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MAA American Mathematics Competitions



Curriculum Burst 99: Unicorns at Play

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Before district play, the Unicorns had won 45% of their basketball games. During district play, they won six more games and lost two, to finish the season having won half their games. How many games did the Unicorns play in all?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Algebra: Interpreting and solving problems algebraically.

COMMON CORE STATE STANDARDS

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.

8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables.

MATHEMATICAL PRACTICE STANDARDS

Make sense of problems and persevere in solving them. MP1

Reason abstractly and quantitatively. MP2

Construct viable arguments and critique the reasoning of others. MP3

MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ENGAGE IN SUCCESSFUL FLAILING ESSAY 1:

SOURCE: This is question # 20 from the 2007 MAA AMC 8 Competition.





THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This feels like an algebra problem in the guise of a word problem. Okay. Deep breath!

Let's give some names to things.

Let W be the number of wins the Unicorns had and L their number of losses. The total number of games they played is W+L .

We are told that 45% of these games are the wins. Okay so the total number of wins out of the total number of games is the same as 45 out of 100:

$$\frac{W}{W+L} = \frac{45}{100}.$$

If they played every $100\,$ games played, $45\,$ of them were wins. If they played $200\,$ games, $90\,$ were wins. And so on.

(And if they played just 50 games, then $22\frac{1}{2}$ were wins?!)

Okay. Next is district play: six more wins and two more losses. Hmm.

The question says they finish the season with 50% of the games being wins.

The new number of wins is W+6 and the new number of losses is L+2. But "50% wins" means the number of wins equals the number of losses.

That's not too hard to interpret algebraically:

$$W + 6 = L + 2$$
.

This gives L = W + 4. Let's just put that back into the first equation. (It feels like something meaningful to do.)

$$\frac{W}{W+L} = \frac{45}{100}.$$

We see:

$$\frac{W}{W+W+4} = \frac{45}{100}.$$

So

$$100W = 45W + 45W + 180$$

$$10W = 180$$

$$W = 18$$

and

$$L=22$$
.

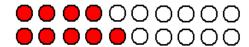
Thus
$$W + L = 18 + 22 = 40$$
.

Just to be sure ... What was the question?

How many games did the Unicorns play in all?

Oh! W+L is the total number of games before district play. So with district play the total is 48 games. That's the final answer!

Extension: We said that "out of every 100 games played, 45 are wins." Since $\frac{45}{100} = \frac{9}{20}$, we could say equivalently that out of every 20 games played, 9 are wins.



Is there a way to solve this problem purely visually, say, using copies of the diagram shown?

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