

Curriculum Inspirations

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MAA American Mathematics Competitions



Curriculum Burst 107: Best Value

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A company sells detergent in three different sized boxes: small (S), medium (M) and large (L). The medium size costs 50% more than the small size and contains 20% less detergent than the large size. The large size contains twice as much detergent as the small size and costs 30% more than the medium size. Rank the three sizes from best to worst buy.

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Algebra; Percentages

COMMON CORE STATE STANDARDS

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 7: [PERSEVERANCE IS KEY](#)

SOURCE: This is question # 22 from the 2005 MAA AMC 8 Competition.



THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question is confusing. There's too much information!

There are three different detergent box sizes: S, M, and L. And we're being told about their costs and their sizes – well, costs and sizes in relation to each other. But all the information is overwhelming. **Deep breath!**

Let's go through it all line by line.

The medium size costs 50% more than the small size...

I don't know what the cost of the small size is. But if its cost is D dollars, then the medium box costs $1.5 \times D$. (That's 50% more, right?)

... and contains 20% less detergent than the large size.

So the medium box has volume 0.8 times the volume of the large size.

Okay, I am getting lost already. Let me make a table:

	Small	Medium	Large
Cost	D	1.5D	
Volume		0.8V	V

Here I wrote V for the volume of the large box, whatever it happens to be. This table is easier to work with.

Okay, what's next?

The large size contains twice as much detergent as the small size and costs 30% more than the medium size.

Okay, the volume of the small box is half that of the large. We can put that in.

	Small	Medium	Large
Cost	D	1.5D	
Volume	0.5V	0.8V	V

And the large costs 30% more than the medium. That's confusing! The cost of the large is 1.3 times $1.5D$. I am just going to write that!

	Small	Medium	Large
Cost	D	1.5D	$1.3 \times 1.5D$
Volume	0.5V	0.8V	V

Now ... what's the question?

Rank the three sizes from best to worst buy.

What makes a good deal? The greatest amount of detergent per dollar spent. Okay. Let's work out the volume-to-cost ratio for each.

$$\text{small} = \frac{0.5V}{D} = \frac{1}{2} \cdot \frac{V}{D}$$

$$\text{medium} = \frac{0.8V}{1.5D} = \frac{8}{15} \cdot \frac{V}{D}$$

$$\text{large} = \frac{V}{1.3 \times 1.5D} = \frac{100}{13 \times 15} \cdot \frac{V}{D} = \frac{100}{195} \cdot \frac{V}{D}$$

Now V/D is just some number. So what we need to do is compare the multiples of this number, whatever it is.

For starters, $\frac{8}{15}$ and $\frac{100}{195}$ are both larger than a half, which

means that the small box is the worst deal. Now, is $\frac{8}{15}$

larger than $\frac{100}{195}$? That is, is $\frac{8 \times 195}{15 \times 195}$ larger than

$\frac{100 \times 15}{195 \times 15}$. Hmm.

$$8 \times 195 = 8 \times 200 - 40 = 1560$$

$$100 \times 15 = 1500$$

So $8/15$ (for the medium box) is largest.

The order of best to worst deal is thus: MLS.

Extension: Why does "cross-multiplying" in algebra class work?

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