Curriculum Inspirations
Inspiring students with rich content from the MAA American Mathematics Competitions
MAA

Curriculum Burst 134: Removing Square Tiles

By Dr. James Tanton, MAA Mathematician at Large

A set of tiles numbered 1 through 100 is modified repeatedly by the following operation: remove all the tiles numbered with a perfect square and renumber the remaining tiles consecutively by starting with 1. How many times must the operation be performed to reduce the number of tiles in the set to one?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the lower high-school grades.

MATHEMATICAL TOPICS

Algebra: quadratic expressions

COMMON CORE STATE STANDARDS

A-APR.4 Prove polynomial identities and use them to describe numerical relationships.

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 7: **PERSEVERANCE IS KEY**

SOURCE: This is question # 22 from the 2002 MAA AMC 10A Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question seems strange, but I think I understand it.

We start off with tiles 1 through 100 and remove tiles $1, 4, 9, 16, \ldots, 100$, that's ten of them, leaving 90, and we renumber these remaining tiles 1 through 90.

Then we remove tiles $1, 4, 9, 16, \ldots, 81$, that's nine of them, and renumber what remains 1 through 81.

I might as well keep going!

We remove $1, 4, 9, 16, \ldots, 81$, nine of them, and renumber the rest 1 through 72.

We remove $1, 4, 9, 16, \ldots, 64$, eight of them, and renumber the rest 1 through 64.

We remove another eight, and renumber the rest 1 through 56.

We remove $1, 4, 9, 16, \ldots, 49$, seven of them, and renumber the rest 1 through 49.

We remove another seven and renumber the rest 1 through 42.

We remove six of them and renumber 1 through 36.

We remove six of them and renumber 1 through 30.

We remove five of them and renumber 1 through 25.

We remove five of them and renumber 1 through 20.

We remove four of them and renumber 1 through 16.

We remove four of them and renumber 1 through 12.

We remove three of them and renumber 1 through 9.

We remove three of them and renumber 1 through 6.

We remove two of them and renumber 1 through 4.

We remove two of them and renumber 1 through 2.

We remove one and are left with one!

Umm. I've forgotten the question!

How many times must the operation be performed to reduce the number of tiles in the set to one?

If I am counting correctly, we moved tiles 18 times.

Extension: Did you notice that we started with a perfect square number of tiles and, after performing the operation twice, we had another perfect square number of tiles? In fact, this phenomenon occurred every time we have a perfect square count of tiles. Coincidence?

Curriculum Inspirations is brought to you by the Mathematical Association of America and the MAA American Mathematics Competitions.

MAA acknowledges with gratitude the generous contributions of
the following donors to the Curriculum Inspirations Project:

The TBL and Akamai Foundations
for providing continuing support

The Mary P. Dolciani Halloran Foundation for providing seed
funding by supporting the Dolciani Visiting
Mathematician Program during fall 2012

MathWorks for its support at the Winner's Circle Level