Curriculum Burst 87: Selecting Committees

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A student council must select a two-person welcoming committee and a three-person planning committee from amongst its members. There are exactly 10 ways to select a two-person team for the welcoming committee. It is possible for students to serve on both committees. In how many different ways can a three-person planning committee be selected?

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QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the lower high-school grade levels.

MATHEMATICAL TOPICS

Permutations and Combinations

COMMON CORE STATE STANDARDS

S-CP.B9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 1: **ENGAGE IN SUCCESSFUL FLAILING**

SOURCE: This is question # 11 from the 2013 MAA AMC 10A Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This sort of question feels like a familiar textbook problem … I think. (There must be a twist!)

What do we know?

Well, the only number we are given is "10," for the number of two-person committees. Hmm. That's not telling us the number of students. It just says that if we choose two of them, there are ten ways to do so.

Oh … so that means there are ten ways to label two people "on" the committee and the rest "off" the committee. I don't know how many people there are, but if there are N of them, we have:

$$\frac{N!}{2!(N-2)!}=10.$$

(See http://gdaymath.com/courses/permutations-and-combinations/ for thinking of "N choose 2" as labeling.)

It seems compelling to try to solve for N from this equation. Can we?

Well, it simplifies:

$$\frac{N(N-1)}{2}=10.$$

This gives:

$$N^2-N=20.$$

Let me multiply through by four. (Huh? Now see http://gdaymath.com/courses/quadratics/ , lesson 2, for why I do this!)

$$4N^2-4N=80.$$

	2N	-1
2N	$4N^2$	-2N
-1	-2N	1

We have:

$$4N^2-4N+1=81$$
$$(2N-1)^2=81$$
$$2N-1=9 \text{ or } -9$$
$$2N=10 \text{ or } -8$$
$$N=5 \text{ or } -4$$

For this problem we must have $N=5$. That is, there are five students.

Umm. What was the question?

> *It is possible for students to serve on both committees. In how many different ways can a three-person planning committee be selected?*

Okay, among five people we need to label 3 as "on" this second committee and 2 as "off." There are:

$$\frac{5!}{3!2!}=10$$

ways to do this too!

Extension: A student council must select a two-person welcoming committee, with one person on that committee named "chair," and a three-person planning committee, also with one person named "chair," from amongst its members. There are exactly 30 ways to select a two-person team for the welcoming committee with chair. It is possible for students to serve on both committees. In how many different ways can a three-person planning committee with chair be selected?

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