Curriculum Burst 42: A Length in a Square

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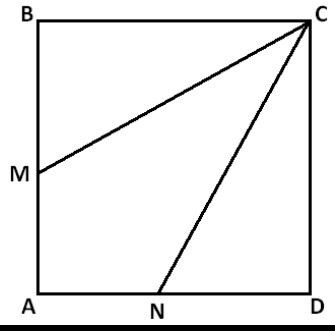
Square $ABCD$ has sides of length 3. Segments CM and CN divide the square's area into three equal parts. How long is segment CM ?



THE QUICK STATS:

Click here for video

MAA AMC GRADE LEVEL

This question is appropriate for the 8th grade level.

MATHEMATICAL TOPIC

Geometry

COMMON CORE STATE STANDARDS

8.G.B Understand and apply the Pythagorean Theorem.
7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.

PROBLEM SOLVING STRATEGY

ESSAY 2: **DO SOMETHING**

SOURCE

This is question # 23 from the 1999 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

The best start …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I have mixed feelings about this question. It's about a square with side-length 3 – not scary –and is asking for the length of the line CM. I find that request scary because I think we need to know where exactly the point M sits on the left side of the square in order to find that length. Is it half way up? Two-thirds of the way up? I don't know.

In order to "DO SOMETHING" let me just list easy things I know that might be relevant:

A square of side-length 3 has area 9.

The question says that the square is being divided into three regions of equal areas.

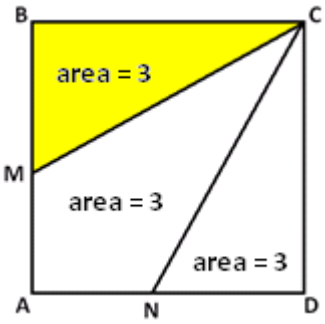
Each region in the square has area $\frac{1}{3}\cdot 9 = 3$.

I don't know a formula for the area of the funny shape in the middle of the square, but I do know:

The area of a triangle is given by $\frac{1}{2}\cdot base \times height$.

This is all well and good, but the question – as I recall – is asking for a length. All I've mentioned so far are areas.

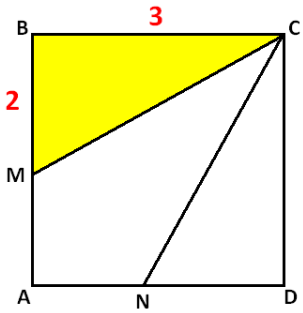
Well … the length CM is an edge of one of the triangles, a triangle with area 3.



Oh! If I think of BC as the base of that triangle (and it has length 3) and BM as the height we have:

$$\frac{1}{2}\cdot 3\cdot BM = 3$$

This gives $BM = 2$. Alright, the point M is one third of the way up the side of the square. (The picture isn't drawn to scale I guess!)

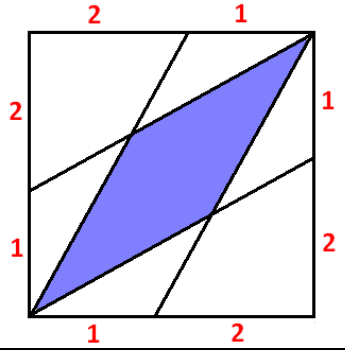


Now I can see how to work out the length CM: Just use Pythagoras's theorem. (We have a right triangle!)

$$CM = \sqrt{3^2 + 2^2} = \sqrt{13}.$$

Lovely!

Extension: (TOUGH!)
What is the area of the shaded region shown?



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