Curriculum Burst 146: All Red Beads

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A bag contains two red beads and two green beads. You reach into the bag and pull out a bead, replacing it with a red bead regardless of the color you pulled out. What is the probability that all beads in the bag are red after three such replacements?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the lower high-school grades.

MATHEMATICAL TOPICS

Probability

COMMON CORE STATE STANDARDS

S-CP.B Use the rules of probability to compute probabilities of compound events in a uniform probability model

MATHEMATICAL PRACTICE STANDARDS

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 1: Engage in Successful Flailing

SOURCE: This is question # 21 from the 2003 MAA AMC 10B Competition.

THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

To understand the problem, let me just think of some ways we can make all the beads red.

One way: Pull out a green and replace it with a red. Then pull out a green and replace it with a red. And then we are done! (The third move does nothing more for us.)

Another way: Pull out a green and replace it with a red. Then pull out a red. Then pull out a green and replace it with a red.

And I see now that there is only one additional option: pull out red, then green, then green.

So I guess we can just work out the chances of each of the three cases occurring separately.

First case (GGR):

The chances of pulling out a green first are $\frac{1}{2}$. The chances of pulling out a second green are $\frac{1}{4}$ (since we have three reds and one green in the bag after the first move). The chances of pulling out a red in the third move are 1. Thus the probability we'd be performing "GGR" is:

$$\frac{1}{2}\times\frac{1}{4}\times 1=\frac{1}{8}.$$

Second case (GRG):

The chances of pulling out a green first are $\frac{1}{2}$. The chances of pulling out a red second are then $\frac{3}{4}$. The chances of pulling a green third are $\frac{1}{4}$. The probability of thus performing "GRG" is:

$$\frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}=\frac{3}{32}.$$

Third case (RGG):

The chances of performing "RGG" are:

$$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{4}=\frac{1}{16}.$$

Thus the chances we are performing one of these three cases is:

$$\frac{1}{8}+\frac{3}{32}+\frac{1}{16}=\frac{9}{32}.$$

Extension: A bag contains r red beads and g green beads. You reach into the bag and pull out a bead, replacing it with a red bead regardless of the color you pulled out. What is the probability that all beads in the bag are red after g such replacements? After $g+1$ such replacements?

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