

Curriculum Inspirations

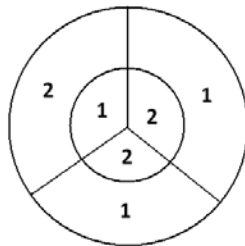
Inspiring students with rich content from the
MAA American Mathematics Competitions



Curriculum Burst 102: A Dart Probability

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On the dart board shown in the figure, the outer circle has radius 6 and the inner circle has radius 3. Three radii divide each circle into three congruent regions, with point values shown. The probability that a dart will hit a given region is proportional to the area of the region. When two darts hit this board, the score is the sum of the point values in the regions. What is the probability that the score is odd?



QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Probability; Area formula for a circle.

COMMON CORE STATE STANDARDS

- 7.SP.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
- 7.G.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 7: [PERSEVERANCE IS KEY](#)

SOURCE: This is question # 25 from the 2007 MAA AMC 8 Competition.



THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This problem doesn't feel too scary to me.

The only way to get two numbers to add to an odd score is to hit a "2" with one dart and a "1" with the other. So I need to know the chances of hitting a two and the chances of hitting a one! These are given by the fraction of area each number covers on the dart board.

So let's work out some areas!

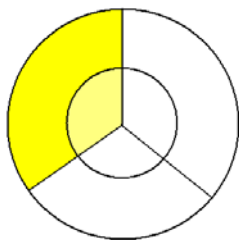
The large circle has radius 6 and so has area:

$$\pi \times 6^2 = 36\pi.$$

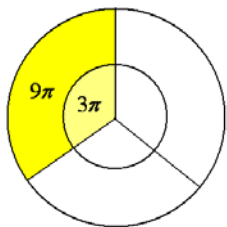
The small circle has area $\pi \times 3^2 = 9\pi$.

Each "wedge" in the small circle is one third of this area, and so each inner wedge has area 3π .

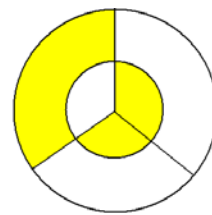
Each "wedge" of the big circle is one third of 36π , which is 12π , but that includes a small wedge too.



So the area of the outer part of this big wedge (that sounds confusing!) is $12\pi - 3\pi = 9\pi$.



Okay. The 2s cover these parts of the board:



That's an area of $9\pi + 3\pi + 3\pi = 15\pi$ out of a total area of 36π . The chance of hitting a 2 is:

$$P(2) = \frac{15\pi}{36\pi} = \frac{15}{36}.$$

The 1s cover the rest of the area, which must be $36\pi - 15\pi = 21\pi$, and the chance of hitting a 1 is:

$$P(1) = \frac{21}{36}.$$

Now ... I could either hit a 1 first and a 2 second (the chances of this are $\frac{21}{36} \times \frac{15}{36} = \frac{7}{12} \times \frac{5}{12} = \frac{35}{144}$) or I could hit a 2 first and a 1 second (with probability $\frac{15}{36} \times \frac{21}{36} = \frac{35}{144}$). So the chances of getting a 1 and a 2 in any order are:

$$\frac{35}{144} + \frac{35}{144} = \frac{70}{144} = \frac{35}{72}.$$

Extension: In this question the odd score we get must be 3, and there two ways to get 3 as a sum of 1s and 2s:

$$3 = 1 + 2$$

$$3 = 2 + 1$$

How many ways are there to write 4 as a sum of 1s and 2s? How many ways to write 5 this way? 10? 500?

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