Curriculum Burst 51: Given a Bye

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There are 100 players in a singles tennis tournament. The tournament is single elimination, meaning that a player who loses a match is eliminated. In the first round, the strongest 28 players are given a bye, and the remaining 72 players are paired off to play. After each round, the remaining players play in the next round. The match continues until only one player remains unbeaten. The total number of matches played is:

(A) a prime number   (B) divisible by 2 (C) divisible by 5 (D) divisible by 7 (E) divisible by 11

QUICK STATS:

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MAA AMC GRADE LEVEL
This question is appropriate for the 10th grade level.

MATHEMATICAL TOPICS
Combinatorics

COMMON CORE STATE STANDARDS
No direct connection – apart from fostering mathematical sophistication and confidence in problem solving.

MATHEMATICAL PRACTICE STANDARDS
MP1   Make sense of problems and persevere in solving them.
MP2   Reason abstractly and quantitatively.
MP3   Construct viable arguments and critique the reasoning of others.

PROBLEM SOLVING STRATEGY
ESSAY 8:   **SECOND GUESS THE AUTHOR**

SOURCE: This is question # 15 from the 2003 MAA AMC 10b Competition.

THE PROBLEM-SOLVING PROCESS:

As always, the correct first step is …

> **STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I am good with the first two sentences. We have 100 players about to play games in pairs, with one player being eliminated each game. But I am confused by the next sentence:

In the first round, the strongest 28 players are given a bye, and the remaining 72 players are paired off to play.

I am guessing "given a bye" means that 28 players don't have to play, and so the whole question is really about 72 players playing games?

After each round, the remaining players play in the next round.

Is this the 28 people? Is that what "remaining" means? Or does "remaining" mean the people who haven't been eliminated in that round of games? That would be 36 players (among 72 people one from each pair moves on).

The match continues until only one player remains unbeaten.

What about those 28 people? I haven't played enough sports in my life to know what "given a bye" means?

The total number of matches played is…

Okay. The answers offered are weird! I am confused enough by the question statement. I'll worry about the answers later on.

So what are we doing here? We've got 100 people playing games, two per game. There are 28 people that are either in or out, I don't understand which, yet we're meant to get down to a single unbeaten player in the end.

Well, the only way that makes sense is if those 28 people did play at some point. We have to eliminate 99 people to leave a single unbeaten player. As each game eliminates one player, there must have been a total of 99 games played. (And option (E) is the only description that applies to the number 99.) So if the author wrote a question that makes sense (and it is just me who doesn't understand the details), this has to be the answer!

I guess we are done. But I am curious if I can make sense of the numbers. Let me play with the actual figures.

In the first round there are definitely 72 players playing. That's 36 games in round one. And in the next round we either have 36 players playing in another 18 games (if the 28 people are truly "out") or we have $36 + 28 = 64$ players playing in 32 games (if those 28 are now back in). Which makes sense?

First case: Round two has 18 games, which means there are 9 winners for round three. That's a problem. We can't split this group into pairs!

Second case: Round two has 32 games, yielding 16 folk moving on. These folk play 8 games yielding 8 to move on. Then 4 more games for 4 more to play again. Then 2 more games and 2 final folk, who then play in a final 1 game. That's a total of $36 + 32 + 16 + 8 + 4 + 2 + 1$ games, and that equals 99!

Okay, "given a bye" means "pushed to round two" without playing in round one. I get it now!

> **A Classic Example:** *The following puzzle is a classic. I can't resist sharing it here.* Two cyclists at opposite ends of a straight section of road 20 miles long ride towards each other each at a steady speed of 10 miles per hour. A bug flying at a speed of 12 mph flies back and forth between the two cyclists, instantaneously reversing direction (somehow!) when it reaches the front wheel of one cyclist to head back to the front wheel of the other cyclist. The cyclists are prepared to crash into each other. How far does the bug fly before being sandwiched between the two front wheels assuming the bug started on the front wheel of one of the cyclists?

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