Curriculum Burst 86: Arithmetic Logs

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> The sequence
>
> $$\log_{12} 162,\ \log_{12} x,\ \log_{12} y,\log_{12} z,\log_{12} 1250$$
>
> is an arithmetic progression. What is x?

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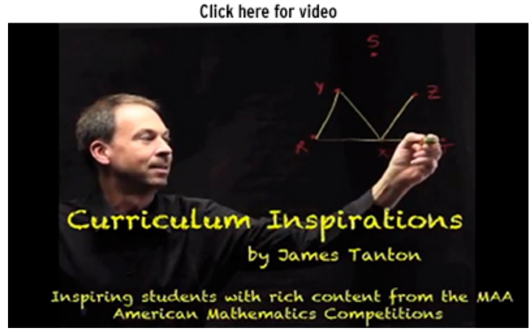

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the upper high-school grade levels.

MATHEMATICAL TOPICS

Logarithms

COMMON CORE STATE STANDARDS

F-BF.B5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 7: **PERSEVERENCE IS KEY**

SOURCE: This is question # 14 from the 2013 MAA AMC 12A Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

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STEP 1: Read the question, have an
emotional reaction to it, take a deep
breath, and then reread the question.
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This question definitely looks scary!

Let's just keep calm and see if we can work our way through it, slowly.

There is a list of (horrid looking) numbers that are, we are told, in "arithmetic progression." This means they increase by some constant amount from term-to-term. So the sequence is basically of the form:

$$A\,,\quad A+d\,,\quad A+d+d\,,\quad A+3d\,,\quad A+4d\,.$$

That looks less scary!

Now, for us, $A=\log_{12}162$ and $A+4d=\log_{12}1250$.
(There are the only actual values we know.)
It's not pleasant, but we see:

$$\log_{12}162+4d=\log_{12}1250\,.$$

Well the only thing I can think to do now is to write:

$$\begin{aligned}4d &= \log_{12}1250-\log_{12}162\\ &= \log_{12}\frac{1250}{162}\end{aligned}$$

I am guessing $\frac{1250}{162}$ simplifies: $\frac{1250}{162}=\frac{625}{81}$. Hmm. Well perhaps not.

Okay, so $d=\frac{1}{4}\log_{12}\frac{625}{81}$.

Oh! This is:

$$d=\log_{12}\left(\frac{625}{81}\right)^{\frac{1}{4}}$$

and $625=25\times 25=5^4$ and $81=3^4$, so this is actually:

$$d=\log_{12}\frac{5}{3}\,.$$

Alright. Feeling good! What was the question?

What is x?

Now $\log_{12}x$ is the next term in the sequence: "$A+d$."

$$\begin{aligned}\log_{12}x &= \log_{12}162+\log_{12}\frac{5}{3}\\ &= \log_{12}\left(162\times\frac{5}{3}\right)\end{aligned}$$

Aah! So $x=162\times\frac{5}{3}=81\times\frac{10}{3}=270$.

Extension: What's x if the five terms were in geometric progression?

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