences, and the American Academy of Arts and Sciences. In 1975 he was awarded the National Medal of Science.

*Don Albers is Associate Executive Director and Director of Publications at the Mathematical Association of America.*



*Receiving the Medal of Science from President Ford in 1975.*

ming on computers. In 1960, he was named chair of the Operations Research

## An Earth Filled with Computers\*

*By George Dantzig*

In my *Linear Programming and Extensions* you will notice that I pay great tribute to Leontief. It was Leontief who around 1932 first formulated the Inter-industry Model of the American Economy, organized the collection of data during the Great Depression, and finally tried to convince policy makers to use the output from the analysis. All of these things are necessary steps for successful applications, and Leontief took them all. That is why in my book he is a hero.

Leontief's model had a matrix structure which was simple enough in concept with sufficient detail that it could be useful for practical planning. I soon saw that it had to be generalized. Leontief's was a

steady-state model and what was needed was a highly dynamic model, one that could change over time. In his model there was a one-to-one correspondence between the production processes and the items produced by these processes. What was needed was a model with many alternative activities. Moreover, the application had to be large scale — with hundreds, perhaps thousands of activities and items. Finally, it had to be computable. In other words, once the model was formulated, there had to be a practical way to compute what quantities of these activities to engage in so as to be compatible with their input-output characteristics and given resources. The model I formulated would be described today as a time-staged dynamic linear

program with a staircase matrix structure. Initially there was no objective function; in other words, no explicit goal. Such goals did not exist in any practical sense because planners simply had no way to implement them.

A simple example illustrates the fundamental difficulty of formulating a planning program using such an activity-analysis approach. Consider the problem of assigning 70 men to 70 jobs. An "activity" consists of assigning the  $i$ -th man to the  $j$ -th job. The restrictions are (a) that there are 70 men, each of whom must be assigned, and (b) that all of the jobs, also 70, must be filled. The level of an activity is either 1, meaning it will be



used, or 0, meaning it will not. Thus there are  $2 \times 70$ , or 140, restrictions and  $70 \times 70$ , or 4900, activities with 4900 corresponding zero-one decision variables. Unfortunately there are also 70 factorial permutations, or ways to make the assignments. The problem is to compare 70 factorial ways and to select the one which is optimal, or “best” by some criterion.

Now in this example 70 factorial is a very big number. To get some idea of how big, suppose we had had an IBM main-frame computer available at the time of the Big Bang fifteen million years ago. Would it—between then and now—have been able to examine all the possible solutions? No! But suppose that an even more powerful computer had been available, one that could have examined one billion assignments per second. The answer would still be no. Even if the Earth were filled with nanosecond-speed computers, all working in parallel, the answer would still be no. If, however, there were ten Earths, all filled with nanosecond-speed computers, all programmed in parallel from the time of the Big Bang until the sun grows cold, then perhaps the answer would be yes. The remarkable thing is that the simplex method with the aid of a modern computer can solve this problem in a split second.



*George Dantzig as a graduate student.*

This example illustrates why, up to 1947 and for the most part up to this day, a great gulf exists between man’s aspirations and his actions. Man may wish to state his wants in terms of an objective to be extremized, but there are so many ways to go about doing the job, each with its advantages and disadvantages, that it has been impossible to compare them and to choose among them that one

which is best. So, invariably, man has always had to turn to a leader whose “experience” and “mature judgment” would guide the way. The leader’s guidance usually consisted in the issuance of a series of edicts or ground rules to those developing the programs. Although such methods are still widely used, the world today is far too complex for such simplistic methods to work, and they don’t.

In late 1946, before we knew that high-speed electronic computers were soon to exist, I had formulated a mathematical model that satisfactorily represented the technological relations usually encountered in practice. However, in place of any explicitly stated goal, or function to be extremized, there were a large number of ad hoc ground rules issued by those in authority to aid in the selection of the solution. Without these it would have been impossible to choose from the astronomical number of feasible solutions.

\*Excerpted from George Dantzig in *More Mathematical People* edited by Albers, G.L. Alexanderson, and Constance Reid, Harcourt Brace Jovanovich, Inc, Boston, 1990.

## In Memoriam

**Ronald C. Biggers**, age 59, died on April 23, 2005 after a stroke. He held the distinct honor of being the first African American to earn a Ph.D. in pure mathematics from the University of California at Irvine. His area of emphasis was algebraic geometry and combinatorial group theory. He taught at many institutions before joining Kennesaw State University in Kennesaw, GA, in 1989. Biggers is survived by Celo Biggers, his wife of more than 30 years, and his two daughters. He was an MAA member for 32 years.

**Edward N. Mosley**, age 66, passed away on June 12 after an extended battle with cancer. He taught at Lyon College for 35 years and was a member of MAA for 43 years. Mosley also served as Governor of the Oklahoma-Arkansas Section. He is survived by his wife, Mary Eleanor, a son, John Mosley, and his brother Dr. James Mosley.

## Study Shows Gains for Women in Mathematics

A recently-concluded study (sponsored by the MAA together with the American Mathematical Society, the American Statistical Association, and the Institute of Mathematical Statistics) shows that women are participating in mathematics in increasing numbers. The study, which was published in the August issue of the *Notices of the AMS*, shows that about one third of all doctorates in the mathematical sciences during 2003-2004 went to women. This continues a long-term trend of increasing participation by women that has persisted since the 1980s, when gender records began to be kept.

The study used the ranking of graduate mathematics departments by the National Research Council to investigate the status of women in the top 48 mathematics departments in the US. Women received 25% of the doctorates at these institutions, up from 21% the previous year. Also noted were the large percentage of women among undergraduate mathematics majors, the increased visibility of women in mathematics competitions such as the Mathematical Olympiad and the Putnam Competition. For the details, see the August issue of the *Notices* or visit <http://www.ams.org/notices/200507/survey.pdf>.

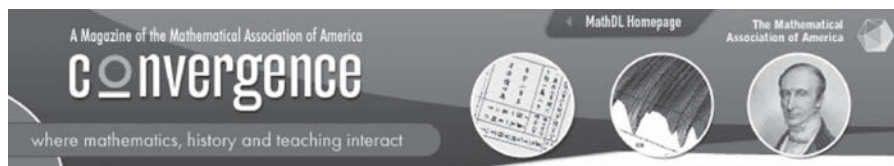
## Convergence: Mathematics, History, and Teaching An Invitation and Call for Papers

By Victor Katz

*Convergence: Where Mathematics, History, and Teaching Interact*, is the MAA's new online magazine about the history of mathematics and its use in teaching. Part of MathDL, the mathematics digital library, *Convergence* is aimed at teachers of mathematics, be they secondary teachers, two- or four-year college teachers, or college teachers preparing secondary teachers. (We describe the range of topics as "grade 9-14 mathematics": algebra, synthetic and analytic geometry, trigonometry, probability and statistics, elementary functions, calculus, linear algebra, and differential equations.) The editors, Victor J. Katz, from the University of the District of Columbia, and Frank Swetz, from Penn State University, Harrisburg, welcome all members to log in to the *Convergence* website at <http://convergence.mathdl.org> and see what the magazine has to offer.

Among the types of material appearing in the magazine are the following:

- Expository articles dealing with the history of various topics in mathematics curriculum. These may contain interactive components and color graphics, to take advantage of the capabilities of the Web. Each article will have a discussion group attached, where readers can share suggestions as to how the material can be used in the classroom and point out strong points and possible pitfalls.
- Translations of original sources, generally accompanied by commentary showing the context of the works. The goal of these translations is always to show teachers how ideas were developed in various cultures and how knowledge of this development is useful to teaching the same ideas to today's students.
- Reviews of current and past books, articles, and teaching aids on the history of mathematics of use to teachers, as well as reviews of websites



providing information on the history of mathematics.

- Classroom suggestions. These may be self-contained articles showing how to use history in the teaching of a particular topic or they may be materials closely related to a main article, showing in some detail how to use the article in a classroom setting.
- Historical problems. These problems will appear in a section entitled "Problems from another time," with new problems appearing frequently.
- What Happened Today in History? Each day, there will be a listing of 2-3 "mathematical events" which happened on that date in history.
- Quotation of the day. A new and interesting quotation about mathematics from a historical figure will appear in this section each day. The reader will also be able to search our database of quotations to find additional ones.
- An up-to-date guide to what is happening around the world in the history of mathematics and its use in teaching. The magazine will report on past meetings and give notice of future meetings. Where abstracts are available for a particular meeting, these will be included. We may also include copies of handouts for easy access, as well as links to the author's webpage, if available.

The magazine is currently free to all, due to the support of the National Science Foundation, but registration is required to access the site. A small subscription

fee will be charged beginning later this year.

Currently, we have a limited supply of articles in our pipeline. Because our goal is to bring out new material on a regular basis, we need a continual flow of articles and classroom suggestions. We therefore welcome your ideas for articles as well as your completed manuscripts. In particular, we welcome short classroom suggestions that can immediately be implemented by teachers.

Materials should be sent both in hardcopy and electronically. Hardcopy should be sent to Victor Katz, *Convergence*, Mathematical Association of America, 1529 18<sup>th</sup> St. N.W., Washington, DC 20036. Electronic files should be sent to Victor Katz at [vkatz@udc.edu](mailto:vkatz@udc.edu). We can take articles in Word or TeX, but please include illustrations (in jpg format), applets, etc. as separate files, and give explicit instructions for both internal and external hyperlinks. If there are many illustrations or applets, it is best to send the electronic version on a CD to the MAA address.

If you have an idea for an article, but do not know how to produce applets for it, we suggest that you contact an expert on your own campus for help. If necessary, however, we can provide help in the editorial office, provided you give us very explicit instructions as to what you need.

If you are interested in writing reviews for *Convergence*, send your name and some indication of the kinds of material in which you are interested to Frank Swetz at [guru6@myway.com](mailto:guru6@myway.com). We also welcome interesting quotations as well as information on mathematical dates to add to our database.

## What I Learned from... Project NExT

By Dave Perkins

Project NExT is an MAA professional development program for undergraduate professors who have recently acquired a Ph.D. You can recognize a Project NExT fellow at any MAA national meeting by the colored dot on his or her nametag. When I learned in spring of last year that I had been accepted as a NExT fellow, I speculated that the award might be little more than a feather in my cap (and an orange dot on my nametag). It is no exaggeration, however, to state that my experiences as an orange dot fundamentally changed my approach to teaching. I am grateful for the chance to tell the stories of two professors who spoke to us and inspired me to change.

Thomas Banchoff of Brown University demonstrated the online system he uses to collect and respond to homework. Programmed by his undergraduate students, this system informs Dr. Banchoff when a student has posted the answer to a homework problem, allows him to reply to the student's work, notifies the student, and so on. Once a certain deadline has passed, the system releases these discussions for public view. Like many professors, I have on occasion allowed students to rewrite their homeworks, hoping that the dialogue thus created will benefit the students in a way that the simple "here is your paper with my comments" model cannot. Dr. Banchoff's approach, however, takes the dialogue approach even further by moving it into the public forum.

Since hearing his presentation a year ago, I have experimented with Dr. Banchoff's idea in two classes. In a special topics class on the constants  $\phi$ ,  $\pi$ ,  $e$  and  $i$ , my students posted their homework answers in a shared folder, accessible to all. Because I did not have software that would unlock the answers at a given time, the students were able to view answers that were posted before the deadline. Thus, I asked them to give each other credit every time credit was due — e.g., "I was stuck, but saw how Shawna factored the polynomial to get started, so I used that idea to fin-

ish the problem myself." If a student made a mistake, I replied in the shared folder, and expected a response. At the end of the year, the students' evaluations of the course included only positive reactions to this approach.

This past semester, I team-taught a course in advanced logic that met once a week. My colleague from philosophy and I chose an article each week (e.g., "Computing Machinery and Intelligence" by Alan Turing) and asked the students to read it at least once during the next two days. While reading, they were to formulate a list of questions that were raised by the material, then choose one and post it to a shared folder. Later, after having read the article more carefully and discussing it in class, they each were to choose another student's question and respond to it with an essay. All of this discourse was public to the rest of the class; a general shared folder collected whatever discussion spilled over — further questions, links to websites, and so on.

The second professor, Sarah-Marie Belcastro of Xavier University, surprised the orange dot crowd with her approach to teaching, an approach I like so much that I plan to teach over half of my classes next year in her style. During her presentation she said, "If all we want is to convey information to our students, we might as well just read a text to them, and in that case, they might as well just read the text themselves." I think that statement is perfectly well put, and I could not agree more.

A month after hearing her speak, I taught a real analysis class with her approach. We read *Elements of Real Analysis* by David Sprecher page by page; at the start of each class, I asked who had written questions into their reading journal, and what the questions were. Usually, other students answered these questions, and occasionally I suggested my own answers. Every now and then, none of us knew what to say, and we spent the class puz-

zling out the answer on the whiteboard. There is still one proof in chapter 8 that none of us can explain! Once again, the student evaluations were all positive, and one bright physics student points to that experience as the reason why he is applying to graduate school next year not in physics, but in mathematics.

These are by no means the only pedagogical impacts that Project NExT had on me. My attitudes toward both undergraduate research and my own research, for example, have also changed greatly. Consider this my thank you to an undertaking that has affected me profoundly, and an encouragement to you to let your new colleagues know about Project NExT. You can find out more at <http://archives.math.utk.edu/projnext/>.

*Dave Perkins is an assistant professor of mathematics at Houghton College in western New York. He is particularly interested in discrete mathematics. His email address is david.perkins@houghton.edu.*

### Correction

A note in the May/June issue of *FOCUS* referred to "ICM 2006, Barcelona, Spain." That, of course, is a mistake: as indicated on page 7 of this issue, ICM 2006 will be held in Madrid. We apologize for the error.



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The infinite possibilities of mathematical literacy.

## The Preparation of Mathematics Teachers: A British View Part 2

By Peter N. Ruane

What should teachers know about mathematics in order to be able to present the subject most effectively in the primary and secondary classroom?

For primary teachers, the problem isn't so simple; they have to teach a range of subjects and can't possibly be expert in all of them. But English primary schools have subject coordinators who assume leadership roles for particular aspects of the primary curriculum. Mathematics coordinators provide guidance in curriculum matters and take the lead for in-school staff development. They should be able to shed light upon the underlying complexities of primary mathematics and have understanding of its continuation in the secondary school. So, what follows is a brief description of the specialist maths components of a primary B.Ed. degree that took a different approach towards extending the subject knowledge of those primary teachers who were likely to become maths coordinators.

Prior to choosing maths as a specialist study, students would have completed several basic curriculum modules (Number, Shape & Space, Data Handling and Algebra), which had a strictly pedagogic emphasis and prompted students to reflect upon their previous learning. But the later specialist modules were founded upon the following principles:

1. Mathematical topics were considered from various perspectives, so complex numbers, for instance, would be viewed traditionally as extended solutions of quadratics; they would be introduced in the form of  $2 \times 2$  matrices and, finally, as a field extension of  $(\mathbf{R}, +, \times)$ . Geometric and wider algebraic relevance would also be explored.
2. Teachers at one level (infant, primary, secondary, higher education) should have insight into the nature of the mathematics that is taught at subsequent and preceding levels.
3. Excursions into 'advanced' topics, wherever possible, emanated from analysis of a relevant aspect of school mathematics.
4. Equal emphasis was placed upon mathematics as a body of knowledge (content objectives) and as a way of knowing (process objectives), and coursework therefore included mathematical investigations and special studies with an historical emphasis.

All well and good, you might say, but how were such principles put into practice? The specialist modules included one on *Pre-Calculus*, another on *Proof, Logic and Boolean Algebra*, and a module on *Geometry*, but the one described below was devoted to *Number*; here's a compressed outline of its contents.

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### NUMBER

**Children's formation of number concept and counting** motivated discussion of cardinality (the Russell definition of natural number), ordinality and a formal description of bijective equivalence of sets (finite or infinite). (For secondary B.Ed. students, the Peano Axioms would be included).

**Classification of natural number:** figurate numbers (square, rectangle, triangle etc), prime numbers and the fundamental theorem of arithmetic and other aspects, such as additive partitioning and digital roots etc.

Initial discussion usually focussed upon primary school activities and then became more analytical, as exemplified by the following assignment given to final year primary B.Ed students as part of their coursework:

#### Assignment

(a) By applying the Fundamental Theorem of Arithmetic, find a natural number,  $x$ , such that:

$$x^4 - 1296 = 0.$$

Hence show that, if  $x$  is a triangle number, it is possible for  $x^2$  to be a triangle number. Is a generalisation possible in this respect? For the value of  $x$  specified by the above equation, determine the triangle number that is nearest in value to  $x^3$  and that which is nearest in value to  $x^4$ .

(b) Again by use of the FTA, decide which of the numbers 1225 and 1485 is a square number. Prove that both are triangle numbers and determine how many other triangle numbers and how many square numbers lie between them.

(c) If the difference between consecutive triangle numbers,

$t_m$  and  $t_{m+1}$ , is 1000, find, for the same value of  $m$ , the difference between the consecutive square numbers  $s_m$  and  $s_{m+1}$ .

Is there any evidence to suggest that triangle numbers occur more or less frequently than square numbers? Is there any validity in the claim that there are fewer of one of these two types of number than the other? Justify your answers to both questions

(d) A group of children are building cuboids amongst which are one with a volume of  $2079\text{cm}^3$  and one of volume  $2464\text{cm}^3$  but they find that these two particular cuboids have bases of equal area. Calculate the dimensions of both cuboids given that they are all specified in terms of natural numbers (greater than 1).

(e) For two natural numbers,  $a, b$ , the concepts of highest common factor (hcf) and lowest common multiple (lcm) can be defined as follows:

- (i)  $x = \text{hcf}(a,b)$  if natural numbers  $m,n$  can be found such that  $a = mx$  and  $b = nx$  where  $m$  and  $n$  are co-prime.
- (ii)  $y = \text{lcm}(a,b)$  if  $a|y$  and  $b|y$  and if, also,  $a|x$  and  $b|x \Rightarrow y|x$

Illustrate these definitions by means of one specific example for each.

(f) Prove that, for any natural numbers  $a,b$ ;

$$a \times b = \text{lcm}(a,b) \times \text{hcf}(a,b)$$

**Note:** Part (d) of this assignment is based upon a hypothetical classroom situation due to the policy of assessing subject knowledge in the context of classroom activities.

**Integers:** The starting point would be to see how directed numbers are introduced in various school textbooks. One such approach portrays them as 1-dimensional vectors and distinguishes between use of the symbols  $+, -$  to denote operations as opposed to their use as labels for directed numbers (e.g. the difference between  $7-3$  and  $7+3$ ). Practical and theoretical importance of the integers would also be considered and the topic would culminate with construction of the system of integers  $(\mathbf{Z}, \oplus, \otimes)$  from equivalence classes of ordered pairs of natural numbers, where  $\oplus$  and  $\otimes$  are the binary operations on  $\mathbf{Z}$  induced from  $(\mathbf{N}, +, \times)$ . So, for the first time in their lives, students would be able to prove that  $-1 \otimes -1 = +1$ . Yes, they found this challenging, but the point was to expose the complexities lying just below the surface of school mathematics

**Rational numbers:** Fractions, equivalent fractions and decimal representation would have been treated in earlier curricu-

lum modules, but the approach now became more serious. For example, whilst it is true that the vast majority of students/teachers may accept that  $1/3 = 0.\overline{3}$ , too many are unable to explain why this is so. But statements like  $1 = 0.\overline{9}$  always generate widespread disbelief, illustrating the sort of intuitive and mathematical leeway to be made up when teaching this aspect of number. Nonetheless, the connection between rational numbers and finite or recurring decimals has to be established and formally proved. Naturally, this involves intuitive ideas of limits and questions of infinite processes (e.g. every rational number is the sum to infinity of a GS).

Many teachers (trainee or otherwise) have a poor grasp of the structural properties of number systems and tend to confine their insights to algorithmic procedures. So, when distinguishing between  $\mathbf{Z}$  and  $\mathbf{Q}$ , say, it comes as a surprise to realise that a concept such as consecutiveness has no meaning in  $\mathbf{Q}$ . And, apart from the algebraic difference between the two systems, there is also the anti-intuitive revelation that  $\mathbf{Q}$  is dense whilst also being countable.

The Euclidean geometric derivation of rational numbers would also be considered, together with many surprising results like ,

$$\sum_1^{\infty} \frac{1}{4^n} = 3 \sum_1^{\infty} \frac{1}{10^n} = \frac{1}{3}$$

which arose from the analysis of a primary school activity of dividing a square into thirds by repeatedly taking quarters. Finally, the (ordered) field  $\mathbf{Q}$  of rational numbers would be constructed from the (well-ordered) domain  $\mathbf{Z}$  of all integers (re: equivalent fractions).

**Irrationals:** Students are strictly averse to irrationals, and symbols such as  $\sqrt{2}$  aren't readily accepted as representing 'proper numbers'. Despite evidence as to the precise geometric existence of such things, they can't resist replacing them by some rational approximation; so  $\pi$  has to be  $22/7$  and  $\sqrt{2}$  has to be something like 1.414. But, having accepted rational numbers as repeating decimals, one has to take account of the set  $\mathbf{Q}'$  of infinite non-repeating decimal numbers. Algebraically, this is shown to be different to  $\mathbf{Q}$  and is not so easily comprehended as a whole (e.g. there is no countable array). But students grudgingly accept it as the lesser-known partner in the marriage  $\mathbf{Q} \cup \mathbf{Q}' = \mathbf{R}$ .

Cantor's diagonal argument introduces the notion of uncountability, and showing that  $\mathbf{Q}$  has measure zero establishes the fact that  $\mathbf{Q}'$  is really the senior marriage partner. Informal ways of showing this include the generation of an

infinite decimal number on a random digit-by-digit basis. Next on the scene is the probabilistic trauma that, if a real number is randomly selected, then  $P(\text{irrational}) = 1$  and  $P(\text{rational}) = 0$ . A similar conclusion is achieved with respect to algebraic and transcendental numbers.

**The Reals.** For older primary school children, there are various calculator activities devised to enhance estimation skills. For example, to calculate the edge-length,  $L$ , of a cube whose volume is, say,  $10 = L \times L \times L$ , children each produce a list of converging ‘guesstimates’. In the absence of silly errors, each list will form a finite rational Cauchy sequence approximating  $\sqrt[3]{10}$ . At this juncture, our student-teachers would be introduced to the definition of real numbers as equivalence classes of rational Cauchy sequences with binary operations induced from those on  $\mathbf{Q}$ . Other aspects of  $(\mathbf{R}, +, \times)$  were considered, such as completeness and its algebraic and practical versatility.

**Complex numbers.** This theme was introduced in accordance with principle 3 given above, whilst the treatment was in line with that described in 1. In terms of equation solving, the complex numbers would be revealed as the most versatile of all systems hitherto discussed and the topic would conclude with the fundamental theorem of algebra.

That concludes a compressed description of the module on Number but, as in all modules, the exploration of ideas was rooted in some aspect of school mathematics. For example, in the pre-calculus module, the main theme was ‘growth’ and, from a textbook intended for 12 year-olds, there was an exercise based upon the notion of repeated doubling for which the book showed the graph of  $y = 2^x$  as being a continuous curve. However, although students (and most teachers) could readily assign values to  $2^x$  when  $x$  is an integer, many are at a loss regarding the meaning of  $2^x$  for non-integral values of  $x$  (say,  $x = \sqrt{2}$ ). Yet, whatever their knowledge of the laws of indices, few teachers seem able to apply it to make links of the sort:

$$2^{1.414} = 2^{\frac{1414}{1000}} = \sqrt[1000]{2^{1414}} \Rightarrow 2^{\sqrt{2}} \approx \sqrt[1000]{2^{1414}}$$

The module on Proof, Logic and Boolean Algebra began with analysis of the range of early school activities described in my article on “What I learned by Teaching Logic and Proof” (FOCUS, May/June 2004). Proof would be discussed in its widest pedagogical context and the logical structure of various forms of proof would be revealed. Applications of logic to switching circuits has particular relevance to the practical work on circuitry that appears in the primary and secondary science curriculum, so here’s a couple of illustrative examination questions used for this module.

**Question 1.**

(a) Using the laws of the algebra of propositions (provided), show that the proposition

$$\sim(p \vee q) \vee (\sim p \wedge q)$$

is logically equivalent to a one-variable statement. Specify which of the laws is being used for each step of the simplifying procedure.

(b) Two groups of children each design a simplest possible electrical circuit for the simulation of the scoring system for each of the games they are playing. Each game is based upon the simultaneous tossing of three coins (1p, 2p and 5p) and the circuits are to be as follows:

For game 1, a bulb lights up if the three coins are either all heads or all tails.

For game 2, a bulb lights up if the three coins show either exactly 2 heads or exactly 2 tails.

(i) Using the symbols  $p, q, r$ , respectively, to denote the statements for which the 1p and the 2p and the 5p coins land as ‘heads’, write down a compound proposition for each of the circuits to indicate when the bulbs are lit. Sketch the two circuits.

(ii) What is the simplest compound statement logically equivalent to the negation of that which represents the circuit for game 2? Give reasons.

(iii) What would be the effect of arranging the two circuits in parallel?

**Question 2**

For each of the parts (i) to (iv) below, you may select methods of proof from: counterexample, exhaustion, contradiction, deductive proof or mathematical induction.

If you employ these (or any other) methods of proof, you should specify the exact point in your argument where this is so.

(i) State, with proof, the number of interior angles in a pentagon that may be right angles. Illustrate your proof by means of clear diagrams.

(ii) Without reference to decimal representation of real numbers, prove that, if  $\sqrt{5}$  is irrational, then is  $\frac{\sqrt{5} + 1}{2}$  also irrational.

(iii) Prove, or disprove, that, for all  $2 \times 2$  real matrices  $A, B$ ,  $(A+B)^2 = A^2 + 2AB + B^2$

(iv) Prove, or disprove, that  $7^n + 4^n + 1$  is divisible by 6 for any natural number  $n$ .

### Geometry

On entry to higher education, most students have a very fragmented knowledge of geometry — even those with good A-level (high school) grades. So the approach here began with many of the practical, visual and investigative activities that characterise the geometry of primary and early secondary years. Lack of space debar adequate description of the work covered, but isometries, shear, stretch, enlargement would be encountered within the context of classroom activities and then formally defined and then applied to the proof of geometric theorems (e.g., Pythagoras' theorem, or a rotational proof of the fact that, in a circle, angles on the same chord are equal). Work on three-dimensional geometry was carried out in the curriculum modules.

### Conclusion

The course described above is not offered as any sort of exemplar to which others might aspire, but merely to indicate how the mathematical preparation of teachers can commence with analysis of school mathematics and be built outwards from there. As for the students themselves, they were a very mixed bunch in terms of prior mathematical background, but the approach of relating more abstract ideas to school mathematics was hugely motivating. So the pay-off is that teacher subject knowledge pertains more strongly to that which they are called upon to teach, and yet there is no implied upper limit regarding subsequent mathematical learning.

Experience with secondary in-service courses revealed many gaps in the basic subject knowledge of *graduate* maths teachers. Most of the contents of the above Number module would be alien to the majority of them and, with respect to calculus, very few appreciative of the ambiguities in Leibniz notation (for example). As for something like circular and hyperbolic functions, there would be appreciation of their behavioural similarities but no awareness of the strong geometric analogy between the two, and little idea of their historical development.

This, then, is the basis of the claim that the mathematical education of teachers should commence with a thorough exploration of school mathematics, together with an introduction to the history of mathematics. The best route for the achievement of such a goal is via appropriately written B.Ed. degrees, which

strongly connect school mathematics with its pedagogy and its historical development. Pursuit of additional mathematical expertise could then be a matter of subsequent in-service professional development.

### Note:

*There were a few transcription errors in part 1 of this article (in the May/June 2005 issue). Two of them were mathematical and have caused some consternation among some readers. Firstly, the equation*

$$\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4}$$

*appeared with an addition sign on the left-hand side instead of the division sign, which obviously made no sense. Secondly, the fraction  $\frac{1}{3}$  should have been shown equal to the infinite repeating decimal 0.333... and not the terminating decimal 0.3333.*

*Peter Ruane (ruane.p@blueyonder.co.uk) is retired from university teaching, where his interests lay predominantly within the field of mathematics education. The first part of this article appeared in the May/June issue of FOCUS.*

We are deeply grateful for the generosity of the following individuals, who have made bequests to the Mathematical Association of America.

Every bequest is a powerful expression of their loyalty, their lifetime involvement, and their faith in the future of the MAA. We remember each of them fondly and with deep personal and professional respect.

Barbara C. Beechler  
*Member for 53 years*

Charles F. Hicks  
*Member for 45 years*

Murray S. Klamkin  
*Member for 56 years*

Kenneth E. Kloss  
*Member for 43 years*



## So You Want To Be A Teacher

By Jacqueline Brannon Giles

In a two-year college, it may be that many of the teachers did not plan to be one when they were in college. Over the years, they may have been business people, engineers, social workers or in other professions. No doubt their need to connect and communicate with people is what drove them to the teaching profession. Or, as in my case, in 1968 a colleague (Dr. Eugene DeLoatch, who later founded the School of Engineering at Morgan State University) looked at my personality and attributes, and decided to encourage me to become a mathematics instructor.

There were great educators in my family: an aunt taught for more than 40 years in the Houston Independent School District; and another aunt worked at Texas Southern University for 30 years, and founded the Texas Southern University Gospel Choir for students that was led by the outstanding musician V. Michael McKay, and had members such as Jennifer Holliday, Finnessa White, and other great singers. So I have always been aware of the role of teachers, since both aunts and my mother nurtured and directed me during my formative years.

My first teacher was my mother (now 81 years young) who was in the first calculus class at Jack Yates High School in Houston, Texas. (One of her classmates was the late Dr. Robert Terry, who was a president of Texas Southern University at the peak of his career.) She patiently played games and puzzles with me when I was a child. She corrected my speech, posture, logic and overall presentation in life, including my choice of clothing and colors. She was a role model and coach. Her opinion was readily given when I had a need, and even when I did not recognize my need. Her keen eye and steady gaze would scan me up and down, and in and out to identify that I had attained the level of excellence that she expected in all of her children.

I claim that this level of attentiveness is needed in classroom teachers. We are beginning to realize that the “whole man

or woman” needs to be attended to. Developing the climate for students to learn often requires the attentive heart of a mother, and the protective demeanor of a father.

Why do I say these things? Well, in the two-year college I have encountered students who were abused, misused, and confused. Of course, there are those students who are gifted, healthy, focused and mature, but we cannot ignore the presence of the others who also need to be thoroughly “taught.” As students in the two-year college, they are seeking a better way, and another chance to make it in life. They are looking for new ways and new paths to reach their goals — goals that are often unclear but certainly different from experiences in their past. We, the teachers, stand before them with a tremendous amount of influence. In some ways, their lives are in our hands for each 50 minute period, and their view of themselves and their potential can be re-shaped, released or distorted and oppressed, depending on what we teachers do.

Even if we did not want to be teachers during our early career, our presence in the two-year college classroom is confirmation of our “call” to be there. And, with each call in life comes a responsibility — in this case, for perpetual preparation and dedication because the students sitting before us may have never experienced the supportive, instructive, corrective and even therapy-type environment we can create for them.

So you want to be a teacher! Yes, because you and I want to do something to assist our nation remain strong and secure. Our role as a mathematics teacher is a role in the “army” at home in the United States. To fail to teach and to learn more about teaching this generation translates into failure to be an effective citizen in the United States. Our mindset should be that all students deserve a chance (and even a second or third chance) to develop into fully functional citizens. We have to believe that even those who have ex-



Jacqueline Brannon Giles

perienced hardships and barriers beyond our comprehension deserve an opportunity to be taught and to be in the presence of someone who points the way to access and to a greater degree of functionality in society.

A conclusion is that our job as mathematics teachers and as educators is not an easy one; it is a challenging one that we do and learn to love to do it well. We realize that our totality of perceptions and experiences are simply an example and that the next generation can build on our core beliefs and extend those beliefs for their generations.

Granted, each generation is dependent on the previous one, yet if taught well, will extend and improve, and move toward a more excellent way for all of mankind. If this vision is set before us, it at least directs us toward a more excellent way, even if our reality is muttered by life’s tendency to drift toward entropy: “a state of disorder and chaos.” Nevertheless, we must aim toward the good, the just, and the excellent and learn to tap into those things that promise true excellence in this professional/academic journey.

*Jacqueline Brannon Giles is on the editorial board of FOCUS. She teaches at the Houston Community College System — Central College. She wrote this article thinking of some of her students and friends who had expressed the desire to be teachers.*

## U.S.A. Mathematical Olympiad Winners Honored

The 34th U.S.A. Mathematical Olympiad Awards Ceremonies took place in Washington DC on Sunday and Monday, June 26 and 27. This event honors the twelve top winners of the annual USA Mathematical Olympiad exam, the premier high-school level mathematical problem solving competition in the United States. The two day celebration began with a Sponsors' Reception at the MAA Headquarters. Representatives of the sponsoring organizations of the American Mathematics Competitions along with members of the MAA Executive Committee, were there to meet and greet the winners and their families. On Monday morning, the winners toured the Cryptologic Museum at the National Security Agency and enjoyed a talk from one of the on site mathematicians on classical cryptography.



The 2005 USAMO winners (in alphabetical order) are **Robert Cordwell** of Albuquerque, New Mexico, **Zhou Fan** of Parsippany, New Jersey, **Sherry Gong** of San Juan, Puerto Rico, **Rishi Gupta** of Cupertino, California, **Hyun Soo Kim** of River Edge, New Jersey, **Brian Lawrence** of Kensington, Maryland, **Albert Ni** of Carmel, Indiana, **Natee Pitiwan** of North Andover, Massachusetts, **Eric Price** of Falls Church, Virginia, **Peng Shi** of Toronto, Canada, **Yi Sun** of San Jose, California, and **Yufei Zhao** of Toronto, Canada.

Dr. Kathie Olsen, Associate Director for Science at the Office of Science and Technology Policy in the Executive Office of the President, was our host in Dr. John Marburger's absence, at the celebratory reception and dinner at the U.S. Department of State. The formal awards ceremony, presided over by MAA President Carl Cowen, took place at the National Academy of Sciences. Dr. Donald G. Saari, professor of mathematics and economics at the University of California-Irvine, delighted the winners and the audience with his USAMO Address titled "*Mathematics is Everywhere.*" The winners received the USAMO Medal,

*USAMO winners from left to right: Zhou Fan, Yi Sun, Robert Cordwell, Hyun Soo Kim, Brian Lawrence, Eric Price, Rishi Gupta, Peng Shi, Yufei Zhao, Sherry Gong and seated on foot: Albert Ni. Standing to the left: Tina Straley, MAA Executive Director and to the right: Carl Cowen, MAA President. Photograph taken at the Einstein Memorial on the National Academy of Sciences grounds, Washington, DC, Sculptor Robert Berks. Photograph courtesy of Robert Allen Strawn.*

named in honor of Gerhard C. Arenstorff, twice a winner of the USAMO and a member of the first USA team in the International Mathematical Olympiad.

Dr. Olsen, in her pre-dinner address, read a letter with laudatory greetings and congratulations from President Bush.

After dinner, Brian Lawrence received the Samuel L. Greitzer/Murray S. Klamkin Award for his superior achievement in the Olympiad exam. Dr. James Carlson, President of the Clay Mathematics Institute, designated Sherry Gong as the seventh CMI Mathematics Olympiad Scholar. Sherry best fulfilled the prize's criteria of elegance, beauty, imagination, and depth of insight. The newest prize is the Robert P. Balles Distinguished Mathematics Student Award, given to each of the twelve winners, in an effort to recognize and reward their high achievement

in the world of mathematics competitions.

The highlight of the evening came when the Akamai Foundation Scholarships were presented to the 1st place winner, Brian Lawrence, 2nd place winner, Eric Price, and the tying 3rd place winners, Peng Shi and Yufei Zhao. These scholarships are in the amounts of \$20,000, \$15,000, and \$10,000 (divided) respectively. By awarding these scholarships, the Akamai Foundation hopes to encourage these and other students to continue their pursuit of mathematics education.

On June 12th the students traveled to Lincoln, Nebraska to participate in the Mathematical Olympiad Summer Program (MOSP) for advanced training for the International Mathematical Olympiad (IMO). This program is funded in part with a grant from the Akamai Foundation.

## Math Youth Days at the Ballpark

By Gene Abrams

About 100 times bigger, and about 100 times louder! That's how I'd compare the environment at the four recent *Sky Sox Math Youth Days* to the environment of my typical math classrooms.

For the fourth consecutive year, the math department at the University of Colorado at Colorado Springs has partnered with the Colorado Springs Sky Sox baseball team and Agilent Technologies to provide mathematics-based activities for kids at the ballpark. The general idea of Math Youth Days is this: throughout the academic year, students in grades 4 through 8 from schools throughout the Pikes Peak region complete various activities which connect mathematics to baseball (samples given below). The culmination of those activities is that the kids (and their teachers) get to spend a whole school day at a real baseball game! (The games have an unusual 10:35 AM start time to accommodate the school schedule.)

There are two math activities that we deliver at the ballpark. The first is called the *Mound Master Competition*. About 30 minutes before the start of the game, eight kids from eight different schools are randomly chosen to come down on the field to participate in this single-elimination contest. Each pair of students is asked a question; the first one to raise her/his hand and give the correct answer moves on to the next round. (If both kids answer incorrectly then the one whose answer is closest to the correct answer moves on. Kids must answer using 'mental math'; no calculators allowed here!) The quizmaster has use of a wireless microphone, so questions can be heard clearly by everyone in the stadium. Students in the stands are encouraged to play along (without shouting out answers), and to root for their classmates! Some sample Mound Master questions are



Students enjoying a day of baseball and mathematics.

given below. Why "Mound Master"? Because the student who wins the contest gets to throw out the ceremonial first pitch of the ballgame!!

The second activity is called the *Stat Star* competition. As students enter the stadium they are given an 8 1/2 x 14 inch sheet which contains information about that day's game, and about baseball in general. One section contains up-to-date player statistics, including such quantities as batting average, home runs, runs batted in (for hitters), and innings pitched, strikeouts, earned run average (for pitchers). Player names and uniform numbers are listed so that kids can keep track of who is on the field during the game. A second section of the Stat Star Sheet contains a handful of information items regarding how some of these statistics are computed. They might specify, for example, that the distance between bases is 90 feet. Or they might be more complex:

A pitcher's Earned Run Average (ERA) is:  $(R / IP) \times 9$  where R = number of runs allowed and IP = number of innings pitched. Earned Run Average is expressed to two places of accuracy. So, for instance, if a pitcher has allowed 50 runs, and has pitched 150 innings, then his Earned Run Average is  $(50 / 150) \times 9 = 3.00$ . [The idea

expressed by ERA is: if the pitcher pitches a lot of complete 9 inning games, how many runs would we expect him to allow each game? About 3.]

The contest works as follows. Starting with the break between the first and second innings, and continuing for the next five half-inning breaks, one of our UCCS math students stands on top of the Sky Sox dugout with the wireless microphone and reads a question. The question is clearly heard by everyone in the stadium. Simultaneously,

an abbreviated version of the question appears on the scoreboard. The schoolchildren are encouraged to work together, work with their teachers, use calculators if necessary, in order to come up with the correct answer. Here are some sample Stat Star questions:

1. Last season my favorite Sky Sox pitcher gave up 90 earned runs in 180 innings pitched. What was his Earned Run Average last season?
2. Henry Aaron holds the major league record for career home runs with 755. In actually circling the bases during those 755 home runs, how many miles did he run? (Give your answer to the nearest mile.)
3. Sky Sox pitcher Jason Young's curveball travels sixty feet to home plate in half a second. How many miles per hour is that? (Give your answer to the nearest m.p.h.)

As you can see, students need to bring some of their own knowledge to the question (e.g. number of feet in a mile), as well as some knowledge of baseball (which they get from the information sheets). Once all five questions have been announced, students are given an inning to complete their answers and write them in the space provided at the bottom of the sheet. They then write their names

next to the answers, tear these off from the bottom of the sheet, and put them in boxes which are circulated through the stands by the UCCS students. Our UCCS students then sort through these answers, distill from them those which have all five answers correct, and place correct answers in a box. The winner of the Stat Star competition is chosen at random from among the correct responses. The winner's name is announced, and s/he gets to come on top of the dugout and receive prizes from both the Sky Sox and from UCCS. More importantly, the winner is showered with wild cheers from her/his classmates! After the game, the Stat Star questions (and answers) are posted at the Sky Sox website at <http://www.skysox.com> for students, teachers, and parents to use as a part of follow-up activities.

All in all, the day is fun and exciting for the school kids (more than 12,000 this year!), their teachers, and the volunteers from UCCS. Both the Sky Sox and UCCS have gotten significant positive feedback regarding all of the activities. The UCCS volunteers provide a valuable community service in an enjoyable environment. The school teachers not only have a good time, but also are able to provide valuable mathematics lessons for their kids. Of course the kids get

the chance to experience, in yet another venue, how mathematics really is all around us.



The Sky Sox mascot looks on as winners from the Stat Star contest are announced.

Over and above all of these positive aspects of the day, in my opinion perhaps the most important thing the kids take away from the day is the following. In large part due to our desire to help destroy harmful stereotypes regarding mathematics ability, prior to each of the Stat Star questions a Sky Sox representative gives a brief introduction of the UCCS math student who will read that question. I believe it makes an extremely

powerful impression on kids when they find out that the reader is "...a captain in the Air Force and a master's degree student in Applied Math at UCCS..." or "... a young woman with a 3.9 GPA. as a

mathematics major, who is attending her first ever baseball game today ...", or "... a high school student who just this morning took a final exam in calculus, and will attend college next year as a math major ...", or, perhaps most compellingly, "... a second year math major and mother of two young students who are in the ballpark today ..."!

*Gene Abrams, is Professor of Mathematics at the University of Colorado at Colorado Springs. He has been a faculty member at UCCS since 1983. He is the author of more than two dozen research articles in mathematics. Along with his col-*

*league Jeremy Haefner, Abrams co-developed the MathOnline program at UCCS in 1998. In 1988 he was named the UCCS campuswide Teacher of the Year. In 1996 he earned lifelong designation as a University of Colorado systemwide President's Teaching Scholar. In 2002 he received the annual Burton W. Jones Outstanding Teaching Award from the Rocky Mountain Section of the Mathematical Association of America.*

**Sample yearlong class or individual projects**

Calculate the total distance traveled by the team for the away games played during last year's Sky Sox season.

Research the salaries of major league baseball players today, including the Colorado Rockies. Compare the salaries of early players (e.g. Babe Ruth) to those of today's players. Make a chart of graph showing salary increases or decreases.

Measure and calculate the average distance your classmates can throw a baseball. (Three throws each.) Make a list of the average distances.

**Sample Mound Master Questions**

If a batter hits a home run, he runs 360 feet around the bases. If he only hits a double (and runs to second base), how far does he run?

The Sky Sox expect 5,000 fans at the ballgame today. If the Sky Sox play 70 home games, and they draw this many fans to each game, how many fans will have attended a Sky Sox game by the end of the season?

The Sky Sox play at the AAA level, one step below the major leagues. Over the years, 75% of Sky Sox players have gone on to the majors. If the Sky Sox currently have 24 men on their roster, how many of the current players would we expect to go on to the majors?

## The Fundamental Theorem of \_\_\_\_\_

By Jeffrey Nunemacher

In 2003 I needed to design a written Honor's Exam in mathematics as part of my duties as the outside examiner for the honor students at a sister institution. As a final question, I wanted something that would touch the whole mathematics curriculum and leave some room for individuality and opinion. I decided to ask the students to state what they considered to be the fundamental theorem in a variety of named areas of mathematics and briefly to defend their choices. Reaction to this question was sufficiently positive — from both students and faculty — that it seems worth posing this question to a broader audience. Thinking about this issue is a good way to solidify one's understanding of a mathematical subject.

What should be the characteristics of the fundamental theorem of a branch of mathematics? Probably we all agree that such a theorem should have far-reaching applications within the branch, should not be too trivial, should not be too esoteric, and should somehow capture the essence of the subject. In addition, it would be nice if the fundamental theorem had a proof founded on basic principles and with some aesthetic appeal. These attributes apply, for instance, to the Fundamental Theorem of Arithmetic, which asserts the existence and uniqueness (up to order) of the prime factorization of any positive integer greater than one.

With this in mind, we might divide undergraduate mathematics into four categories: (1) those for which there is a generally agreed upon fundamental theorem that is called "the fundamental theorem of \_\_\_\_\_"; (2) those for which a best choice can be made and defended; (3) those for which there are several good candidates to be called the fundamental theorem; and (4) those areas for which it is not possible to identify a single most fundamental theorem. Let us briefly consider each of these categories.

Into category (1) fall arithmetic, classi-

cal algebra, single-variable calculus, projective geometry, and Galois theory. All these areas of mathematics have a named fundamental theorem. One could also include the theory of Abelian groups, even though it is a rather narrow sub-area within abstract algebra. In case you have never studied projective geometry (it used to be the most common sort of geometry to encounter in the undergraduate curriculum), the Fundamental Theorem of Projective Geometry asserts that given two four-tuples of non-collinear points in the projective plane, there is a unique projective transformation mapping one to the other. As an undergraduate at Oberlin in the late sixties, I well recall learning each of these fundamental theorems and trying to understand why they were deemed fundamental.

Into category (2) I place multivariable calculus, (ordinary) differential equations, complex analysis, classical statistics, and perhaps Euclidean geometry. In each case I think there is a definite best choice (with which you may certainly disagree). For multivariable calculus it is the general Stokes's Theorem, which relates integration over a set to behavior of some kind of antiderivative on the boundary of the set. In dimension 1 this result reduces to the Fundamental Theorem of Calculus, and in dimensions 2 and 3 it yields Green's Theorem, the Divergence Theorem, and the classical Stokes' Theorem.

For ordinary differential equations the fundamental theorem is the basic existence and uniqueness theorem, which guarantees that every smooth system of ODEs has a smooth local solution (the flow). Since we now typically use software to sketch these solutions so that we can study the geometric behavior of the flow, it is clearer than ever that this theorem, which asserts the existence and uniqueness of the flow, is the central result of an ODE course. For complex analysis most mathematicians would declare that the fundamental theorem is

Cauchy's Theorem, which leads directly to the Cauchy Integral Formula and the Residue Theorem. Actually when I teach complex analysis, I prefer instead a broader theorem, which includes Cauchy's result. I call it the Grand Equivalence: the class of analytic functions can be defined on an open disk in the complex plane by any one of the following conditions: complex differentiability (differential calculus); Cauchy's Theorem (integral calculus); the Cauchy-Riemann equations (partial differential equations); power series (infinite series); and conformality (geometry), at least away from points where the derivative is zero.

For classical statistics the fundamental theorem is surely the Central Limit Theorem, which relates the Gaussian distribution to a well behaved sum or average of a large number of independent random variables. It is this result which sheds light on the behavior of large samples and allows the construction of the most common kinds of confidence intervals and significance tests.

Finally, for Euclidean geometry the natural candidate is the analog of the projective fundamental theorem. My candidate is the assertion that given any two three-tuples of non-collinear points in the Euclidean plane, there exists a Euclidean transformation, which maps one to the other, if and only if the respective distances are preserved, and such an isometry is unique whenever the three distances are distinct. If one takes seriously Klein's view that geometry is the study of those properties of a space which are preserved under the action of a group, then this assertion is the basic result upon which all of the rest of Euclidean geometry depends (even though a typical high school student never encounters it!).

Let us now consider Category (3), consisting of areas for which there are several reasonable alternative choices. Into this grouping I place linear algebra, basic real analysis, mathematical logic,

group theory, and advanced real analysis (measure theory and Lebesgue integration).

Here are three possible choices for the fundamental theorem of linear algebra with a one- sentence reason for each.

(1) Every nonzero vector space has a basis. If linear algebra is the study of vector spaces, then the space is completely described once a basis is known.

(2) Strang’s Fundamental Theorem (see his *Linear Algebra and Its Applications*): Given any matrix there are associated four fundamental subspaces, which have dimensions determined in terms of the rank and certain orthogonality behavior. If linear algebra is the study of matrices or linear mappings, then the geometric action of a rectangular matrix on Euclidean space is fully described by this theorem.

(3) Every square matrix is similar to a matrix in Jordan canonical form. This statement includes diagonalization theory and is the key result in many matrix calculations in linear algebra, physics, and elsewhere within mathematics. I find it hard to choose any one of these over the others. Of course, some maintain that linear algebra is a subject without any theorems, but that position is too extreme for most of us.

For basic single-variable real variables, if we have in mind the structure of subsets of the real line, then a reasonable choice is the compactness and connectedness of a closed bounded interval. But if we focus on functions rather than on sets, we might select the fact that a real-valued continuous function on a closed interval attains its maximum and minimum as well as all values in between them.

These two results are related, but they are not the same theorem. Which is more fundamental? Notice that unless we construct the reals from the rationals, we cannot select the completeness of  $\mathbf{R}$  as our fundamental theorem, since we desire a theorem and not a postulate.

In mathematical logic there are two possible choices, both due to Gödel. The Completeness Theorem guarantees that in any first order theory anything which is true (in all models of the theory) is also provable. This result shows the strength of the concept of first order logic. But the Incompleteness Theorem guarantees that in any (simple) theory strong enough to encompass arithmetic there exist true statements which are not provable. This result focuses on the limitations of classical logic and is the single result of logic most important to non-logicians.

Here are two candidates for the fundamental theorem of group theory of very different levels of sophistication. The First Isomorphism Theorem decomposes any group homomorphism in terms of its kernel and, in particular, determines all homomorphic images of a group. This result focuses on the nature of mappings between groups. The very difficult classification theorem identifies all finite simple groups. Thus it establishes the building block from which all finite groups are made.

Finally, here are two candidates for the fundamental theorem of advanced real analysis. The Dominated Convergence Theorem provides a useful and simple condition under which the Lebesgue integral behaves nicely with respect to convergence. It is likely to be the most used result from this theory in applications. But it can be argued that the complete-

ness of the spaces  $L^2$  and  $L^1$  is a more fundamental result. This fact of completeness establishes that the extension to these natural classes of Lebesgue integrable functions has filled all the holes left by consideration of only continuous or Riemann integrable functions.

What is left to fall into Category (4) (those with no obvious choice for a fundamental theorem)? Here is a short list: (point set) topology, combinatorics, probability theory, graph theory, numerical analysis, and number theory. Some would argue that the fundamental theorem of combinatorics is Newton’s binomial theorem (perhaps, with its generalization for non-integral exponents). While this result underlies the solution of many different counting problems, I remain unconvinced that it deserves the title of the fundamental theorem.

If number theory is merely the natural continuation of arithmetic, perhaps we should regard the Fundamental Theorem of Arithmetic as the correct result as well for number theory. That choice, however, seems too mundane when we contemplate the Prime Number Theorem or Wiles’s proof of Fermat’s Last Theorem. I leave you to ponder the appropriate fundamental theorem in these areas.


*Jeff Nunemacher enjoys teaching a wide range of mathematics courses and some computer science at Ohio Wesleyan University, where he has been chair for twelve years. He was educated at Oberlin and Yale and has previously taught at UT Austin, Oberlin, and Kenyon. (email: jlnunema@owu.edu)*

*Editor’s note: Responses to this article are encouraged!*

<b>FOCUS Deadlines</b>			
	November	December	January
Editorial Copy	September 16	October 15	November 15
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# 2005 Award Winners for

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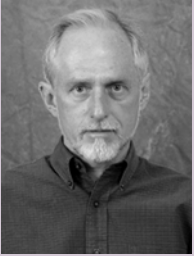
Chris Meyer  
Pacific Lutheran University

NORTH CENTRAL



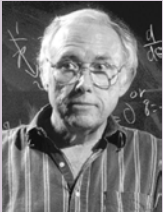
Ivy Knoshaug  
Bemidji State University

IOWA



Keith Stroyan  
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# Distinguished Teaching





## “I Like Change” An Interview with Tina Straley

By Don Albers

Tina Straley was named Executive Director of the MAA in January 2000. Her path to Washington and the position of Executive Director had its start in Brooklyn with stops in Florida, Alabama, and Georgia. It's unlikely that there is a standard path to becoming an executive director, but in the interview that follows we examine her trajectory and learn what attracted her to the job. During her first five-year term, she was at the center of growth and many changes, especially in the area of professional development. Now in her second term, she is excited about further growth of professional development programs and starting the new conference center. For someone attracted to initiating new things and change, the MAA has proved to be a good match for her.

The following interview was done in Dr. Straley's office in March of 2005.

**Don Albers:** Where were you born?

**Tina Straley:** I was born in Brooklyn, New York. We lived in a six story apartment building of mostly young families. I remember my friends there, and I remember the neighborhood park. My aunt and uncle and two cousins and my grandparents also lived there. But then we all moved to Florida when I was seven.

**DA:** This had a real nuclear family aspect to it. Do you remember things that you particularly enjoyed as a little girl before going to Florida?

**TS:** I always enjoyed drawing. My sister Carol, who is older than my cousins and I, used to play school with us. She was the teacher. We used to write stories, tell stories, and do drawings. Although this was in Brooklyn, behind the apartment building was a big empty lot. We used to catch butterflies and lightning bugs there. To us the lot was a field. There was a little shack on the field with nothing in it. The kids decided to make it a clubhouse.



*Young Tina minus green paint.*

Somebody got paint and brushes. I was very small, and I was also one of the youngest. My mother dressed me that day in brand new blue jeans, and remember I said I was the smallest. We only had green paint. All the big boys painted the roof and the paint dripped everywhere. I went home covered from head to toe in green paint.

**DA:** How did your mother react to a green Tina?

**TS:** That day she was not pleased, but she never punished me for anything. My mother was the kind of person that you could confide in. She was supportive, and she had a wonderful sense of humor. She loved laughing. If everybody went to a restaurant and laughed, then for her it was a good time and she thought the food was excellent. If there was no laughing, then the food couldn't have been very good either.

**DA:** Were your parents also born in the U.S.?

**TS:** Yes, both in New York. My grandparents all emigrated from Eastern Europe in the big wave of immigration from 1890 to 1910.

**DA:** Did your mother work outside of the home?

**TS:** When I was young, she was a stay-at-home mom. When I turned 12, she went to work. She was a saleswoman in women's clothing.

**DA:** And your father, what stands out in your memory about him?

**TS:** He was a good guy. Everybody loved him. In Florida, he was a food sales manager. His first product was beer, later soft drinks, and then a whole line of packaged food. He traveled extensively throughout South Florida. He knew everybody because not only would he meet the people working in the stores, but the people who came to shop as well. He always carried candy because he had a real sweet tooth. When I was small, he'd always give me little candies. To my daughter Jessica, I think that stands out most. Whenever she saw him, Grandpa Al always had some little hard candy for her.

**DA:** You had an early interest in drawing and still take art classes. Did you have an early interest in reading, too?

**TS:** It was not until I was in school that I became interested in reading. I first remember reading books on my own in third grade. I read the *Bobsey Twins* and *Gulliver's Travels*. I also read one Nancy Drew after another. We moved a lot and books were hard to move. We had a set of encyclopedias that had been bought for my sister and some assorted books my mother must have liked. In addition to the ones I mentioned, there were short stories by Guy de Maupassant, plays of

Shakespeare, a set of encyclopedias, and a few other books. One was a book called *Nobody's Boy*, which was very sad. By the time I was in the fourth grade I had read all the children and teen books we had. In fourth, fifth, and sixth grades I read the encyclopedia, the plays of Shakespeare, and the short stories of de Maupassant, not all, of course, but a good number of them. By sixth grade, I was an avid reader.

### Prison Windows

DA: Was school fun at that stage of life?

TS: I didn't like going to school. I liked doing well in school. But I didn't like being in school. I remember feeling the windows looked like prison windows. I didn't like school until activities became available. But I still didn't like the classes. I liked all the activities. That's why I went to school. And yet I was a good student. I worked hard, and I got good grades, and I didn't realize it at the time, but I look back now and I think I was always pretty competitive for grades and positions.

DA: How did you choose a college?

TS: I went to a public college because I had assumed that that was all we could afford. I went to a good one—The University of Florida.

DA: What was it like?

TS: Big and intimidating. Adjusting to a big state university having come from Miami Beach (which was a very close knit community) was like going from a small town to a big city. My best friend from high school was my roommate, and the other students from Miami Beach formed a community. In that sense it was certainly not going alone. I had a ready made support system. I had a good time, and I did well.

### Are You a Math Major Yet?

DA: As a freshman, did you have an idea what you were going to major in?

TS: I started out as a double major in English and math, because I really didn't

know. I considered chemistry, but never really pursued it. I always wanted to do something in art, so I think it may have been the first or second semester that I



*Math major Tina ready for the next challenge.*

decided I would try architecture because I thought it would combine mathematics and art. The first course in the architecture program was on building materials. Talk about a bad way to start. It was about cement, and I decided then I didn't want to be an architect. I dropped the course, the only course I ever dropped in all of college.

I continued to double major in English and in mathematics. In the summer between my freshman and sophomore years, I visited my sister who had recently moved to Atlanta.

DA: So when she invited you to Atlanta, you went up right away.

TS: I was looking for a job for the summer, and I was having trouble finding a job in Miami. I went up to Atlanta, and I did get a job. I had a fabulous time, so I decided to just stay in Atlanta. A friend of ours, the Dean of the Business School, encouraged me to study actuarial science at Georgia State University.

DA: At that point were you still majoring in English and math?

TS: When I went to Georgia State, I just continued along in calculus and beyond. However I had to start over in the sophomore English sequence. Therefore, I was ahead in math, but I was no longer ahead in English.

DA: So that helped you make the choice.

TS: I got to interesting courses in mathematics faster than in English. And truthfully, I liked calculus a lot more than any other course. I loved the ideas of infinity and infinitesimals.

At the end of my sophomore year, I had to declare a major. I thought, "I bet if I major in math, I could always get a job." There's actually more to the story. I had a scholarship from the State of Florida to study actuarial science at Georgia State. That was part of my reason for leaving Florida and going to Georgia State. As long as I was studying applications of calculus, which comprised the first actuarial exam, I liked it. At the beginning of calculus class, everyday, Mr. Eason, my instructor, walked across the front of the room, he'd look at me and ask, "Are you a math major yet?" I'd say, "No." Then he'd shake his head and start class.

Classes were five days a week. He'd come in the next day, and he'd start class the same way. "Are you a math major yet?" "No." Shake his head, and start class.

I had been taking actuarial science for a couple of quarters. I met with the actuarial science advisor to plan my schedule. For the junior year and beyond, the program was all business courses. I asked, "When do I take history and languages? When do I take philosophy, if I have to take all of these business courses?" He said, "When you're a math major." He was being sarcastic. I looked at him and said, "bye." I went directly to the dean's office and changed my major to mathematics. In the calculus class the next day, Mr. Eason asked, "Are you a math major yet?" And I said, "Yes." And he said, "Good."

DA: Was Mr. Eason a good teacher?

**TS:** Excellent. He was a country guy. He used to come to class with a straw in his mouth. Maybe it was because he couldn't smoke in class.

The next course I took at the end of my sophomore year, the first upper division course, was real analysis. The next was set theory, which I loved.

**DA:** You eventually ended up doing universal algebra.

**TS:** Right. I liked set theory, linear algebra, and modern algebra. I minored in philosophy because I loved logic. When I was going into my senior year, I started getting nervous about what I would do after graduation. I went to school that summer and took the four education courses I needed to get certified to teach high school mathematics. I practice-taught one of the terms of my senior year because by this time I actually had gotten ahead in credits. I graduated at the end of winter quarter.

**DA:** Had you been thinking graduate school?

**TS:** When I graduated, Georgia State nominated me for a Woodrow Wilson fellowship and they had just started a new masters degree program in mathematics. I was dating my future husband. He was in the masters degree program at Georgia State. I realized if I applied for the Woodrow Wilson, I'd have to go someplace else. So I never applied.

The head of the department at Georgia State asked me if I would like an assistantship there, so I said yes. My bachelor's degree was okay, but not great. However, Georgia State had hired three new Ph.D.s just out of graduate school to offer this masters program. They were all gung-ho. Once I finished the masters program, I had a really strong background. I went back home to Florida, and I taught in the high school I had attended for one year. Then I got married and went back to Atlanta, where I taught for a year at Spelman.

**DA:** In that period, the mid-60's, were there many white faculty at Spelman?

**TS:** No, not a lot, but not what I would call few either. The students were excellent. The faculty was small and collegial and very supportive. The President and the faculty encouraged the students to pursue Ph.D.'s and return to Spelman as faculty and role models.



*Tina with her daughter Jessica, who recently completed her Ph.D. in English.*

**DA:** So you were there for one year.

**TS:** Right. And then Wilt and I went to Auburn to graduate school.

**Auburn**

**TS:** Then my husband Wilt and I started graduate school at Auburn. Wilt wanted to work in topology, and Auburn had an excellent faculty in topology.

**DA:** Many of the Moore refugees went to Auburn from UT-Austin after R.L. Moore's forced retirement from the University of Texas-Austin.

**TS:** Auburn was one of the principal places the Moore folks went. I was interested in algebra, and I very naively thought every place will have algebra.

Auburn was the place that Wilt wanted to go, plus I had an NSF traineeship there, so we went to Auburn. When I got there I was very surprised to find that the algebra was almost totally lacking. I started working in topology.

Lo and behold Auburn hired Curt Lindner from Emory in combinatorics and universal algebra. Lindner had just gotten his degree under Trevor Evans, a leader in universal algebra. Evans had given a wonderful talk at Auburn. It just made me wish that I was in a place where I could be studying that topic. And then Curt gave a talk as part of his interview on similar topics. And I just kept my fingers crossed that they would hire Curt and he would take the job, and he did. I started studying with him right away. The next year, I finished my degree.

**DA:** That's fast.

**TS:** Yes. I finished in three years. Curt led a seminar of advanced students reading research papers. I read a paper by Jean Doyen. Because it was in French, I had to translate it; so, therefore, I had to think about every word. Curt said, "Gee, that would be such a great result if it could be generalized," and that's what I did. That was the first chapter of my dissertation and my first published paper in the *Journal of Combinatorial Theory*.

**DA:** Did you and Wilt finish at the same time?

**TS:** No. I didn't have to teach, and I got onto this problem very early. It's easier when you have a well-defined problem that you can solve; once you solve it, you're pretty much done. I went on and did other generalizations as part of the dissertation, but the first generalization was the big result. Wilt had heavy teaching obligations at some points. He took five years, an average length of time.

**DA:** So you had two years to do other things while he was finishing.

**TS:** Right. And again, Auburn was wonderful to us. They gave me a full-time instructorship for two years.

DA: Then he was done. What was the next stop?

Kennesaw

TS: Well, it was 1973, and the job market was awful. When I had finished two years earlier in 1971, I actually got letters from mathematics departments at major research universities asking me to apply. No one was writing letters to anybody to apply in 1973. We needed two jobs in the same location and we had a new baby. We went back to Atlanta and took positions not at all where we thought we would have ended up. I went to Kennesaw Junior College, just temporarily, on a one-year appointment. And Wilt took a job in the business school at Georgia State under the condition that he would study business courses.

DA: Wow. That's quite a condition.

TS: And he did. He actually completed two masters, one in finance and one in international business, and he changed his career to the quantitative areas of business. I took a job at a two-year college, thinking for just one year, and then I'd look for a real job.

Well, I had a baby, a nice group of colleagues, and pretty good students. I stayed there a second year and a third year. Kennesaw Junior College converted to a four-year college after just a few years. We had a faculty of young Ph.D.s, and we created a whole program from scratch. We could do anything we wanted. It was fun. Kennesaw grew at six to ten percent a year. We kept adding students, adding programs, and I kept getting promoted. There were so many opportunities to be creative and to do new things. What I thought was just a stop-gap position for one-year turned out to be a great place for a career. Again I lucked out.

DA: And you had a young daughter to care for.

TS: Right. I was divorced and a single mother, so it was hard to leave a place that treated me really well. The roots of my career were strange, but at that point it was actually on a pretty nice trajectory. Kennesaw was growing, and I was growing with it. The thing that I had to give up was research. At first I wrote papers



Tina with family: left to right: Tina, Father, Mother, Carol, Jessica, and Wilt.

with Curt and on my own. I had to do my work on weekends and during vacations because the demands of the job at Kennesaw were very great. At a place that is growing and changing, in addition to the normal academic committees, we always had new responsibilities, courses, policies and procedures, and programs to develop.

DA: As the institution grew, you said you were getting promoted.

TS: Most of my years at Kennesaw I was the ranking female on the faculty. And there weren't that many men promoted to full professor before me.

DA: As you said, you were getting promoted. When did more serious leadership roles begin to develop for you at Kennesaw?

TS: In 1987, I became department chair. But when you say leadership, it didn't just start there. As we transitioned from division to department, there was a leadership group in the math department of

about five of us, with Chris Schaufele as department chair. I was the first faculty member to chair the college-wide promotion and tenure committee. That was before I was a chair. In the winter of 1987 we split the department between mathematics and computer science. I became acting chair, and then after a search, I became chair.

DA: Then later you went to NSF.

TS: I went to NSF in 1993.

Earth Algebra

DA: Before coming to NSF, what stands out as your biggest accomplishment?

TS: It's hard to say just one thing. But it was clear that college algebra was a disaster. In a department meeting, the curriculum committee chair presented suggestions, cosmetic

changes, for revising college algebra. I thought, "I've been in this same discussion hundreds of times. I had read the *NCTM Standards*; this was 1989 or 1990. I told the department, "I want to throw out the course and start over." And I repeated the recommendations from the *Standards*. I said, "I want group work. I want real world problems. I want writing. And I want technology."

"Now, who's willing to create a new course?" Chris Schaufele wanted to revise the business calculus course that followed college algebra. I said, "first you've got to change college algebra." He and Nancy Zumoff headed up the project. Their challenge was how to make college algebra interesting.

Nancy and Chris came up with the idea of the environment being the context of the courses. That was brilliant. They created Earth Algebra. I had contacts at Georgia Power Company through the state mathematics coalition. They gave us a \$6,000 grant to pay part timers. Nancy and Chris each had a two-course reduction to develop the course in the

spring, followed by summer stipends from Kennesaw. I worked with them on the first grant proposal and some on subsequent proposals. It's a long story.

The project received both FIPSE and NSF funding. In fact, when NSF funding came through, we already had funding from FIPSE. We received additional funding from NSF/DUE. Nancy and Chris had funding from both agencies for the next 10 years for Earth Algebra and other courses with similar design.

**NSF**

**DA:** When you went to NSF in 1993, what were you thinking? What was motivating you? Were you just looking for something new? Were you getting bored at Kennesaw?

**TS:** I had been department chair for six and a half years, and that is really long enough.

**DA:** Your daughter Jessica was settled at Cornell.

**TS:** She was going into her junior year. I went to NSF and she went off to Oxford University, England.

**DA:** She would have been about 20 in 1993. So she was fairly independent.

**TS:** Yes. It was a good time for me personally. The department was in good shape, and it was time to make a change. One person shouldn't be department chair forever.

**DA:** But why NSF?

**TS:** Because of Earth Algebra, and the fact that I had reviewed at NSF several times. As department chair I had brought teacher preparation in secondary mathematics into the department. NSF was looking for someone with that interest. My plan in life has always been when an opportunity comes along, I take it. I've never planned out my career. But there's something worthwhile about being ready to capitalize on new opportunities. In the case of NSF, here was an opportu-

nity to do something new, to do something exciting, to make a difference.

So I went back to Kennesaw, but in a totally different position.

**DA:** That sounds right for someone who likes change.

**TS:** That's right. I like change. The portfolio that I was given was a bit ambitious. I was overseeing technology, the library, sponsored programs, assessment, space assignments. There were some things in there I told the VP I didn't want, but I got them anyway. The next year, I also became Dean of Graduate Studies. In four years I was able to restructure the position into a really great job and worked on building up scholarship, external funding, and graduate programs.

**The MAA Calls**

**DA:** Well, it wasn't too much later that you became aware that Marcia Sward was going to retire as Executive Director of the MAA.

**TS:** Right. That was the beginning of 1999.

**DA:** How did that develop? You had been very active in the MAA at the section level prior to that.

**TS:** I also was active at the national level. I was the chair of the local organizing committee for the MAA's first solo MathFest in Atlanta, and I was chair of the program committee for the UCLA MathFest. I was active both at the national and the section level. I had been Newsletter Editor and Chair of the Southeastern Section. I was also working on the MAA books program as Notes Editor.

**DA:** You may not have had much of an idea of what an executive director did.

**TS:** Very little, except that I thought a lot of Marcia. When I heard that she was retiring from this position, I told her how sorry I was that she was leaving. I remember telling her how much MAA meant to me. She thanked me for that. That was at the Joint Mathematics Meetings in San Antonio.



*The Executive Director does not always wear evening clothes to the office.*

**DA:** This was 1993-1995. You were there two years as a rotator and then you returned to Kennesaw.

**TS:** Right, but not to what I was doing. You can't go home again. You're different, and they're different. Kennesaw was keeping my position for me, but I realized after two years, I couldn't go back in the department and say, hey, I'm back. That's not fair to the department; they had moved on and I had, too.

I told my Dean, Vice President, and President that if I came back to Kennesaw I would want to do something different that would capitalize on what I had learned. I had started a third year at NSF when I got a call from Kennesaw and, as they say, got an offer that I couldn't refuse to become associate vice president. And again, it seemed like a great opportunity.

DA: So did you begin thinking about the position then?

TS: No, not at all. In San Antonio Wade Ellis stopped me and said, "You know, Marcia Sward is retiring as Executive Director." He said that he was on the search committee for Marcia's successor. I thought he was going to ask me if I had some good ideas for people to nominate for Executive Director. Instead he told me the search committee wondered if I would be interested in applying. I was totally blown away. I would never on my own have applied for this position if people didn't suggest it to me and ask me to think about it. So I did. I applied.

DA: Has it turned out to be different from what you expected?

TS: Of course I had no idea of what the job would be like, seeing it from the outside and not having worked here. I remember talking to you about it during the interview. For example, I realized that in being the executive director I would have a level of responsibility that I didn't have within a state university system, where there was always someone I was reporting to. As MAA's Executive Director, I wouldn't have that kind of support.

I knew that I would have to deal with everything from budgets to personnel, that the entire operation would report to me. I never thought of myself as a business person. I never thought of myself as anything but an academician in some cog of the academic wheel. I was concerned about how I would handle these new responsibilities. Raising money through a grant and worrying about the bottom line of an organization's finances are two different things. I never had to worry about how to pay the bills. And that's just an example of the feeling that you're out there on a limb in this position, responsible for the people who work for you, and responsible to all the members. I never had that sense of responsibility before.

I knew that those things would be part of the job, and I was truly concerned about my being able to do that job.

DA: What was the biggest attraction of the job?

TS: The biggest attraction of the job was the MAA. This is the association that I had been involved with for a long time, and in which I have felt very much at home. To me, the MAA was my professional family, my professional support network. I have very strong feelings about this association and the members. Coming to work for an association that I already cared that much about was the top attraction.

DA: You have outlined your biggest fear of the job. In practice, what have you had the most fun with?

TS: I love creating new things. I like change. I like coming up with ideas and then working with a group on those ideas; the ideas get refined, and they get better. I love setting new things in motion, going in new directions. That's what I still like best about the job. Surprisingly, the running of the business has turned out to be something I like as well.

DA: What's been the toughest part of your job?

TS: The toughest part is personnel. We're very lucky that we have an excellent staff of very dedicated people. But there's been some change to get to that, and some hires that didn't work out, and some people who were here who have left. Most of the staff have been, even from the very beginning, just excellent, highly qualified, dedicated people who are wonderful to work with. Firing somebody is the worst thing I have had to do.

### Big Goals

DA: You've now completed your first five years, and you recently signed a contract for five more years. What do you hope to accomplish in the next five? What are your big goals?

TS: My number one priority has been and remains professional development. Five years ago we had a well established and highly appreciated publications program. We still do. In fact, it's bigger and

better. But we had virtually nothing in the area of professional development programs. We had national and sectional meetings with mini-courses and short courses. But in the whole area of ongoing professional development activities, we had little to offer. Some individuals, on their own, had obtained grants for the MAA to deliver workshop programs; and I, with Brian Winkel, was one of those people.

I came here with an interest in delivering programs. We did not have a director of programs. We had a very small programs department of one full-time person and two half-time people. That department supported committees, placement tests, sections, student chapters, and liaisons. But they were not involved in delivering programs.

We've come a long way. We have a full spectrum of programs now. But it's very highly dependent on National Science Foundation funding. I would like to get it to the point of being a self-sustaining program integral to the MAA. I think we're headed in that direction. What Michael Pearson, as Director of Programs, has done is fantastic, and he has wonderful ideas for how to grow the program and how to bring it to self-sustenance.

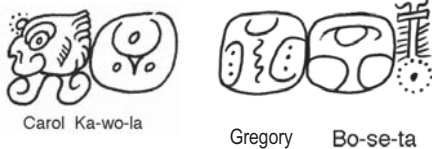
DA: You'll soon have the new conference center. That should help out a bit, too.

TS: That will be a great step forward. In fact, I hoped that I would be here for two five-year terms, and I'm delighted that I've been invited to stay on a second term. It would really hurt to leave at this point when we're just getting the conference center off the ground. This is something that I've dreamed of, and I'm, again, very fortunate to have the kind of support that I got from you, Don, and from Jerry Alexanderson, and the generous financial support of Paul and Virginia Halmos to make this dream come true. What an opportunity! And it's my dream to see it come to fruition.

*Don Albers is Associate Executive Director and Director of Publications at the Mathematical Association of America.*

**Third Annual Mathematical Study Tour—Home of the Ancient Maya**

The Maya tour was our second tour with the MAA and we plan on participating in others. We enjoy the mathematics history strand that is a core of these tours. It becomes clear that mathematics has a significant impact on the development of cultures. The excellent leadership has developed outstanding tours that are both educational and fun, and the tours have left enough free time to enjoy relaxing in the areas visited. We have also enjoyed making new friends in the mathematics community through these tours.



*Gregory Dotseth, Professor Emeritus,  
Mathematics Department,  
University of Northern Iowa  
and Carol Dotseth*



As a participant in all three MAA study tours, I was struck by the focus of this trip. In Greece, the topics covered a time span from Pythagoras through issues in mathematics education in Greece today. In England, our study ranged from the construction of Stonehenge to decoding messages during World War II. This year in Mexico we immersed ourselves deeply into the contributions of a single culture, with special emphasis on the classic era of Mayan civilization. The sites were both beautiful and extensive and the guides were superb, offering insights only practicing archaeologists can give. Additionally, what a treat it was to be able to experience new flavors in the regional cooking!



Linda Li-ni      Joel Ch'o-le

*Joel Haack, Interim Dean, College of Natural Sciences  
Professor, Department of Mathematics,  
University of Northern Iowa  
and Linda Haack*

I was excited by this trip because I have always had a fascination with ancient civilizations and how they made sense out of their world. It's great fun to travel with a mathematics group. These individuals frame questions particularly well and organize fragmented information into coherence. I returned with new respect for the Mayan civilization and new appreciation for the archeologists who unearth the remains and interpret their findings. An unanticipated bonus was gaining some insight into contemporary issues in Mexican society and especially issues that affect the more than 7 million Mayans in Mexico now.



Joan Ch'o-na

*Joan R. Leitzel  
President Emerita, University of New Hampshire*



The Maya trip was an exciting adventure for this budding historian. Our three guides gave us the most current insights into Maya cosmology. Numerologist Christopher Powell described the construction of prevailing ratios found in Maya architecture. Archeo-astronomer Alonzo Menendes demonstrated how building alignments provided sight lines for celestial events. Finally, Alfonso Morales led us through the complex structure of Maya writing. I'll be reporting results at the Mediterranean Studies Conference this July as well as the undergraduate Mathematics Colloquium at Dowling College in the fall.

*Sandra Monteferrante  
Professor, Department of Mathematics  
Dowling College  
and Steve Frohock*



Sandra Sa-na-ba



Steve Tzi-bi

Where do I begin to talk about this trip? I went because I knew that I would probably never be adventurous enough to do something like this by myself. What did I learn? The list is long, but I think what I will take away the most is a much greater appreciation for the Mayan people. Their science and mathematics were much more advanced than I ever imagined. My fellow travelers were awesome! It was a grueling trek some days and I never heard a single complaint from anyone. In addition, our guides (Chris, Alfonso & Alonso) were so knowledgeable and helpful. My personal thanks go out to everyone.



Herb He-ba

*Herb Kasube  
Professor, Department of Mathematics  
Bradley University*



I teach the History of Math course both at the undergraduate and graduate level, and I always include a section on the Mayan numeration system. So I felt that this MAA Study tour would enhance my knowledge on the subject as well as get to see first hand some of the great Mayan archaeological sites. Well, both expectations were exceeded!!! The expertise of Alfonso, Alonzo, and Christopher is indeed admirable. It was also a distinct pleasure to meet all the wonderful people on the trip. What a great bunch!!! I'm looking forward to future study tours.



Phillip Pi-li-pi

*Phillip Scalisi  
Professor and Chair, Department of Mathematics  
Bridgewater State College*

I embarked on the tour to learn about Mayan mathematics which I believed I'd be able to incorporate into an inservice course for teachers. But I was also looking to fellowship with a group of mathematicians with similar interests. Needless to say, both missions were realized. As to the trip itself, of course there were the wonderful sites at Chichen Itza and Palenque that I'd been reading about, but there were some lovely gems of surprises along the way, such as the bumpy ride through the jungles to Bonampak to view the beautiful, still vibrantly colored stelae depicting Mayan court life. I was so impressed by our guides and teachers — Alfonso, Alonso, and Chris. Their love for their work and these enthusiasm for sharing what they knew with us was a joy. As with all great teachers, they inspired me to learn more!



Carol Ka-wo-la

*Carole Lacampagne, Retired, U.S. Department of Education*



It was extraordinary to watch Alfonso read through a long heiroglyph text almost like reading the newspaper.



Robert Wa-bu-ta

*Robert Bumcrot  
Retired, Department of Mathematics  
Hofstra University*

*Name glyphs drawn by Cristin Cash  
of the Maya Exploration Center*



## Archives of American Mathematics Spotlight: The New Mathematical Library Records

By Robin Howard and Kristy Sorensen

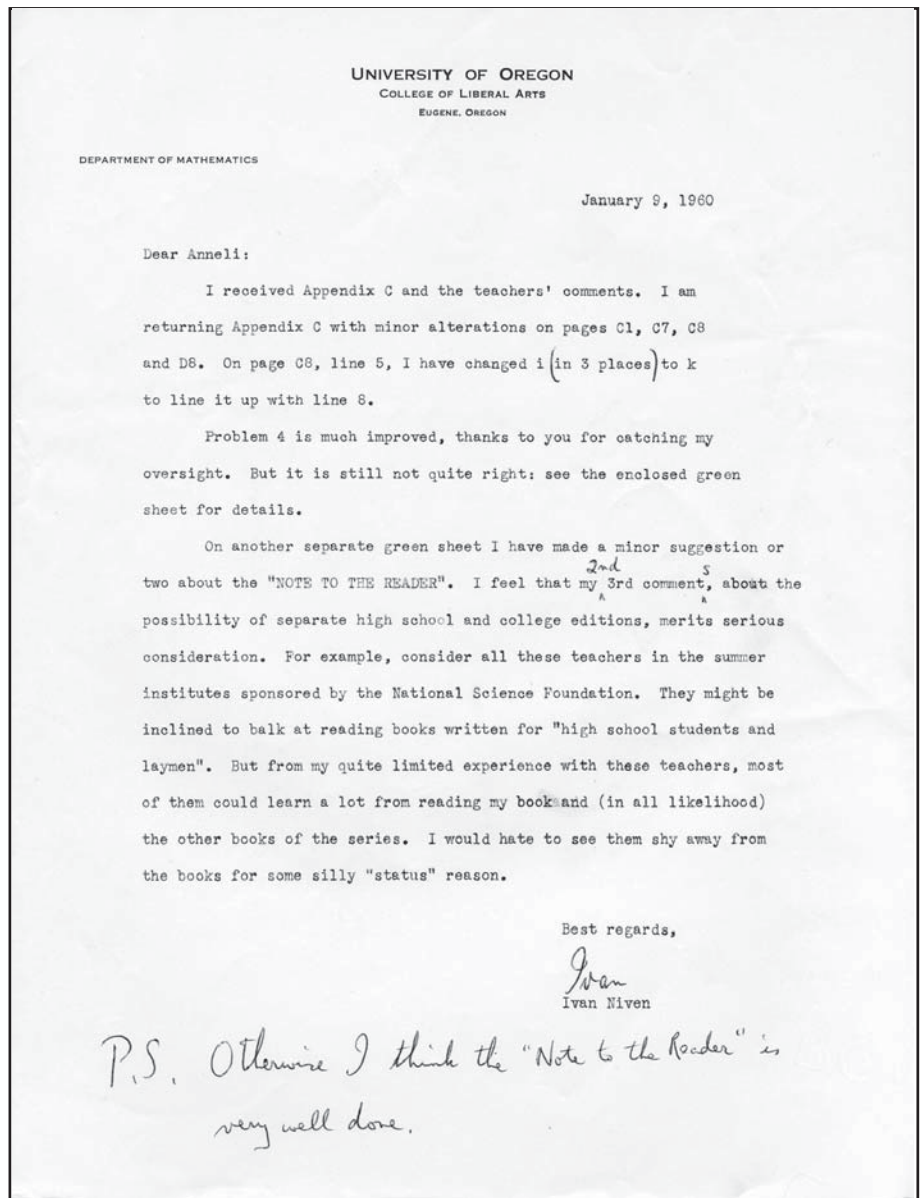
The Archives of American Mathematics is pleased to make an online inventory of the *New Mathematical Library* Records available to researchers. This collection documents the work of editor Anneli Lax to bring engaging mathematical texts to young students of mathematics, and provides an inside look at the work of mathematical publishing.

The *New Mathematical Library* (NML) is a series of monographs on various mathematical topics. They are not textbooks, but are meant as supplements for the interested high school or early college student. The monographs are written by individual mathematicians, and at the NML's beginning, most of the authors had not written for the high school level prior to their work in the series.

The first monographs appeared in 1961 and were originally published for the School Mathematics Study Group Monograph Project, begun in 1958 to remedy the perceived shortage of well-written mathematical materials for young people. Initially published by Random House and the L.W. Singer Company in conjunction with Yale University, the Mathematical Association of America took over publication in 1975.

The NML was intended as a temporary project, set to come to an end after the publication of approximately thirty monographs or after commercial American publishers began to produce similar books for high school students. Instead, books are still being published in the NML series as of 2003, though at a slower pace than during its height in the 1960s.

Anneli Lax, the NML's technical editor for almost forty years, was born Anneli Cahn on February 23, 1922, in the Kattowitz, part of Germany at the time, but part of Poland soon after. Her family left Kattowitz for Berlin in 1929 to escape discrimination against Germans, but in 1933, to escape discrimination



A letter from Ivan Niven to Anneli Lax regarding the publication of his book, *Numbers: Rational and Irrational*, the first book in the *New Mathematical Library* series. From the *New Mathematical Library Records*, ca. 1929, 1957-1997, *Archives of American Mathematics*, Center for American History, The University of Texas at Austin.

against Jews, moved to Paris, Palestine, and finally, in 1935, to the United States. She studied mathematics at Adelphi College and, following her graduation in 1942, she became an assistant researcher

in New York University's Aeronautics Department and joined NYU's Courant Institute as a graduate student in 1943.

NML transition from  
Random House to MAA

Sept 18, 1974  
1:20 PM

Dear Jason:

We negotiated this morning in Peter Leddy's office. Al Willcox of the Math. Ass. of America raised the question of R.H. possibly making a gift of the NML to the MAA and deriving tax benefits and wonderfully favorable publicity for such a generous act. Peter Leddy said that his division could not get tax benefits since it makes no profits, but he did not object to Al's plan of contacting Carol Newsom who might discuss the gift notion at the top of the corporation. If all this fails, the MAA will probably still make an offer and hopes to reach agreement with R.H. The MAA is very much interested in the series.

A handwritten note by Anneli Lax describing a September 18, 1974 meeting at which the transfer of the New Mathematical Library from Random House to the Mathematical Association of America was discussed. From the New Mathematical Library Records, ca. 1929, 1957-1997, Archives of American Mathematics, Center for American History, The University of Texas at Austin.

While at NYU, Lax met and married mathematician Peter Lax. In 1955 Lax received her PhD and in 1961 NYU appointed her to the faculty of the Department of Mathematics, where she stayed until her retirement in 1992. Dr. Lax accepted the position of technical editor of the *New Mathematical Library* series at its inception in 1958, and remained editor until her death in 1999. The MAA renamed the series the *Anneli Lax New Mathematical Library* in her honor in the year 2000.

This collection reflects the progress of the *New Mathematical Library*, specifically under the editorship of Anneli Lax. It includes correspondence with authors and publishers, outlines and drafts of monographs, and various production records.

The online inventory for the *New Mathematical Library* Records is available here:

<http://www.lib.utexas.edu/taro/utcah/00387/cah-00387.html>

The Archives of American Mathematics is located at the Research and Collections division of the Center for American History on the University of Texas at Austin campus. Persons interested in conducting research or donating materials or who have general questions about the Archives of American Mathematics should contact Kristy Sorensen, Archivist, [k.sorensen@mail.utexas.edu](mailto:k.sorensen@mail.utexas.edu), (512) 495-4539.

Web page: <http://www.cah.utexas.edu/collectioncomponents/math.html>

## What I Learned... about Online Assignment Management

By Glenn Ledder

Online assignment-management (OAM) systems allow instructors to create assignments or exams that are posted on the web, taken by students on their own time, and computer-graded to provide instant feedback for students and a record for instructors. Interest in these systems can be measured, perhaps, by the number of talks at mathematics meetings on issues related to their use.

The Department of Mathematics at the University of Nebraska-Lincoln uses an OAM system called EDU, from Brownstone Learning; however, most of these principles apply equally well to other systems. (EDU, as well as Maple T. A. and several publisher-hosted products associated with textbooks, is founded on common code architecture developed by Professor John Orr of our department.)

I have been using OAM long enough to compile an impressive list of mistakes and bad choices as well as a smaller list of good practices. In the hope that others might profit from my errors without having to repeat them, I have collected here what I believe are the ten most important principles for OAM use. Some of the items in the list are specifically for exams or for daily assignments, while others apply equally to both. Here, then, are the top ten things I've learned about Online Assignment Management:

### 10. Use matched sets of questions

When several different cases occur in a problem class, it is good to require students to work a problem from each case. An example would be two problems of the form "Find the (exact)  $x$  coordinate of the global minimum of  $f(x) = 3x^3 + bx^2 + cx$  on  $[-1, 1]$ ," where  $b$  and  $c$  are chosen so that the global minimum occurs at a critical point for one of the questions and at an endpoint for the other one.

### 9. Use a mastery protocol

In a mastery protocol, students are required to successfully work one problem from each of several categories. This means that students repeat only those problems they have not yet worked correctly, and it also allows the instructor to set up question hierarchies (see item #5). This is qualitatively different from the case where a student must get a certain number correct to pass and has multiple attempts at passing. In the latter case,

students who do not pass must repeat all of the questions on their next attempt. Mastery protocols do not work well for exams, but they are ideal for short assignments.

### 8. Choose the right material for online instruction

Online instruction is fine for routine computation and ideas that can be developed through examples, but it is no substitute for a live class meeting for the teaching of nuanced ideas, problem-solving strategies, or techniques in which individual steps need a separate presentation. The typical homework problem from a textbook is too complicated to be of instructional value as an all-or-nothing computer-graded problem.

### 7. Write good questions

Many of my students try to learn the material only when they have exhausted all other ways to pass an exam. It is good instructional practice to help students reach this point quickly. Randomized parameters allow for an enormous variety of answers to problems that look almost identical, eliminating any hope of passing an exam by memorizing the answers. Grouping problems by category has a more subtle effect. Many students who fall short of a passing mark on an exam will quickly repeat the exam, hoping to get an easier set of questions. The more closely different settings of an exam are related, the more likely students will come to appreciate that trying to get an easy version is hopeless. This desirable uniformity of appearance and difficulty comes from having an exam constructed by random selection of one problem each from several standardized categories.

### 6. Set high standards and allow retakes

Our standard educational system is built around one-shot exams. These allow us to categorize students by level of achievement, but they encourage students to work towards performance rather than learning (so that the goal is to complete assignments and get good grades rather than to learn the material), and may let them move on without mastering fundamental material. An alternative plan is to set high performance standards and require students to repeat an exam until they achieve the standards. This plan does not distinguish the stronger students from the weaker, but (see point #8) it is not necessary to do that on material that is well suited to online testing. Another objection is that allowing retakes can encourage students not to study for an exam. I solve this problem by basing the students' grades on the date at which they pass the exam and limiting them to one attempt per day. Students have three days in which they can receive the full 30

points for passing my exam. After that, the number of points they get decreases by one each day. This policy is very effective at motivating students to pass the exam quickly.

### 5. Use a question hierarchy

Students learn best from success that build on previous success. Success rates can be improved by using a hierarchy of questions that allow students to progress from easy questions to hard ones. For example, consider the problem of finding the

derivative of  $\frac{3-2\cos x}{4+7\sin^2 x}$ . Experienced students should find this

problem to be straightforward; however, it does require students to successfully combine the quotient rule, the chain rule, and the formulas for the derivatives of the trig functions. Students who have just learned one of these components are not likely to be able to do this problem successfully. I use a hierarchy of

problems in which students must first differentiate  $\frac{2x}{5-4x}$ ,

$\frac{3x}{3+4\sin x}$ , and  $\frac{1+4\cos x}{6+2\sin x}$  before tackling the desired problem.

The average student will probably have to work seven problems to get through the four-question hierarchical assignment. The same student would probably also need seven tries to succeed at the corresponding single-question assignment, but she will have got only one question right in the process instead of four.

### 4. Encourage students to rework missed problems before trying an exam or assignment again

Students who take the time to analyze their mistakes show much greater improvement in their next attempt than students who don't; however, few students use this seemingly obvious method of study. I suspect that the problem is the all-too-common performance orientation. To students whose goal is to complete the assignment rather than to learn the material, time spent studying is time not spent on the task of completing the assignment. Even students who have a learning orientation often have not developed good study techniques. It is worth spending a little bit of class time teaching students how to learn mathematics.

### 3. Use a short time window

Mathematics lectures generally build on previous material. The standard calculus course includes daily lectures, a small amount of daily or weekly homework, weekly quizzes, and monthly exams. Most students fall behind during the first week and only catch up when studying for an exam. This means that most lecture material is delivered to students who have not

learned the background material from the previous class meeting. Online instruction has the potential to ameliorate this difficulty, provided assignments are timed so that each is due before the next class meeting. There is no harm in extending the short time window as needed, but the standard practice should be to require online work to be done in time to prepare the students for the next class.

## 2. Minimize instructor commitment

No matter how useful OAM is, most faculty are not going to use it if they perceive it as a large time commitment. I set up an administrative structure that utilizes EDU's facility to have class folders created as copies of existing folders, with the daughter folders inheriting any materials kept in the parent folder. I have a master course folder that contains the question banks, the single online examination used by all sections, and the non-credit assignments used by students to practice their skills or study for exams. I also have an enhanced master course folder that is a copy of the master folder with the addition of a set of assignments-for-credit used by some of the course instructors. These course folders can be reused each semester just by changing the assignment dates. Each individual class has its own folder that is copied from the appropriate master folder. I find that a one-hour training session is sufficient to teach instructors how to do the only tasks that they must actually perform themselves: changing assignment dates, observing and re-grading student work, and downloading the gradebook.

### 1. Give minimal credit

Few of my students will take the time to do an assignment that does not count for credit, no matter how interesting or valuable the assignment. Yet the same students will do an inordinate amount of work for minimal credit. Last semester, I gave two points for each of my 30 web assignments, out of 600 total points for the course; my students completed 75% of the total number of assignments possible in the course. My colleague offered the same assignments for no credit, and his students completed only 2% of possible assignments. The credit I gave for the assignments had only a minimal direct impact on student grades; not a single student passed because of the availability of a few easy points. The indirect impact was much larger: my students were better prepared for class and were able to get more out of the limited class time available for the course.

*Glenn Ledder teaches at the University of Nebraska-Lincoln. He can be reached by email at [gledder@math.unl.edu](mailto:gledder@math.unl.edu).*

## Letters to the Editor

### Another Mac Lane Story

Since the next issue of FOCUS will contain an extended obit of Saunders Mac Lane, I thought your readers might enjoy the following anecdote.

In the Fall of 1959, he was teaching an algebra course to undergraduates at the University of Chicago, using (of course!) Birkhoff-Mac Lane's *A Survey of Modern Algebra*. A student asked him why some of the exercises were marked with an asterisk. He replied that those were the ones Birkhoff couldn't do!

Jack Driscoll

### Las Vegas and Saunders Mac Lane

Today's early morning e-mail from my neighbor wanted me to click on an internet link of an article from the Boston Globe. Its heading read: "Saunders Mac Lane, developed key Algebraic theory; at 95." I said, my god, this man is doing fundamental research in mathematics at 95! But this reality lasted only for a second. In his death on April 15, 2005, Mac Lane may have set a record as the longest living and active mathematician of the 20th century!

While replying my Bostonian neighbor, I said, "He is the man responsible for not letting the Joint Annual Meetings of the AMS and MAA take place in Las Vegas! Very stubborn man to the end."

I continued to muse over my comments. How can you crack a long standing mathematics problem or develop a new concept without persistence and perseverance? Never! One's being stubborn in dealing with colleagues may not be a positive quality. However, in certain areas of "pure" mathematics, there may be a correlation between originality and stubbornness. It is not a litmus test, though in my sample, it worked with 95% accuracy!

During the first and only Joint Meetings held in Las Vegas in Jan 1971, Mac Lane

pushed a resolution that has ruled Las Vegas out as a conference site! I was shocked to discover it when my interest in the Meetings grew in 1980's. A couple of math bigwigs essentially told me, "You forget Las Vegas as long as Mac Lane is alive!" Once I wrote him a very persuasive letter on this issue. He graciously replied in his long hand, but refused to change his mind.

One may talk of his legacy both mathematical and non-mathematical. They are two separate things. One does not have to study category theory to know its impact on many branches of mathematics including computer science. His influence on mathematics organizations is deep for the number of people who know him directly (his colleagues and PhD students at Level 1) and indirectly (his students' students, Level 2, 3, 4, and 5!). To give an idea of his hold over math organizations, I never got a reply from any officer of the AMS or MAA of my written suggestions for holding a Joint Meeting in Las Vegas! It was essentially a one man crusade that I gave up five years ago.

Yes, it was five years ago when I last saw Mac Lane at a Joint Meeting. He spoke at a panel discussion on Philosophy of Mathematics. People were standing wall to wall in a big hall to hear him! He had notes written and rolled up in yellow sheets. Most of the time, he rambled on. In fairness, that was a great mathematical performance at 90!

Satish C. Bhatnagar  
University of Nevada Las Vegas

### A Visit to Einstein's Papers

May I add a footnote to Herbert Kasube's interesting suggestion, "Why You Should take a Mathematical Study Tour" (FOCUS, Dec 2004, Vol 24, Page 9)?

Back in 1999, fulfilling a long-standing desire, I visited the Einstein archives at the Hebrew University in Jerusalem. Einstein, in his will, specified that his papers be kept there.

For me, a science and technology teacher, it was a very special event to have the opportunity of seeing first hand some of these original items, and thereby to experience a degree or closeness to Einstein himself. Later on, I was able to share this experience with my students.

I subsequently decided to write about I had seen, as well as my reactions. These thoughts, together with a selection of archival photographs were published in an article entitled, "Albert Einstein's Personal Papers: A Physics Teaching Resource". This appeared in the British journal, *Physics Education*, January 2000 (volume 35, No.1, page 69).

Samuel Derman  
New York City College of Technology

### More on Baley Price

Like Steve Carlson, I obtained three degrees from the University of Kansas, all in mathematics. So I was pleased to see the article honoring G. Baley Price in the May/June 2005 issue of FOCUS. But I wish to note that G. Baley chaired the KU Mathematics Department for a longer period than 1959–1970 as indicated in Steve Carlson's article. Dr. Price also chaired the department throughout 1954–1959 during which time I completed my BA and MA degrees. I first met him in the fall semester of 1954 when my College Algebra and Trig instructor (William Hartnett, later of SUNY Plattsburgh) encouraged me to ask Dr. Price about opportunities in mathematics. The result of that conversation was that I enrolled in Analytic Geometry the next semester, and then proceeded to major in mathematics. I had never given a thought to becoming a math major before this.

Unfortunately I don't know when G. Baley first served as Chair of the KU Math Department. I have tried to find that information in the 788 page "History of the Department of Mathematics, University of Kansas, 1866–1970" which Dr. Price published in 1976. I may have

missed it, but it appears to me that G. Baley did not consider the precise dates of his chairmanship to merit publication, though the fact that William Scott (later of the University of Utah) served as Acting Chair in 1959–1961 while Dr. Price was fully occupied with CBMS is recorded.

W. M. Greenlee,  
Professor Emeritus  
University of Arizona

**Report on a Meeting of the Northeast Section**

I greatly enjoyed the June 17, 2005 meeting of the Northeast Section of the MAA at Bates College in Lewiston, Maine.

The college campus was lovely. It was fun to see and hear presentations by recent college seniors on the isoperimetric problem in Gauss space and prion disease as well as presentations by graduate students on internet search algorithms and by professional mathematicians on subjects as varied as transcendental numbers, the logistic equation, paying employees fairly, bubbles, code breaking, Pascal’s triangle applied to trigonometry, and the history of mathematics. I’ve probably forgotten something or someone, but I found all the presentations and conversations to be of interest.

Part of the fun of these conferences are the “hallway” or “unplanned” discussions. I enjoyed discussing the book “The Pea and the Sun” by Leonard M. Wapner A.K. Peters, Wellesley, MA 2005 with a math book editor and an industrial mathematician. The book explains the axiom of choice and a proof of the Banach-Tarski Theorem.

I remembered there being a claim in this book that Cantor dust can be placed in a one to one correspondence with all the real numbers between 0 and 1 inclusive. This surprised me until I thought about it because Cantor dust is sparse and consists of all the real numbers between 0 and 1 inclusive that can be represented as ternary fractions without any 1s in the numerators. The author explains how

to do this mapping on page 129 of his book, but I had not fully understood on my very first quick read through the book. I understood much better after thinking about the claim and discussing the key ideas of the book with others.

So, here’s a way to do the mapping in my own words.

1. Convert each real number base 10 between 0 and 1 inclusive to its base 2 equivalent number.
2. Convert each 2 in the denominator of the equivalent number to a 3 and convert each 1 in the numerator of the equivalent number to a 2.

For example,

$$\frac{1}{2^1} + \frac{1}{2^2} + \frac{0}{2^3} + \frac{0}{2^4} + \frac{1}{2^5} + \dots$$

maps to

$$\frac{2}{3^1} + \frac{2}{3^2} + \frac{0}{3^3} + \frac{0}{3^4} + \frac{2}{3^5} + \dots$$

and vice versa.

I returned home having met some interesting people, enjoyed many fine conversations and presentations, and with a much deeper understanding of my reading of a very interesting explanation of the axiom of choice and Banach-Tarski Theorem.

Randall J. Covill  
rcovill@woodchuckhill.mv.com

**Helping Incarcerated Mathematicians**

*Editor’s Note: The letter that follows was addressed simultaneously to the editors of FOCUS and of the Notices of the American Mathematical Society.*

Dear Editors,

I read with much interest the March 2005 FOCUS Short Take “Math Set Him Free” and the March 2005 Notices article on Andre Weil’s 1940 letter on Analogy in Mathematics. Like William White, the

subject of the FOCUS piece, and Andre Weil (at the time he wrote the letter), I am an incarcerated mathematician. (To be precise, Mr. White only completed a minor in mathematics, but in light of the fact that he’s incarcerated, that’s close enough for me). I’d like to share with FOCUS’s readership and the Notices’ readership that there is interest in mathematics in prison. To be fair, Mr. White, Professor Weil and I are in .a very small minority. The vast majority of prison inmates, at least in the United States, have no interest in mathematics and only slightly less appreciation of math than the general public. A large number of prison inmates have not graduated high school. Nonetheless, the number of inmates who enjoy mathematics is a positive number; indeed, it is currently at least 2! The number of inmates who are at least curious about mathematics and are at the high school algebra level or above is greater. Those of us who actually do problems for fun or for coursework (i.e., Mr. White and I, hopefully among others) know this from personal experience. In prison, there is very little privacy, and most inmates have at least a passing curiosity about anything another does which is outside the norm.

Spring is coming, and with that, for many of you, spring cleaning, office reorganization, and so on. As you find books, papers, or publications you don’t need, give some thought to donating them to a prison library in your state. Books at the high school algebra level (or lower) would be most useful, to a prison’s general population, especially those preparing for the GED. I personally have a fondness and high opinion of the traditional high school geometry course (with emphasis on proofs) and would love to see one or two geometry textbooks in my library. I also lament the absence of this course in the curriculums of junior colleges and other institutions offering remedial mathematics courses, but that is another letter. I think pre-calculus and calculus texts would be welcome in a surprisingly large number of prison libraries. And post-calculus mathematics texts (even graduate-level texts), while not likely to be checked out by a significant portion of a prison’s population, has a chance of being a godsend to that one

person in the population looking for a book on analysis. Or number theory. Or groups. Allowing for a moment of selfishness, I'd be delighted to see a text or two on differential equations, number theory, and "second-level" linear algebra (canonical forms and the classical groups). If you have a mind to donate old texts or materials, I strongly recommend contacting the librarian or the director of education of the prison you intend to donate to. They can guide you on procedures and on what materials would be most useful. If there's a math-

ematician in their population, they are likely to know him.

As a closing thought, I'd like to encourage the MAA to offer Mr. White a complimentary one-year membership as a means of congratulating him on his recent achievement of earning a degree. Like most of us (mathematicians), his interest in mathematics likely stems from the satisfaction of facing the challenges a good problem provides. The Problems sections and articles of any of the MAA journals would provide the challenge and

pleasure that most mathematicians enjoy. In prison, where intellectual challenge is in short supply and monotony is the norm, the MAA journals would be more of a welcome relief than you could imagine.

Calvin A. Curtindolph  
New Lisbon Correctional Institute  
New Lisbon, WI

## Finding Common Ground in K-12 Mathematics Education

By Michael Pearson

The MAA hopes to help encourage and facilitate constructive discourse between mathematicians and mathematics educators in order to seek common ground in their mutual efforts to improve K-12 mathematics teaching and learning. The success of two pilot meetings (one at NSF in December 2004 and a second at the MAA offices in June 2005) with two mathematicians (R. James Milgram and Wilfried Schmid), three mathematics educators (Deborah Loewenberg Ball, Joan Ferrini-Mundy and Jeremy Kilpatrick) and a moderator from the business community (Richard Schaar) demonstrated that such common ground does exist among individuals who are thought to be strongly aligned with different sides in what has come to be known as the 'Math Wars.'

These meetings resulted in a document designed to serve as a starting point for future conversations. Agreeing that "All students must have a solid grounding in mathematics to function effectively in today's world," the group started with three fundamental premises:

1. Basic skills with numbers continue to be vitally important for a variety of everyday uses.
2. Mathematics requires careful reasoning about precisely defined objects and concepts.
3. Students must be able to formulate and solve problems.

From there, the group explored a number of topics, including the importance of automatic recall of basic facts, the use of calculators in lower grades, instructional methods and teacher knowledge, and found significant points of agreement. The full text of the document is available on the MAA website at [www.maa.org/common-ground](http://www.maa.org/common-ground).

Other groups have met with similar intentions of focusing serious effort on what is essential in the K-12 mathematics curriculum and how best to achieve some level of consensus between various constituencies. We expect further articles and reports will become available that help the mathematical community participate more effectively in guiding our schools towards providing students with the skills they need to succeed, both in higher education and the workplace. Watch these pages for future communications from the MAA regarding these efforts.

*Michael Pearson is Associate Executive Director and the Director of Programs and Services at the Mathematical Association of America.*

## U.S. Team Survives Hurricane to Place 2nd in International Mathematical Olympiad

By Steve Dunbar

Merida, Mexico - July 18, 2006 - The 2005 International Mathematical Olympiad (IMO), 46th in the annual series of mathematical competitions for high school-age students, announced the medal winners today. At this year's IMO, 513 of the best young mathematicians from 93 countries, making it the largest IMO ever, competed in solving 6 problems posed in a grueling nine-hour test administered over two days (July 13 and 14). The competition which poses six math questions, each worth a total of 7 points, would challenge even the finest professional mathematician.

The U.S. finished 2nd overall with a total of 213 points out of a possible 252 points. China came in 1st with 235 points, Russia was 3rd with 212, Iran placed 4th with 201 points and Korea placed 5th with 200.

Upon word of their victory, Steve Dunbar exclaimed, "This is an extraordinarily strong performance by the U.S. team, since this is the first time that these six team members have represented the U.S. at the International Mathematical Olympiad. Congratulations to Team Leader Zuming Feng and Deputy Leader Melanie Wood for preparing the team and presenting their solutions to the judges."

Members and competitions results of this year's team are:

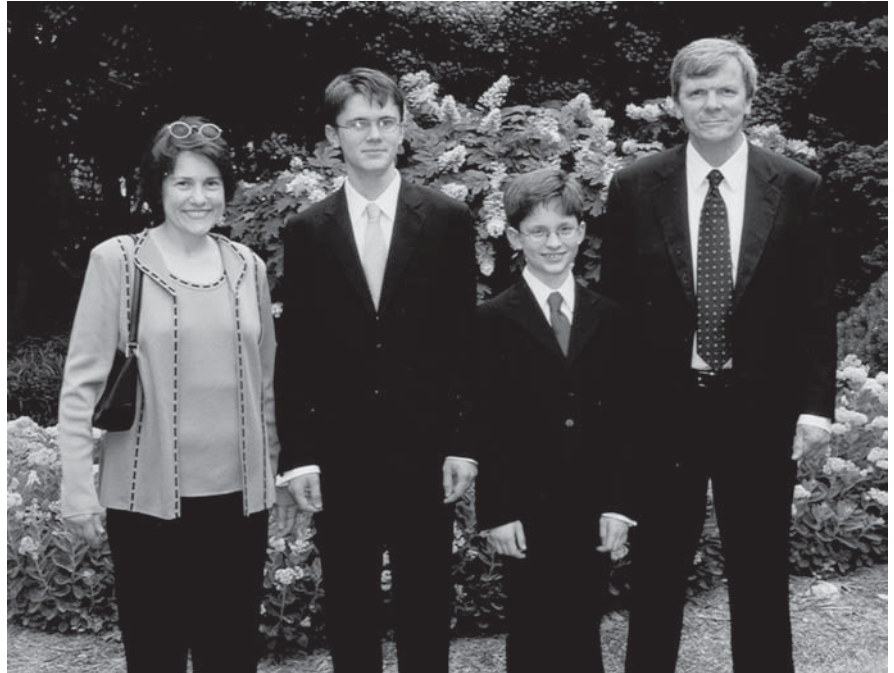
**Brian Lawrence, Perfect Paper, Gold Medalist.**

Attends Montgomery Blair High School, Silver Spring, Maryland.

**Eric Price, 41 points, Gold Medalist.**

Graduated from Thomas Jefferson High School of Science and Technology, Alexandria, Virginia.

**Thomas Mildorf, 39 Points, Gold Medalist.**



*Pictured left to right: Vivian, Brian, Scott, and Jim Lawrence at the USAMO Awards Ceremony in June. Brian Lawrence had a perfect paper at the International Mathematical Olympiad. Photograph courtesy of Robert Allen Strawn.*

Graduated from Thomas Jefferson High School of Science and Technology, Alexandria, Virginia.

**Robert Cordwell, 36 points, Gold Medalist.**

Graduated from Manzano High School, Albuquerque, New Mexico.

**Sherry Gong, 28 points, Silver Medalist.**

Attends Phillips Exeter Academy in Exeter, New Hampshire.

**Hyun Soo Kim, 27 points, Silver Medalist.**

Graduated from the Academy for Advancement of Science and Technology, River Edge, New Jersey.

The U.S. Team is sponsored by the Mathematical Association of America (with support by other mathematical societies, the University of Nebraska and the Akamai Foundation. Transportation is provided through a grant from the Army Research Office. Additional contributions come from 19 organizations and companies in the mathematical sciences. The team is chosen through a four-stage process of mathematics testing by the MAA's American Mathematics Competitions Program

*Steve Dunbar is the Director of the MAA American Mathematics Competitions.*



## The Missouri Collegiate Mathematics Competition

By Alvin Tinsley and Curtis Cooper

The tenth annual Missouri Collegiate Mathematics Competition was held in conjunction with the spring meeting of the Missouri Section of the MAA on the campus of Missouri Western State College in April. The contest is sponsored by the Missouri Section and is held at the site of the spring section meeting. After ten years, the competition continues to be quite popular with students of Missouri colleges and universities, and, of course, the level of student participation in the state meeting has increased dramatically.

Ten years ago, when the contest was in incubation, it was reasoned that in order to appeal to the interests of students and institutions of higher learning at all levels the examination questions should have a wide range of difficulty. The easiest should be at the level of typical calculus and discrete mathematics problems, while the hardest would approach the difficulty of those on the Putnam Exam. It was further decided that the contest would consist of two sessions lasting two-and-one-half hour each, the first to be held from 7:30 pm to 10:00 pm on the Thursday evening prior to the first day of the spring meeting, and the second on Friday morning from 8:30 am to 11:00 am. The competition would be among teams of up to three students, and a college or university could enter two official teams to be accompanied by one or more sponsors from their institution. At the request of a number of colleges and universities, unofficial teams were later allowed to enter, but they were ineligible for awards.

The first two questions in each session are of the easier type mentioned above and are intended to be solvable by all teams in the competition. The following are examples of these questions: one is a traditional parabola question and the other is a discrete mathematics question.

*Let  $P \neq (0,0)$  be a point of the parabola  $y = x^2$ . The normal line to the parabola at  $P$  will intersect the parabola at another*

*point, say  $Q$ . Find the coordinates of  $P$  so that the area bounded by the normal line and the parabola is a minimum.*

*The numbers  $\pm 1, \pm 2, \dots, \pm 2004$  are written on a blackboard. You decide to pick two numbers  $x$  and  $y$  at random, erase them, and write their product,  $xy$ , on the board. You continue this process until only one number remains. Prove that the last number is positive.*

In addition, there is typically a number theory question on the test. The following is an example:

*Find all integer solutions  $(x,y)$  to the equation  $xy = 5x + 11y$ .*

The last question is typically a challenging analysis question. The following is an example:

*Prove that in the MacLaurin series for  $\tan \theta$ ,  $-\pi/2 < \theta < \pi/2$ , every coefficient is non-negative.*

The contest is governed by a committee of seven college and university mathematics faculty members from around the state. The committee members submit questions to the chair and once per year they meet to finalize the exam. Tests for the coming contest and the following year are prepared at the meeting. Questions need not be original, but they very often are. Some may be found in journals and problem books, and some are modifications of such problems.

The competition is held in a room which will accommodate approximately 30 teams which participate annually. Each team has its own table, each student receives a copy of the exam, and scratch paper is provided as needed. Snacks and drinks are available to the students during each session. At the end of each session, the solutions which are secretly coded, are collected, separated by problem number and graded by the committee members. One grader is responsible

for assessing all the solutions for a given problem number, and zero to 10 points are assigned to each team. The results are reported to the scorer, and he only knows where the teams rank relative to each other. The results of the two sessions are combined to determine the top three teams and the ranking of all teams.

The interest in the competition among institutions of all levels has been fairly constant over the past 10 years. Many four-year colleges, all the state universities, and the large research based universities participate annually. One would anticipate that the later institutions would dominate the awards, but that has not always been the case. Often one very good student can carry his team to a high ranking and even first place.

The expenses for an institution's entry in the competition and for the student's travel and boarding are borne by the institution. The contest entry fee is intended to cover the cost of materials, duplicating, and two meals for the students.

The team achieving the highest total score for the two sessions receives a traveling trophy which their department displays for the year prior to the next contest. In addition, each team member receives a plaque indicating that she/he was a member of the winning team for the indicated year. Each participant in the competition receives a certificate of participation, and the awards are presented during the section meeting banquet on Friday evening.

As mentioned above, snacks and food are made available to the students during their participation. To encourage the competitors to mingle and get acquainted, a pizza party is given following the second session. In addition, bowling lanes are reserved for friendly competition among teams that wish to participate after the banquet. A photographer takes pictures of the teams in competition, and a group photograph is shot

prior to the pizza party. The pictures are uploaded to the contest webpage before the end of the meeting.

For those who are interested, the contest rules and pictures are available at the following web address: <http://www.mathcs.cmsu.edu/~curtisc/contest>.

The popularity of the competition has exceeded the expectations of the organizers. A department chair at one of the participating universities has observed that the competition is the most significant activity of the Missouri Section.

Questions concerning the competition may be directed to Prof. Curtis Cooper who serves as chair of the contest committee. His email address is [cnc8851@cmsu2.cmsu.edu](mailto:cnc8851@cmsu2.cmsu.edu).

*Alvin Tinsley and Curtis Cooper teach at Central Missouri State University in Warrensburg, MO.*

## Found Math

*Great Moments in Public Education*

[www.boston.com/news/odd/articles/2005/05/23/sports\\_fans\\_cry\\_foul\\_on\\_math\\_question/](http://www.boston.com/news/odd/articles/2005/05/23/sports_fans_cry_foul_on_math_question/)

A state math exam for North Carolina seventh-graders included a question on football that asked students “to calculate the average gain for a team on the game’s first six plays.” But there was a problem:

*The team opened with a 6-yard loss, a 3-yard gain and a 2-yard loss, which would have made it fourth down with 15 yards to go for a first down. The team’s fourth play was just a 7-yard gain, yet it maintained possession for a 12-yard gain and a 4-yard gain on two additional plays.*

A state official defends the flawed question:

*Mildred Bazemore, chief of the state Department of Public Instruction’s test development section, said the question makes sense mathematically and was reviewed thoroughly.*

*“It has nothing to do with football,” Bazemore said. “It has to do with the mathematical concepts that you’re studying.”*

It seems mathematicians are out of touch with the real world — and even with the world of sports.

(From the *Wall Street Journal*’s “Best of the Web” online column, May 23, 2005; see <http://www.opinionjournal.com/best/>; reprinted with permission.)



the MAA’s 4th Annual Mathematical Study Tour



## Journey to CHINA

June 6 - June 21, 2006

**Travel to the Land of Cathay and  
Explore Its Ancient and Modern Culture**

**Contact Information:**

Lisa Kolbe  
Development Manager  
[lkolbe@maa.org](mailto:lkolbe@maa.org)  
202-293-1170

Full details, itinerary, and registration form will be available September 1, 2005 on MAA Online [www.maa.org](http://www.maa.org)

## Short Takes

Compiled by Fernando Q. Gouvêa

### Seven Mathematical Scientists Elected to the NAS

Among the 72 new members elected by the National Academy of Science are seven mathematical scientists: Malcolm H. Chisolm of the Ohio State University, Iain M. Johnstone of Stanford University, Sergiu Klainerman and János Kollár of Princeton University, Stanley Osher of the University of California, Los Angeles, Margaret H. Wright of the Courant Institute, and Adi Shamir of the Weizmann Institute of Science. Alas, not a one is a member of the MAA; we congratulate them nevertheless.

### New Journal to Focus on the History of Mathematics Education

The first issue of the *International Journal for the History of Mathematics Teaching* is set to appear early in 2006. With a distinguished editorial board headed by Gert Schubring and Alexander Karp, the new journal will be published twice a year by the Teachers College at Columbia University.

IJHMT is an outgrowth of the success of Topic Study Group 29, on *The History of Learning and Teaching Mathematics*, at the International Congress on Mathematics Education held in Copenhagen in 2004, which suggested that there was a need for “a permanent and stable international forum for scholarly research in the history of mathematics teaching.” The journal is calling for submissions of papers. For more information, contact Alexander Karp, IJHMT, Program in Mathematics, Box 210, Teachers College, Columbia University, 525 West 120<sup>th</sup> Street, New York, NY, 10027, or they can be reached by email at [ijhmtteaching@yahoo.com](mailto:ijhmtteaching@yahoo.com).

### Science and Engineering Indicators

The NSF's *Science and Engineering Indicators 2004* (a biennial report to Congress) was first published more than a year ago, but the data have recently been

updated. The report, which can be found online at <http://www.nsf.gov/sbe/seind04>, contains data on elementary and secondary education, higher education, labor force, research and development, and other aspects of Science and Engineering in the United States. The report is an excellent source of raw data for those interested in Science Policy.

### The Chudnovsky Brothers and the Tapestries

An article by Robert Preston in the April 11 issue of *The New Yorker* tells of how the Chudnovsky brothers helped create a digital image of the famous tapestries from The Cloisters, a part of the Metropolitan Museum of Art. The project, which involved knitting together a large number of digital images of portions of the tapestries, proved to be particularly challenging because of minute changes in the position of the fibers while the photographs were being taken. The article can be found online at [http://www.newyorker.com/fact/content/articles/050411fa\\_fact](http://www.newyorker.com/fact/content/articles/050411fa_fact).

### USA Team Takes Second Place at International Young Physicists' Tournament

The USA team tied for 2nd place in this year's International Young Physicists Tournament at the University of Zurich, placing among the top teams for the first time in eighteen years. Five students represented the United States: Phillip Schwartz and Daniel Kerr of the Wildwood School in California and Jonathan Bohren, Robert Kirkham, and Divya Krishnan of the Rye County Day School in New York.

The Tournament is a theoretical and practical competition for teams of high school students, who work on 17 physics problems and then present and defend their findings in “Physics Fights.” Rounds consist of three teams competing against each other, each having a chance to report, oppose, and review. The

final round in this year's competition pitted the USA team against teams from Germany and Belarus to address the following problem: “Granular material is flowing out of a vessel through a funnel. Investigate if it is possible to increase the outflow by putting an ‘obstacle’ above the outlet pipe.” For more about the competition and the USA team, visit the international site at <http://www.iypt.ch> and the USA site at <http://www.usaypt.org>.

### Concern about Test Questions Continue

Given the increasing number of mathematics and science examinations and standardized tests, educators are facing the need to create more and more test problems. An article in the 24 April 2005 of *The New York Times* highlighted the fact that a significant number of these problems are flawed in one way or another: “ambiguous, imprecise, or just plain incorrect.” As a result of pressure from critics, some states (Texas is one that is mentioned in the article) are hiring university professors to review the test questions. The article is available online at <http://www.nytimes.com/2005/04/24/education/edlife/guernsey24.html>, but the *Times* requires payment for access.

### Teaching Awards for the MAA Financial Team

Two members of the MAA's Budget and Audit Committees, Dan Maki and Jim Daniel, have received teaching awards from their institutions. Dan Maki received the 2004 Indiana University President's Award for Distinguished Teaching. Jim Daniel was named to the University of Texas at Austin's “Academy of Distinguished Teachers” (see <http://www.utexas.edu/faculty/academy>). The two, who make up the whole of the Audit Committee and two-thirds of the Budget Committee, are thus as distinguished as teachers as they are in their service to the Association.

### Mathematical Music, Anyone?

Visit <http://www2.collegehumor.com/movies/149448/> for a delightful performance by The Klein Four Group, who sing *A Finite Simple Group (of Order Two)*, a piece M. Salomone based on mathematical double-entendres. For more about the group, including more songs, lyrics, and the Klein Four Store, visit their web site at <http://www.math.northwestern.edu/~matt/kleinfour/>.

### Milken Family Foundation Focuses on Teacher Quality

The “No Child Left Behind” Act requires schools to meet specific goals with respect to teacher quality. In its May 4 issue, *Education Week* reports that in order to help schools meet these requirements, the Milken Family Foundation and the Broad Foundation have created a new foundation “to help urban schools adopt proven strategies for improving the quality of their teaching staffs.” The Milken and Broad Foundations have provided more than ten million dollars to create the *Teacher Advancement Program Foundation*. The foundation is expected to continue the work of Milken’s “Teacher Advancement Program” (TAP), which emphasizes “multiple career paths; instructionally focused accountability; ongoing, applied professional growth; and performance-based compensation.” The new foundation will be headed by Lewis C. Solmon.

The *Education Week* report is online at <http://www.edweek.org/ew/articles/2005/05/04/34milken.h24.html>, and the press release from the Milken Foundation is at <http://www.mff.org/newsroom/news.taf?page=447>.

### A Mathematical Puzzler from *Car Talk*

As the show’s many dedicated fans know, NPR’s *Car Talk* regularly includes “puzzlers”, problems of various kinds that are sometimes about cars, sometimes just tricky logic questions, and occasionally mathematics problems. On one of the weeks during which FOCUS was being prepared, the puzzler was “The Chicken Nugget Conundrum”:

*There’s a famous fast-food restaurant you can go to, where you can order chicken nuggets. They come in boxes of various sizes. You can only buy them in a box of 6, a box of 9, or a box of 20. So if you’re really hungry you can buy 20, if you’re moderately hungry you can buy 9, and if there’s more than one of you, maybe you buy 20 and you divide them up.*

*Using these order sizes, you can order, for example, 32 pieces of chicken if you wanted. You’d order a box of 20 and two boxes of 6. Here’s the question: What is the largest number of chicken pieces that you cannot order? For example, if you wanted, say 37 of them, could you get 37? No. Is there a larger number of chicken nuggets that you cannot get? And if there is, what number is it?*

It’s probably too late to send in the answer and get a \$26 gift certificate from their Shameless Commerce Division, but it’s still a nice problem.

### *Philosophia Mathematica* Has New Publisher

*Philosophia Mathematica* has been taken over by Oxford University Press, the world’s biggest and most significant publisher of philosophy. *Philosophia* is the sister journal of *Historia Mathematica*. It was published over the last 12 years by the Canadian Society for the History of Philosophy of Mathematics. As the names indicate, *Historia* (published by Elsevier) is a History of Mathematics journal, while *Philosophia* deals with the Philosophy of Mathematics. Congratulations to editor Robert Thomas for his success with the journal. For more about *Philosophia*, including a searchable archive, visit its web page at <http://www.philmat.oupjournals.org>.

### Richard Semmler Featured in *Washington Post* Article

An article in the June 11 issue of *The Washington Post* profiles Richard Semmler, who teaches mathematics at Northern Virginia Community College. Describing Semmler as a professor who “finds fulfillment in emptying his pockets,” the article lauds his generosity in

giving half or more of his annual income to charity. Semmler’s largesse is made possible by his very simple lifestyle. “If I didn’t do all of the things I was doing, I would probably have a new car every two years and I would have a huge house with a huge pool,” Semmler told the *Post*, “but I would not do it that way. I want to do it this way.” According to the article, Semmler estimates that he has given away \$770,000 so far, and he intends to reach one million before he retires.

### International Conference on Mathematics Teaching

The 3rd International Conference on the Teaching of Mathematics will be held between June 30 and July 6, 2006, at Istanbul, Turkey. Following on the success of earlier conferences held in Samos, Greece (1998) and Crete, Greece (2002), the 2006 conference intends to focus on “new ways of teaching undergraduate mathematics.” The conference chairs are Ignatios Vakalis of Capital University and Deborah Hughes-Hallett of the University of Arizona, and the conference will be co-sponsored by the MAA. For more information, including information on how to submit a paper, visit <http://www.tmd.org.tr/ictm3>.

**Sources:** New NAS members: press release. IJHMT: email communication, call for papers. Science and Engineering Indicators: email communication, NSF web site. Chudnovsky Brothers: email communication, *New Yorker* web site. International Young Physicists’ Tournament: press release. Test Questions: NASSMC Briefing Service, *The New York Times*. MAA Financial Team: email communication. Mathematical Music: email communication. Milken Foundation: NASSMC Briefing Service, *Education Week*, Milken web site. Mathematical Puzzler: heard on air, email communication, *Car Talk* web site. *Philosophia Mathematica*: email communication. Richard Semmler: *The Washington Post*. ICTM: email announcement.