Curriculum Burst 85: A Reflected Triangle

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A triangle with vertices $(6,5)$, $(8,-3)$, and $(9,1)$ is reflected about the line $x = 8$ to create a second triangle. What is the area of the union of the two triangles?

Click here for video

QUICK STATS:

MAA AMC GRADE LEVEL
This question is appropriate for the lower high-school grade levels.

MATHEMATICAL TOPICS
Geometry: Coordinate Plane, Area.

COMMON CORE STATE STANDARDS
G-CO.A   Experiment with transformations in the plane

### MATHEMATICAL PRACTICE STANDARDS
MP1   Make sense of problems and persevere in solving them.
MP2   Reason abstractly and quantitatively.
MP3   Construct viable arguments and critique the reasoning of others.
MP7   Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 8:   **SECOND-GUESS THE AUTHOR**

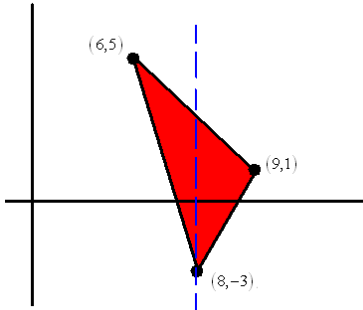**SOURCE:** This is question # 16 from the 2013 MAA AMC 10A Competition.
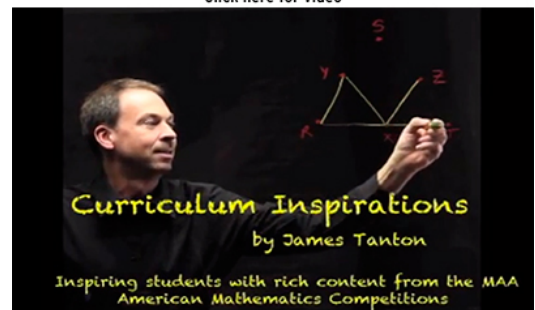
THE PROBLEM-SOLVING PROCESS:

As always …

I definitely need to draw a picture for this. For starters I need to see the triangle and the line $x = 8$ about which the figure will be reflected.


The point $(9,1)$, currently one unit to the right of the vertical line, will be reflected to $(7,1)$ on unit to the left. The point $(6,5)$ will go to $(10,5)$.

I am a little worried about where $(7,1)$ lies in relation to the left side of the original triangle. But since this is a competition problem I am guessing this is going to be "nice." The author of this problem wouldn't want me to be wasting a lot of time working out a complicated area.

In fact, what would be nicest of all is if the point $(7,1)$ lies <u>on</u> the left side of the triangle. Does it? Hmm.

Ooh! Look at the slopes. (I am ignoring positive versus negative in my picture.)

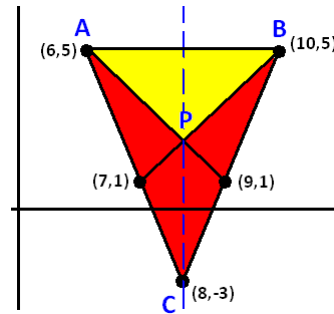two over, eight up   one over, four up
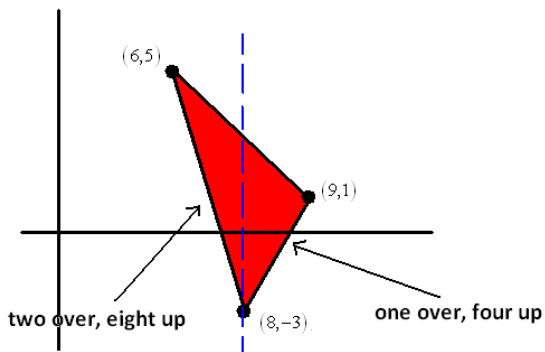
So the picture is symmetrical at least near the point $(8,-3)$ and the reflected triangle lands like this:
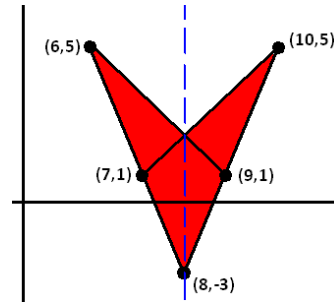
So the question wants this total red area. I think it is easiest think of this area as follows.

Label points as shown:

Then:
$$red\ area = area\ \Delta ABC - area\ \Delta ABP$$

The base of these triangles $\overline{AB}$ has length 4. The height of ΔABC is 8. I guess we need to know the coordinates of point P to figure out the height of ΔABP.

Well, $P = (8, y)$ for some value y, and
$$slope\ \overline{PB} = slope\ \overline{CB} = \frac{4}{3}.\ \text{So}\ \frac{5-y}{2} = \frac{4}{3}\ \text{giving}\ y = \frac{7}{3}.$$

This means the height of triangle ΔABP is $5 - \frac{7}{3} = \frac{8}{3}$.

Okay, we're there!
$$red\ area = \frac{1}{2} \cdot 4 \cdot 8 - \frac{1}{2} \cdot 4 \cdot \frac{8}{3} = \frac{32}{3}.$$

Extension: If everything in this question remained the same, except the point $(6,1)$ was changed to $(6,100)$, and the point $(9,1)$ to $(9,a)$, what value a makes the method of its solution essentially the same?

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