

Curriculum Inspirations

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MAA American Mathematics Competitions



Curriculum Burst 133: Very eight-centered

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The mean, median, unique mode, and range of collection of eight integers are all equal to 8.
What is the largest integer that can be an element of this collection?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the lower high-school grades.

MATHEMATICAL TOPICS

Statistics: Measures of central tendency.

COMMON CORE STATE STANDARDS

S-ID.A Summarize, represent, and interpret data on a single count or measurement variable.

MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 10: [GO TO EXTREMES.](#)

SOURCE: This is question # 21 from the 2002 MAA AMC 10A Competition.



THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

There is a lot of eight-ness going on here:

Eight numbers.

Mean = 8

Mode = 8 (Unique mode, no less.)

Median = 8

Range = 8

I need to make sense of each of these pieces of information. I am not sure where to begin.

Let me describe all this in my own words:

There are eight numbers and the number 8 is repeated the most times among them (unique mode). The average of the numbers, which is their sum divided by eight, is 8. So the sum of all eight numbers is 64.

That's the first three pieces of information.

The median is 8. This means that if I arrange the numbers in order, the average of the middle two numbers is 8.



All the numbers to the right are thus bigger than or equal to 8 and all the numbers to the left less than or equal. But since there are at least two 8s, the middle two numbers must themselves be 8.



The final piece of information: the range is 8. This means the biggest and smallest numbers differ by 8. Okay, we have a data set that looks like:



with $a \leq b \leq c \leq 8$ and $8 \leq d \leq e$. The question wants the largest feasible value of the largest number: $a + 8$.

Well, let's focus on largest values and ask: *Could* $a = 8$? In this case the data set appears:



Is this possible?

The numbers are meant to add to 64, so $d + e = 8$. This is impossible as d and e are each at least 8. Thus a cannot be 8.

Can $a = 7$?



Here b and c each equal 7 or 8. Also, $b + c + d + e = 26$. Now $b + c = 14$ or 15 or 16 , so $d + e = 12$ or 11 or 10 which is impossible. So a cannot be 7.

Alright, marching along. Can $a = 6$?



Here $b + c + d + e = 28$ with $b + c = 12, 13, 14, 15,$ or 16 . What about $b + c = 12$? This means $b = c = 6$ and $d + e = 16$. Ooh, does $d = e = 8$ work?



YES! (And by luck we didn't ruin the mode of 8!)

So that's it: The largest possible value in the data set is 14.

Extension: For which values of N is a puzzle of the following form meaningful?

Suppose N integers have mean, median, (unique) mode, and range of N . What is the largest possible value in the data set?

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