Reflections on an Impoverished Education

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> *"When I use a word," Humpty Dumpty said, in a rather scornful tone, "it means just what I choose it to mean—neither more nor less."*
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> *"The question is," said Alice, "whether you* can *make words mean so many things."*
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> *"The question is," said Humpty Dumpty, "which is to be master—that's all."*
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> *(Carroll, 1960, 269)*

If ever there was one, "quantitative literacy" is a case in point for Alice and Humpty Dumpty. As "The Case for Quantitative Literacy" makes abundantly clear, that phrase means many things to many people. Indeed, as often happens with complex ideas, the phrase may mean different things to the same person at different times. There I serve as a prime example.

School Mathematics

I grew up in the days when "mathematics" meant "school mathematics," divorced from the real world. My high school algebra course consisted of pure symbolic manipulation, for example, and my geometry course focused on producing proofs. Similarly, although my advanced algebra, trigonometry, and precalculus courses contained a few problems that pretended to deal with real-world phenomena, for the most part the contexts of these problems were so idealized as to render any applications meaningless. (When I was a student, probability and statistics, likely locuses of real-world problems, were nowhere to be seen in the secondary school curriculum.)

There was also extensive tracking. The main track aimed for college mathematics. Those not on the college preparatory track were shunted to dead-end courses in shop math or business math that offered neither "real" (that is, abstract, college preparatory) mathematics nor useful skills. Students who left the college preparatory track typically left mathematics as soon as they could. From ninth grade on the attrition rate was roughly fifty percent: only about half the students who completed mathematics at any grade level enrolled in a mathematics course the following year.

The same held in college, only more so. My undergraduate courses in probability and statistics dealt largely with probability distributions; the real world was not really present. Later on, in the 1970s, when as a new faculty member I taught the first generation of "new, improved" courses in elementary statistics, they too were removed from what most people would consider everyday reality. Hypothesis testing, for example, typically dealt only with narrow questions such as whether a batch of ball bearings produced at a factory was defective. To make things worse, the relevant numbers (means and standard deviations for sample sets of ball bearings) were given to students. The only computational tools available were paper and pencil, so working with real data sets was out of the question.

In sum, the only mathematics studied in my day was abstract, formal school mathematics. The real world (and thus quantitative literacy) was something else altogether. Of course, I lived in the real world. There I regularly used mathematical ideas, although not in any way that was obviously derivable from my formal training. I designed bookcases that had to fit exactly right. I made major purchases and worried about the accumulation of interest. I had to make sense of quantitatively based claims in the media. For example, was the chemical alar, which had been used on apple crops, really dangerous? Did electric power lines cause radiation damage? And what about major political issues? The stakes were large, the costs of quantitative illiteracy enormous.

Consider, for example, Ronald Reagan's "voodoo economics." Challenged in the 1980 presidential debates to explain his budgetary proposals, Reagan responded by waving his arms: "There's a line that goes like this [moving his arm in an upward direction, from left to right] and another line that goes like this [moving his arm in a downward direction, from left to right]. When those two lines cross, we'll have a balanced budget." This was unmitigated nonsense. Who knows what the lines represented, what the point of intersection meant? It did not matter. Perhaps intimidated by the mathematics, or charmed by his performance, interviewers did not follow up. The next day, newspapers reported that Reagan won the debate on economics.

The consequence, eight years later, was that the United States had gone from a budgetary surplus to the largest national debt in history. Some years later, California voters opted to invest in prisons rather than schools; Californians will pay the price for that decision in the years to come. Although it may be stretching the notion of quantitative literacy a bit, the fact is that a trends analysis would have pointed out the difficulties with this kind of policy. The absence of mathematical sense-making makes a big difference in the real world. (For a series of vignettes describing the disjunction between the ability to do formal reasoning, as in school mathematics, and the ability to use mathematics as a form of sense-making, see Schoenfeld, 1990.)

Quantitative Literacy

So quantitative literacy counts—big time. Generally speaking, I am comfortable with the description in the case statement. It is clear to me that quantitative literacy includes the various elements described in the statement: confidence with mathematics; a cultural appreciation of mathematics; the ability to interpret data, to think logically, to make decisions thoughtfully, to make use of mathematics in context; and more. Likewise, the expressions and skills seem well chosen. The case statement is entirely consistent with my general sense of what quantitative literacy should be: the predilection and ability to make use of various modes of mathematical thought and knowledge to make sense of situations we encounter as we make our way through the world.

Of course, that definition still begs a host of questions. Does quantitative literacy differ from what we learn, or should learn, in mathematics classes? Should we test for it as an exit skill from high school? From college? Can we identify certain courses that might serve as proxies—if you pass the course, then you meet the quantitative literacy requirement? In the context of my previous experience, classroom realities, and demographic data, these all at one time seemed to be very reasonable questions.

Convergence

I have now come to think about these issues differently. Recently I served as one of the writers of the National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* (NCTM, 2000). That experience—the goal of which was to outline a vision of mathematics education for the decade to come—provided me with the opportunity to reconceptualize my views of mathematics instruction and, concomitantly, of issues related to quantitative literacy. I now believe the following:

1. In the past, "quantitative literacy" and "what you learn in mathematics classes" were seen as largely disjoint. Now, however, they should be thought of as largely overlapping.
2. Every student should be enrolled in mathematics courses every year he or she is enrolled in high school.
3. Over the four years of high school mathematics, all students can and should become quantitatively literate and learn the mathematics that will prepare them for college.

I believe these three goals are both reasonable and desirable. Let me explain why. The reasons have to do with a convergence of the needs of the general citizenry for quantitative literacy and the needs of those who will ultimately pursue careers in mathematics and the sciences.

Once again, I will use my own experience as a case study. My Ph.D. is in mathematics; by most standards, I was very "well trained." Nonetheless, the mathematics education that I received was in many ways impoverished. Let me count the ways.

First, it was not until I was long into my career as an undergraduate that I encountered any situations that could really be called "problem solving." Most of the tasks I was assigned consisted of the application of tools and techniques I had just been shown. The idea of confronting a situation and making sense of it was not part of my education. Nor was learning any of a wide range of problem-solving techniques (e.g., heuristics).

Second, the mathematics I studied was "pure"; nary an application was to be seen. I never confronted an ill-defined situation, decided which aspects of it were inherently important, characterized them mathematically in a model, analyzed the properties of the model, drew conclusions about the situation on the basis of the model, or analyzed the reasonableness of those conclusions. I never had the opportunity to critique such models, or discover what makes for a good model and what makes for a bad one.

Third, I neither saw nor worked with any data in high school, and I worked only with "cooked" data in college. Fourth, with the exception of producing formal proofs on demand, the extent of the mathematical communication that was required of me was to produce a series of scribbles and to draw a box around the correct answer. I would have been a much better-trained mathematician if each of these issues, and others, had been addressed.

Interestingly, every one of the items I would have profited from as a mathematician-in-training is absolutely essential for literate citizenship. First, everyone needs to be an adaptive learner and problem solver. In the real world, problems do not come neatly packaged with methods of solution attached; our job is to figure out how to approach them. Second, as the case statement makes clear and as I argued above, literate citizenship calls for making a plethora of informed decisions—about interest rates, about situations that are inherently probabilistic, about the nonsense spewed by politicians. The best way to learn to make sense of applied situations, and to learn to assess claims made by others, is to have lots of practice building and assessing mathematical models.

Third, not only are we inundated by data but we now have access to technologies and techniques that enable us to operate on real data sets. Students can gather and analyze sets of data from real-world situations. Such skills will prepare them for grappling with data when they need to and for interpreting data that confront them. Fourth, it has long been understood that getting the right answer is only the beginning rather than the end of being effective on the job. The ability to communicate our thinking convincingly is equally important. Where better than in mathematics classes to learn this skill?

In short, the mathematical skills that will enhance the preparation of those who aspire to careers in mathematics are the very same skills that will help people become informed and flexible citizens, workers, and consumers. Moreover, a fair amount of mathematics can be motivated by interesting problems and learned in the process of solving them. Our goal should be to build a solid core of mathematics instruction that will serve both the mathematical and quantitative literacy needs of all students, while providing a solid base for those who desire the further study of mathematics.

Who should be responsible for this instruction? In the best of all possible worlds, instruction in all subject matters should touch on quantitative literacy whenever appropriate. Patterns of sense-making in science are often heavily quantitative, likewise in the social sciences. The study of history has been transformed through economic (i.e., quantitative) analyses. Questions of authorship—for example, did Shakespeare really write a particular play?—have been addressed by examining whether the frequency of word usage in the play in question differs significantly from Shakespeare's word usage in his well-known plays. It would be lovely to see "quantitative literacy across the curriculum" join "writing across the curriculum" as an accepted responsibility of all those who teach, at all levels.

My guess, however, is that writing across the curriculum is typically more rhetoric than reality and that infusing quantitative literacy throughout the curriculum would be an uphill battle, viewed by some as cultural imperialism. Thus, while inviting help from colleagues in other disciplines, I see the *Principles and Standards for School Mathematics* as the primary mechanism for achieving broad-based quantitative literacy. Four years of appropriately designed mathematics should ensure that all students emerge from high school quantitatively literate and prepared to pursue further study of mathematics if they so desire. For students who go on to postsecondary education, additional doses of mathematical sense-making—whether encountered in college mathematics or in other courses—couldn't hurt.

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