

Curriculum Inspirations

Inspiring students with rich content from the
MAA American Mathematics Competitions



Curriculum Burst 98: Consecutive Squares

By Dr. James Tanton, MAA Mathematician in Residence

Pick two consecutive positive integers whose sum is less than 100. Square both of those integers and then find the difference of the squares. Which of the following could be the difference?

(A) 2 (B) 64 (C) 79 (D) 96 (E) 131

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Algebra; Geometric Thinking

COMMON CORE STATE STANDARDS

6.G.A Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

MATHEMATICAL PRACTICE STANDARDS

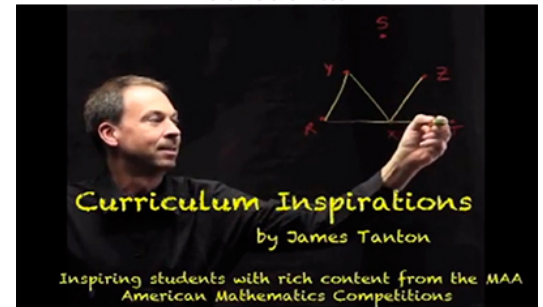
- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 6: [ELIMINATE INCORRECT CHOICES](#)

SOURCE: This is question # 19 from the 2007 MAA AMC 8 Competition.

[Click here for video](#)



THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Let me read this question carefully.

Pick two consecutive positive integers whose sum is less than 100.

Okay, “two consecutive integers” mean two numbers next to each other, like 37 and 38, or 5 and 6. And they have to sum to less than 100. (So we can’t have 55 and 56, say. Their sum is too big.) So I guess 49 and 50 are as big as we can go?

Square both of those integers and then find the difference of the squares.

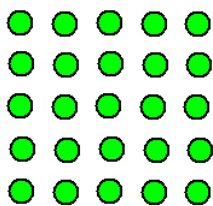
So I am meant to work out $38^2 - 37^2$ and $6^2 - 5^2$ and $50^2 - 49^2$, and so on.

Which of the following could be the difference?

Alright. So only one of the five choices given could be a difference of two squares. Hmm. Is that right?

What is a square number?

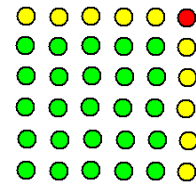
I’m a visual person and to me a square number is, literally, a picture of a square: a square array of dots. Here’s a picture of 5^2 :



A picture of 6^2 has six rows of six dots.

This question wants the difference of two squares.

If we merge a picture of 5^2 with a picture of 6^2 :



we see the two squares differ by an outer “L” of dots. Here the L is made of a column of 5 dots, a row of 5 dots, and one extra corner dot. The difference between 6^2 and 5^2 is “double five plus one extra dot.”

Ooh. This will always be the case for a picture of two consecutive squares: Their difference is double something plus one. This difference is sure to be odd!

But look at our options:

(A) 2 (B) 64 (C) 79 (D) 96 (E) 131

Since the difference is odd, options (A), (B), and (D) are out! So the answer (C) or (E).

I bet the answer is (C) since the numbers we choose must sum to something smaller than a 100. This probably means the difference isn’t too big.

Hmm. Is it possible for two consecutive squares to differ by 79?

The difference is “double something plus an extra dot.” Is 79 double something plus one?

$$79 = 78 + 1 = 2 \times 39 + 1.$$

The small square must be 39^2 and the next square is 40^2 ! (Check: $40^2 - 39^2 = 1600 - 1521 = 79$.)

YEP! The answer is (C).

Extension: No two square numbers (not necessarily consecutive) can differ by 2. Can you see why? Can two square numbers differ by 4? By 44? By 14?

Extra: In general, which numbers equal the difference of two square numbers?

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