

Curriculum Inspirations

Inspiring students with rich content from the
MAA American Mathematics Competitions



Curriculum Burst 126: Mixed Colors

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A mixture of 30 liters of paint is 25% red tint, 30% yellow tint and 45% water. Five liters of yellow tint are added to the original mixture. What is the percent of yellow tint in the new mixture?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Number sense: Percentages. Algebra.

COMMON CORE STATE STANDARDS

7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.

MP2 Reason abstractly and quantitatively.

MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 1: [ENGAGE IN SUCCESSFUL FLAILING!](#)

SOURCE: This is question #17 from the 2007 MAA AMC 8 Competition.



THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

The words in this question are not confusing, but the idea of the question is!

We have 30 liters of liquid which is a mixture of red tint, yellow tint, and water. (I guess I am going over the question again in my mind.) We add five more liters of yellow tint – that makes a total of 35 liters of liquid – and we want the fraction (percentage) of yellow tint in this 35 liters.

Well, I am nervous about adding percentages and liters. Maybe I should just figure out how much of each type of liquid – red tint, yellow tint, and water – there is.

$$\begin{aligned}\text{Red Tint} &= 25\% \text{ of } 30 \text{ liters} \\ &= \text{one quarter of } 30 \text{ liters} \\ &= 7.5 \text{ liters.}\end{aligned}$$

$$\begin{aligned}\text{Yellow Tint} &= 30\% \text{ of } 30 \text{ liters, plus an} \\ &\quad \text{additional } 5 \text{ liters.}\end{aligned}$$

Let me just pause for a moment and try to work out this percentage.

Now 10% of 30 liters is 3 liters, so
 $30\% = 10\% + 10\% + 10\%$ of 30 liters is $3 + 3 + 3 = 9$ liters. There are 9 liters, plus an additional five liters, of yellow tint.

$$\text{Yellow Tint} = 9 + 5 = 14 \text{ liters.}$$

And finally:

$$\begin{aligned}\text{Water} &= 45\% \text{ of } 30 \text{ liters} \\ &= 3 + 3 + 3 + 3 + 1.5 = 13.5 \text{ liters.}\end{aligned}$$

So we have 7.5 liters of red tint, 14 liters of yellow tint, and 13.5 liters of water. (Check: $7.5 + 14 + 13.5 = 35$ liters. Good!)

I think this is all coming together beautifully.
Umm, what was the question?

What is the percent of yellow tint in the new mixture?

Well, there are 14 liters of yellow tint in the 35 liters. That fraction is $\frac{14}{35} = \frac{2}{5}$, which is 40%. Done!

Extension 1: If an additional N liters of yellow tint are added to the 35 liters, find a formula, in terms of N , for the percentage of yellow tint in the new mixture.

What value of N gives a volume of liquid that is 50% yellow tint? (Does the answer surprise you?)

Extension 2: I have 30 liters of paint in a bucket and a second empty bucket. I pour half the liquid from the first bucket into the second. I then pour half the liquid in that second bucket back into the first bucket. I then pour half the liquid now in the first bucket back into the second bucket. And so on, always pouring half the liquid from one bucket into the other, alternating which bucket I pour from turn to turn.

After doing this a million times, what can you say about the volume of paint in each bucket?

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