Mathematics and Numeracy: Mutual Reinforcement

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"The Case for Quantitative Literacy" offers a rich and diverse view of the numeracy needs of society, a view that encompasses much of mathematics and statistics. But the case statement also acknowledges that quantitative literacy has had and continues to have many meanings. In this response, I will focus on one particular aspect of quantitative literacy—the analysis of data—to draw some distinctions between quantitative literacy and mathematics. I believe these distinctions may facilitate a clearer understanding of what each area has to offer students and thus help educators give adequate emphasis to some of the special perspectives of each.

Working Definitions

The case statement makes a persuasive call for increased attention to numeracy in schools and colleges. In contrast, this response presents recommendations for mathematics education, some explicit, others implicit. Because quantitative literacy is so closely linked to school mathematics, those of us who teach mathematics need to consider very carefully the relationship between mathematics and quantitative literacy.

Mathematics is a well-known subject with a venerable history. A common dictionary defines "mathematics" as the science of (a) numbers and their operations, interrelations, combinations, generalizations, and abstractions and (b) space configurations and their structure, measurement, transformations, and generalizations. Numbers and shapes, arithmetic and geometry: these everyone recognizes as the essential foundation of mathematics.

In contrast, it appears from the case statement that the meaning of quantitative literacy is not well established. Indeed, the case statement devotes many pages to descriptions of elements, expressions, and skills that collectively describe quantitative literacy. Taking the words at their face value, we might summarize by saying that quantitative literacy is the ability to understand and reason with numerical information. That ability enables people to be comfortable with numerical data and to use them in meaningful ways, in particular to make well-reasoned decisions.

Using data to make decisions is rather different from the science of numbers and shapes. Yet even the latter well-accepted formulation has generated a wide variety of working definitions of mathematics among mathematicians, mathematics education researchers, and mathematics teachers, to say nothing of philosophers, engineers, scientists, and sociologists. So we should not be surprised when an emerging notion such as quantitative literacy produces the extraordinary variety of illustrations found in the case statement.

To compare mathematics with quantitative literacy, we need to focus not on the variety of definitions but on a few elements that are characteristic of each. For example, in mathematics, reasoning and proof play a central role. A critical feature of mathematics is understanding why assertions based on assumptions must be true. In contrast, numeracy frequently requires inferences based on estimates and approximations, on incomplete or sometimes inaccurate data.

Another distinctive characteristic of mathematics is that its assertions are about relationships among abstractions, whereas the inferences of quantitative literacy are almost always about something real. The first of these mathematical abstractions, both historically and pedagogically, is whole numbers. Other kinds of numbers are also studied, and these several number systems form the basis of many further abstractions, especially in algebra and analysis.

As the dictionary definition suggests, space configurations provide yet another source of abstractions that become objects of mathematical study. Other patterns can also serve as the basis for mathematical abstractions and assertions about their relationships. Quantitative literacy applies results from these mathematical abstractions to gain insight into concrete situations.

The Role of Numbers

Numbers, which are themselves abstractions, play important but different roles in mathematics and quantitative literacy. In quantitative literacy, numbers describe features of concrete situations that enhance our understanding. In mathematics, numbers are themselves the objects of study and lead to the discovery and exploration of even more abstract objects.

As the case statement makes clear, quantitative literacy offers people a quantifiable perspective for understanding the world. It does this primarily by reasoning with information obtained through measurements of quantifiable phenomena. The range of situations that can be quantified is very broad and constantly expanding. Familiar examples going back hundreds of years include the sizes of physical objects and the duration of events. More recent common examples include income distribution in populations, the differential effects of various medical treatments, and the relative popularity of different programs, policies, or politicians. In all these settings the role of numbers is descriptive: numbers help us understand and compare features of real-world situations and enable us to make decisions based partly on those features.

In mathematics, numbers play a very different role. The goal of a mathematical study of numbers is to obtain a better understanding of how numbers work within their own structures, of the properties they exhibit or lack. In contrast to quantitative literacy, in which numbers are descriptors of characteristics of the objects being studied, in mathematics numbers themselves are the objects of study.

When properties of specific numbers are studied, mathematicians quickly discover that it makes sense to look not at individual numbers but at systems of numbers and then to form even more abstract structures based on these systems. Reasoning about those general structures can provide insights that eventually may be applied to real-world problems, but mathematicians often are more interested in the abstract structures than in the concrete situations. Whereas the power of quantitative literacy derives from its faithfulness to real problems, the power of mathematics comes from its abstractness and consequent generality.

Historically, the need to measure objects and other phenomena provided a natural motivation for the introduction and development of numbers. The ways in which numbers are used to measure and compare objects and to analyze their properties are the first steps toward quantitative literacy. At the same time, learning how to compare numbers and combine them to form new numbers in meaningful ways are among the first steps in acquiring mathematical knowledge and ways of thinking.

Contrasts and Comparisons

From their common beginning in numbers, quantitative literacy and mathematics necessarily diverge, especially in school education. The elements and expressions enumerated in the case statement provide examples of these differences. This divergence of emphasis does not require completely separate instructional paths, but it does require that teachers give adequate and separate attention to the goals of each. At times, numbers and abstractions of number systems should be studied to understand and master efficient computational algorithms and to see what properties they satisfy. At other times, elements of this abstract knowledge need to be applied to understand models of complex concrete situations.

These different perspectives lead to different roles for the various types of reasoning employed in quantitative literacy and in mathematics. Although deductive, inductive, and analogic reasoning are important in both, the first seems more central to mathematics while the second and third play more prominent roles in mathematical modeling and quantitative literacy.

Mathematics provides many examples of knowledge that is important for understanding real-world phenomena, such as the order relation on real numbers, efficient computational algorithms, derivation of indirect dimensions from direct measurements, and use of scientific notation to express very large and very small numbers.

On the other hand, not all mathematical results, even about numbers, are directly applicable to or motivated by concrete situations. The important Fundamental Theorem of Arithmetic is a good example. It asserts that every integer greater than 1 is a unique product of prime numbers (except for reordering). Although this theorem turns out to have unexpected applications in logic and cryptography, these are neither the reasons it was discovered nor the primary reasons for learning why it is true.

Increased Demand

It is easy to understand the case statement's call for more emphasis on quantitative literacy. Technology has moved us into the information age. Technological improvements have broadly expanded both what can be measured and the accuracy of measurements. Increasingly sophisticated mathematical techniques convert complex measurements into tables containing massive amounts of data. These changes in turn confront decision makers with overwhelming amounts of data. The Internet, itself a product of technology, has made previously inconceivable quantities of data available to almost anyone who wants them. Understanding how these data are created—both technologically and mathematically—is an important prerequisite for being able to use data meaningfully.

This expanded availability of data has affected many areas of society. Fifty years ago most people who had knowledge of the natural sciences and engineering understood the fundamental importance of measurement and data. But there were very few such people. At that time access to huge quantities of statistical data was quite limited because of the unacceptably high costs of acquiring raw data and analyzing them to produce meaningful summaries.

Relatively inexpensive and widely distributed technology now enables the collection and analysis of massive amounts of data. One consequence is the dramatically increased role of quantitative data analysis in the social sciences. Beyond the social sciences, data are also used, for example, in political campaigns, to inform government decisions, and by businesses to develop product designs and marketing strategies. These and similar uses profoundly alter our cultural climate. Some understanding of the meaning and sense of data, including their acquisition and manipulation, is required for intelligent participation in a society in which decisions increasingly rely on interpretations of data.

In this sense, as the case statement argues, quantitative literacy provides an important complement to mathematics—the one emphasizing reasoning with data, the other reasoning with numbers and shapes. Although quantitative literacy and mathematics share a common root in numbers, they diverge in emphasis and subject matter. Yet they remain inextricably interconnected, especially in education. As students progress through their education, they should be given many opportunities to see how each supports the other.

Specifically, as students acquire mathematical sophistication they need opportunities to apply what they learn to the analysis of quantitative data and the understanding of real-world phenomena. Reciprocally, their study of increasingly complex natural and cultural phenomena should provide contexts for developing deeper mathematical understanding. The classic example of this reciprocity is velocity and acceleration, but analyses of opinions and behavior now are quite common as well. In this way a balanced educational program can enable numeracy and mathematics to reinforce each other.