Curriculum Burst 111: Different Darts

By Dr. James Tanton, MAA Mathematician in Residence

> Five friends compete in a dart-throwing contest. Each one has two darts to throw at the same circular target, and each individual's score is the sum of the scores in the target regions that are hit. The scores for the target regions are the whole numbers 1 through 10. Each throw hits the target in a region with a different value. The scores are: Alice 16 points, Ben 4 points, Cindy 7 points, Dave 11 points, and Ellen 17 points. Who hits the region worth 6 points?
>
> (A) Alice   (B) Ben   (C) Cindy   (D) Dave   (E) Ellen

QUICK STATS:

MAA AMC GRADE LEVEL
This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS
Organization of Data

COMMON CORE STATE STANDARDS
7.SP.8b   Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

MATHEMATICAL PRACTICE STANDARDS
MP1   Make sense of problems and persevere in solving them.
MP2   Reason abstractly and quantitatively.
MP3   Construct viable arguments and critique the reasoning of others.
MP7   Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 10:   **GO TO EXTREMES**

SOURCE: This is question # 18 from the 2004 MAA AMC 8 Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

> **STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I am lost reading this question! Let's make sure I get it straight.

Five friends compete in a dart-throwing contest. Each one has two darts to throw at the same circular target, and each individual's score is the sum of the scores in the target regions that are hit. The scores for the target regions are the whole numbers 1 through 10. Each throw hits the target in a region with a different value.

So, there are five people each throwing two darts. This means there are ten scores. Each score is one of the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$. Since each dart hits a different region of the target, each dart hits its own number. So all ten numbers appear as scores.
The total score of each person is the sum of the two numbers their darts hit. (Is that all the information? Did I get it right?)

The scores are: Alice 16 points, Ben 4 points, Cindy 7 points, Dave 11 points, and Ellen 17 points.

Okay.

Who hits the region worth 6 points?

Oh dear! How can I know?

It seems natural to focus on some extreme score. Ben got the lowest score: 4. Ellen got the highest: 17.

Oh, 4 can only come from $1+3$. So I know Ben's darts.

Ellen could be $7+10$ or $8+9$. I don't know her darts.

Okay, since small seems to be fruitful, what's the next smallest? Cindy with 7. With the numbers that are left, this can only be $2+5$.

Next smallest is Dave with 11. This has to be $4+7$.

Alice with 16 has to be $6+10$ and she is the one that hit the six!

Awesome!

> **Extension:** The scores $4, 7, 11, 16, 17$ represent five numbers for which there is only way to write each as a sum of two numbers from the set of numbers one to ten using each number exactly once. (Got that?!)
>
> Is there another set of five numbers with this curious property? A third set?