Numeracy, Mathematics, and General Education

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"The Case for Quantitative Literacy" argues that quantitative literacy (QL) is not merely a euphemism for mathematics but is something significantly different—less formal and more intuitive, less abstract and more contextual, less symbolic and more concrete. Is this a legitimate and helpful distinction?

I believe that the distinction between quantitative literacy (QL) and mathematics is indeed a meaningful and powerful one. For me, the key area of distinction is signaled by the term *literacy* itself, which implies an integrated ability to function seamlessly within a given community of practice. Literacy as generally understood in the verbal world thus means something qualitatively different from the kinds of skills acquired in formal English courses. For one thing, it is profoundly social, and is therefore a moving target because its contents depend on a particular social context. For instance, it is easy to imagine literacies being quite different from one another in different historical periods or cultural contexts. So a literacy is not just an applied version of a discipline. Instead, it would seem to flow out of a specific set of symbolic and communication needs embedded deeply in a particular social environment or community of practice.

Another important point is that literacies are for the most part practiced invisibly and subconsciously by members of a community, not pulled out selectively and applied deliberately to a particular set of circumstances. In practicing QL, therefore, we would expect that an individual would not consciously say "Oh, this is mathematics" and enter

a different ("learned") way of thinking and acting. Instead, he or she would simply act competently without invoking a disciplinary context at all. A final and related point is that, although it may have different aspects such as prose, document, and quantitative, "literacy" is really all one concept. Thus QL is presumably practiced together with other literacies in most actual circumstances, whereas mathematics as a discipline can be practiced on its own.

For all these reasons, I think that the case can indeed be made that QL is different from mathematics as customarily understood. But I also believe that these differences are not easy for most people to grasp at first. This may be in part because they are not always able to recognize their own use of quantitative concepts and tools in everyday life. It also may be in part because early exposure to mathematics presents it as a distinctly different activity from natural forms of communication. Reading and writing thus appear to be expected extensions of everyday life in ways that are not necessarily true of mathematical concepts.

For example, the notion of approximation is inherently legitimate in verbal expression: we choose words to get a point across, and the particular form in which this occurs—unless it is grossly inappropriate—rarely inhibits people from communicating with one another. In contrast, I suspect that most people's early exposure to mathematics strongly imprints the idea that it is somehow illegitimate to improvise and approximate in the quantitative realm: things are either "right" or "wrong" and must be "precise" to be of any use. As a result, I suspect that the ready analogy of quantitative facility to other forms of literacy is not apparent to a lot of people.

> *The idea that everything in mathematics is either right or wrong causes many students great difficulty because they prefer to think in shades of gray, not just black or white. This helps explain why mathematics serves as a "critical filter" that blocks students with weak mathematical skills from rewarding careers. Just how important is it that all students master formal mathematics? Might context-rich quantitative literacy be a more reasonable alternative?*

Certainly I grant the premise of this question—that mathematics coursework does in many cases largely determine student entry into particular disciplines and careers. But I'm not entirely sure that QL in

all cases represents a substitutable alternative to formal mathematical training. We're really talking about different things here, and both may be important for certain college majors and careers. All students, regardless of major or career aspiration, need context-rich QL as an integral part of their education. But other mathematical topics and contexts not explicitly addressed by the case statement may also be required for success in particular fields.

The problem here, I believe, lies much less in the distinction between QL and formal mathematics than in the fact that particular topic areas within the latter may require a different kind of treatment. Students in courses of study that generally require calculus as a prerequisite, engineering and physics for example, probably do not need to master everything that is typically addressed in a traditionally taught calculus course. But they do need to gain formal and operational mastery of particular concepts and tools that would not properly be considered part of the general QL domain. Similarly, students entering business and social science programs (as well as biology and medicine) ought to know far more about probability and statistics than is addressed by QL, at least as described in the case statement, but they may not need to be familiar with all the topics generally covered in a college statistics course.

Thus the pipeline problem for me is more about the way regular college mathematics courses typically are organized and taught than about the more basic distinction between QL and formal mathematical training. At minimum, addressing the question as posed requires (1) more modularization to allow tailored prerequisite experiences to be offered (perhaps "just in time" as students encounter particular discipline-related applications) and (2) far more experience with applications and real-world problem solving than is generally provided. As I understand it, "reform calculus" points in this direction, but I don't see these kinds of applications much in courses such as statistics or college algebra. Certainly, an enhanced and universally required QL component might prepare students to do well in such redesigned college mathematics courses, which would in turn provide more effective preparation for later disciplinary work.

At the same time, greater emphasis on direct application and understanding—regardless of the concepts being taught—would go a long way toward alleviating classic "math anxiety" for many students. The notion

of QL is especially helpful here because it emphasizes that quantitative concepts are a part of everyday life. This linkage might build confidence for many students, because they could be shown to already possess some understanding, no matter what level of QL they currently have attained. Stressing the connections between quantitative tools and familiar situations would similarly reduce the feeling that numbers are part of an impenetrable foreign language.

In short, I believe most QL skills should be fully developed as a prerequisite to postsecondary education, though, like appropriate verbal and writing skills, this cannot be assumed in the short term. Students entering college with particular major and career aspirations, however, need solid backgrounds in selected areas of formal mathematics as well as QL, both taught in ways that make them applicable and engaging.

Standards and Assessments

Many people share your desire that QL skills be acquired by the end of secondary school. Do you think that current efforts to improve public education by developing state-based standards and assessments are likely to lead to improved QL skills?

I believe that depends a lot on the standards and assessments. In general, I like the direction taken by such efforts as the New Standards project, in which the statements put forward are explicit and performance-based, enabling both teachers and the public to gain some idea of what they mean. But to remain true to quantitative literacy as described in the case statement, standards really must be described in "ability" terms rather than in content or knowledge-based terms. The approach also needs to be one in which each standard is clearly illustrated by reference to the types of real-life, concrete problems students are expected to formulate and address.

This, in turn, means that the assessments used in such an approach must be authentic and complex—for instance, requiring students to shift contexts in applying concepts and to operate in real-world settings. More profoundly, in contrast with what (I think) is prevalent in K–12 practice, the emphasis should not simply be on passing high-stakes exit tests, however well constructed. Instead, standards of the type I am thinking of must be embedded in routine faculty assessments of students' classroom work,

and students should be fully aware of what constitutes "good performance" on such tasks.

> *We can all hope that the standards movement will, somehow, lead to improving the QL skills of secondary school graduates and college entrants. But this raises a related question: Just how important is quantitative literacy in the priorities of colleges and universities? Should QL be required as prerequisite to admission to a four-year degree program? Is there any consensus on the level of QL that a college should require as part of general education?*

I believe QL is an extremely important topic for colleges and universities to address. My basic position is that a high level of QL should be a condition of college admission. As I previously admitted, however, this is unrealistic given current levels of mastery—an unfortunate situation that is just as true of more established forms of literacy.

I do not believe, however, that most college faculty share the same degree of anxiety with regard to shortfalls in QL that they have with respect to verbal skills, especially writing. This is a particular problem because it tends to reinforce the kind of disciplinary "steering" already noted in your questions. To put it bluntly, many college faculty either do not recognize current shortfalls in the QL skills of incoming students or do not consider such skills to be important for later success in many fields of study. In my experience, faculty in the sciences and in some of the social sciences tend to complain about deficiencies in both quantitative and communications skills among incoming students, while those in the humanities and many other social sciences are concerned only about the latter. Even fewer faculty would embrace the idea that a particular level of quantitative facility is an integral part of aesthetics or civic life, as emphasized in the case statement. This is the biggest problem with respect to achieving consensus that some level of QL ought to be a requirement for college admission or a required component of general education.

That said, I don't believe that it would be difficult for interested college faculty to quickly arrive at consensus about domain content and an appropriate level of QL as part of a general education requirement. Indeed, I think the case statement does an admirable job of identifying the particular topical ingredients that such a requirement might contain. Given the typical distributed politics of general education at most colleges and

universities, those uninterested would likely let those interested settle the "math part" among themselves as long as the resulting requirement remained roughly the size and scope of its predecessor.

I also don't believe that this is the only way to structure general education. Indeed, the past fifteen years or so has seen the emergence of many interesting alternative designs for general education that might allow more systematic attention to QL. One emphasizes problem-based courses that embed students immediately in practical settings or tasks, the nature of which automatically raises issues of QL together with other cross-cutting literacies and abilities. MIT's course on "Time," which systematically examines differing concepts of time and culminates with students building a workable clock, is a classic example. Other designs essentially turn the standard curriculum upside down by offering major-type courses early, and teaching literacy skills such as QL on an "as-needed" basis as particular applications and contexts come up. Still others define core abilities such as those contained in QL from the outset and interlace them throughout the curriculum—a structure best exemplified by Alverno College. These examples give me confidence that we are capable of constructing curricular designs for general education that can address the problem of coherence, but advocates of QL will need to recognize that they must make common cause with the proponents of other important literacies to make the case for such designs.

Why do you think college faculty outside the natural and social sciences are reluctant to support quantitative literacy? Does their reluctance represent a realistic assessment of the future needs of students or the legacy of their own educational experiences?

I don't think faculty outside the sciences and social sciences are reluctant to support quantitative literacy as much as they are indifferent to it for one reason or another. As my earlier answer suggested, most faculties tend to see both communication and quantitative literacy in "prerequisite skills" as opposed to "educated person" terms. That is, they view such instruction (typically provided by freshman-level courses in English or mathematics) largely from the perspective of what skills they, as faculty teaching subsequent courses, desire their students to have.

Humanities faculty are therefore very disturbed by deficiencies that will prevent student progress in their own courses and, at least at the moment, these are unlikely to include QL skills. Often, in fact, they see these deficiencies as things they will have to remediate themselves, at the expense of material they planned to teach. Social sciences and natural sciences faculty, on the other hand, view quantitative deficiencies (at least in part) from the same perspective that English faculty view deficiencies in writing—as something that they will have to do something about to get on with their own business. I'm not sure that either camp really views any of this from a "societal needs" perspective.

The problem is in many ways just as deep for other literacies. Writing faculty, for instance, often have a hard time understanding that the developmental paradigm of multiple cycles of "write and revise" that they teach does not always correspond well to a real world in which initial drafts must be quick and to the point. Similarly, both hard scientists and historians often rebel at "topics" courses that tend to treat their subjects not as rigorous courses in their own right but as tools for the functional citizenship or cultural literacy needed to understand contextually rooted, real-world issues. In fact, what they take to be this "lite" view of their own disciplines is often what they think they have a duty to root out. So the notion of embedding QL in general education is part of a larger issue of what role this portion of the curriculum ought to play in the first place. As I noted earlier, alternative conceptions of general education, conceptions based on interdisciplinary or problem-based courses and consciously structured to promote and reinforce key abilities such as QL as part of multiyear sequences of learning experiences, hold considerable promise for alleviating this situation, but they are far from the norm.

A Broad View of Quantitative Literacy

Setting campus politics aside, I'm curious what you think about the actual importance of numeracy. How important is quantitative literacy for understanding the arts, humanities, and public affairs? Are there any significant differences between what tends to be taught in mathematics courses and what you would expect of a numerate citizen?

I believe that QL as described in the case statement is integral to a deep understanding of all academic fields and, indeed, constitutes a condition for real intellectual discourse. But missing for most of the students (and instructors) that I encounter at the college level are three basic faculties that the statement addresses: (1) the ability to "see" mathematical functions and quantitative relationships in a graphic or structural form (and the reverse), (2) a reasonable sense of probability (manifest at a minimum in the ability to distinguish reasonable propositions from typical bookmakers' odds), and (3) the ability to estimate or approximate an answer to a multistep problem involving one or more shifts in order of magnitude. I'm not as familiar as I ought to be with what is typically taught in freshman-level mathematics courses, but I don't think these abilities feature prominently.

With regard to broader support for QL, I think conditions differ substantially among the various disciplinary families noted in the question. With respect to public affairs (and here I admit to being trained originally in survey research and econometrics), I think there is a lot of support for the position that facility in interpreting graphic representations of data, understanding basic notions of statistical confidence, and being able quickly to recognize inappropriate uses of data to support a public policy position constitute critical aspects of functional citizenship. For the arts and humanities, though, I think that the case is more difficult to make for both college faculty and the general public. The case statement makes a reasonable attempt to provide points of connection in the arts. At the same time, an evolving quantitative sense is part of the story of history and technology, which students will likely understand.

For me personally, the most compelling rationale for serious attention to QL is in some ways revealed by your question's (almost unconscious) reference to "significant differences." As I take the meaning of the question, your use of the term is analogous, rather than strictly mathematical. As such, it constitutes an excellent illustration of a broader view of QL itself. In just the same way, I recall being asked in a senior honors oral many years ago to connect the concept of the derivative, which had just come up in an interrogation about what I had learned in the required calculus and analytic geometry sequence, with my fascination with rapid patterns of societal change in Germany at the turn of the century, which

had been presented in a course in which I really thought I had "learned" something. The resulting "aha" in my head at that time remains one of the most powerful connection-making experiences of my college career. This analogic use of quantitative concepts is, quite simply, not currently taught in formal mathematics courses—or, indeed, anywhere at the college or secondary level. Emphasizing it more consciously in college course work could, I believe, powerfully deepen both historical and aesthetic understanding.

> *Although the use of mathematical language as analogy and metaphor in ordinary discourse is widespread, it is rarely if ever addressed directly in formal courses. In fact, mathematical scientists tend to be very critical of what they see as "sloppy" uses of precisely defined concepts. Does this difference reflect another aspect of the "two cultures" divide? What do you think can be done to bridge it?*

In telling the story about my senior honors exam, I took a risk of precisely the kind you speak. I thought a lot before putting it before a mathematical scientist because of that "typical" reaction, but I think the root of the answer to the question you pose is that such risks have to be taken to bridge the gap, which I do think is there.

We in the nonmathematical world often are very reluctant to take such risks because of the "one-down" attitude that we feel about being "imprecise" whenever something faintly quantitative comes up in the presence of a mathematical scientist. It is safer just to avoid the subject. (Let me hasten to add that, at least in my experience, this attitude is not projected by the mathematicians in any active way; it comes instead from the perceived aura of the discipline felt by those of us outside it.) Certainly, all of us experience a version of the same concern you describe with respect to our own disciplines when terms and concepts are taken out of context and applied to things with which they have no business at all.

But surely the objective is less to root out all analogical uses of disciplinary terms than to teach students to distinguish good analogies from bad ones and, perhaps more tellingly, to be able to recognize and articulate precisely the places where a particular analogy works and where it fails. From a teaching perspective, moreover, it seems to be exactly such situations that define "teachable moments" in which an imprecise use can

be probed to see if the underlying understanding is really there or, if it isn't, to try to develop it.

You raise an interesting point by noting that such uses of concepts and terms are "rarely addressed in formal courses." I'm not sure they should be—or even could be. Instead, I think they almost always arise in cross-disciplinary discourse or in practice settings in which folks are trying to get a handle (and almost any handle, at first) on a complicated, ill-defined problem. Rather than trying to engineer topics like this that you can't really teach in any formal way, we need to shape courses in many fields to make sure that such situations arise frequently, and then see where they lead.

Similarly, as faculty, we need to take risks in stretching concepts out of context and testing the resulting uses on one another when trying to stake out common ground in general education. Otherwise, by the logic of curricular politics I outlined above, we end up entirely isolated from one another. Taking risks in the presence of students—for instance in introductory interdisciplinary or problem-based college courses—is a powerful introduction to what academic discourse ought to be about.

Quantitative Literacy Across the Curriculum

Speaking of faculty taking risks, is it reasonable to teach quantitative literacy "across the curriculum" as writing often is taught, or does it require special expertise? Which teachers would be best suited to help students become quantitatively literate?

Yes, it does make sense to teach QL across the curriculum. Indeed, I can't conceive of any other way it could be done effectively without turning it into a "discipline" instead of a "literacy," but the analogy with writing also points out some of the substantial difficulties involved, especially in a college setting.

One is the notion of "special expertise" mentioned in the question. A major challenge in implementing writing across the curriculum, for instance, is the fact that faculty do not all know automatically how to coach or assess writing effectively, so substantial efforts at faculty development are generally required. I believe that the same level of effort is required for quantitative literacy and needs to be dedicated to both mathematical scientists and faculty in other disciplines.

The parallel with writing also suggests the need to consciously structure assignments and exercises across the disciplines so they simultaneously reflect meaningful specific subject-area applications and reinforce agreed-upon cross-disciplinary QL skills. This implies a "matrix" design for the curriculum in the early college years—something quite compatible with efforts such as freshman learning communities or similar linked course approaches.

Regarding who should teach QL, I see need for two kinds of expertise. One, of course, is provided by faculty in the quantitative disciplines, especially in areas such as business, science, and the social sciences. Many of these faculty already demonstrate QL in their research and writing. Further development efforts might allow them to more consciously model it in their classrooms and embed it in student assignments. Another type of expertise is probably also required, however, in the form of more specialized "QL coaches" to staff mathematics labs and instructional development centers. These individuals, like writing coaches, would have to be specially trained to assist faculty in developing the kinds of pedagogy and materials best suited to foster QL and to help students overcome typical difficulties with QL. This second role probably requires a unique kind of preparation and individuals with a typical mathematics background might or might not be best suited for it. The trick here, as in the case of writing, is to build an attractive career path for such individuals because they will be largely outside the disciplinary mainstream.

Most of what we've talked about in this interview concerns QL as a pedagogical or curricular issue associated with teaching college students. I wonder, in conclusion, if you have any thoughts about QL as a broader issue of public policy? Should it be such an issue? And, if so, how might it be raised and sustained?

I'm glad you asked that because our work at the National Center for Higher Education Management Systems (NCHEMS) often involves discussions with public policymakers and business leaders about the skill sets that ought to characterize the workforce and citizenry of the twenty-first century. And I'm happy to say that most of the folks we interview on these topics see a strong role for the kinds of abilities discussed in the case statement. As you might expect, we see this most prominently in the employment community, for two reasons. First, more and more workers,

even at entry level, are encountering technology and thus need the ability to comprehend manuals and training materials that require strong numeracy and quantitative skills. Second, more and more businesses are using quality management processes that require every worker to acquire a basic understanding of such concepts as sampling, variation, and significant difference. Together these forces mean strong advocacy for QL skills as part of a larger literacy package.

What I find particularly interesting about the NCHEMS discussions, though, is that they strongly reinforce the point I made at the outset: as a "literacy," QL is not practiced in isolation nor can it be separated from a particular social context. Indeed, we find that most employers and policymakers have a hard time making lists of the attributes they want to see in workers and citizens without subtly combining things that academics of all types like to separate, such as "verbal and quantitative," "cognitive and attitudinal," "academic and vocational." It's a refreshing perspective that, at least to me, constitutes one of the strongest validations of the concept of QL itself.