Quantitative Literacy for the Next Generation

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"The Case for Quantitative Literacy" is a well-informed, well-documented, and persuasive statement on the importance of quantitative literacy. When we see an argument made that adults need to know something that evidence shows they do not know, we naturally turn to the schools for the answer. Accordingly, the case statement concludes that educational policy should promote a strong emphasis on quantitative literacy. But before an already full school curriculum takes on a new concern, we must ask if this concern has been addressed previously. If quantitative literacy already has been addressed, perhaps it is more difficult to learn than we thought. Or we may think we are addressing quantitative literacy when we are not, or not doing it well or well enough. If quantitative literacy has not been addressed to the extent we would like, we need to suggest specific steps that might be taken by teachers and curriculum developers to remedy the situation.

Few of the contexts described in the first paragraphs of the case statement—increases in gasoline prices, changes in SAT scores, risks of dying from colon cancer, numbers of refugees, costs of cell phone contracts, interest on car loans, team statistics in sports, odds in competitions, locating markets in business, analyzing soil, measuring drug dosages, etc.—are ever mentioned, let alone studied, in the mainstream school mathematics curriculum. Most of these contexts, however, did not even exist a hundred years ago when the school curriculum was, for all practical purposes, codified. Quantitative literacy today thus involves many applications that are so new (relative to the pace of educational change) that the school mathematics curriculum has not caught up with them.

Schools should not engage in self-flagellation for being behind current uses of mathematics. Continual updating is a necessary circumstance in such a vibrant subject. School mathematics cannot be expected to foresee new applications of mathematics; it must lag behind them. But we should be concerned if the adults of tomorrow do not possess the quantitative literacy that we know is needed today.

Most of the adults who do possess quantitative literacy learned it outside the mathematics classroom in much the same way they learned about computers. They developed some number sense, arithmetical confidence, and experience with data by being interested in sports and calculating sports statistics, by working in a store and dealing with money in and money out on a regular basis, by handling their own taxes and financial affairs, by building models, or by engaging in any of a large number of other activities. They could not have learned quantitative literacy in the mathematics classroom because historically the mainstream mathematics curriculum has devoted extraordinarily little attention to the connections of mathematics with the real world.

Yet today a large number of adults do not possess quantitative literacy. Whatever these adults may have learned in school, it did not include the prerequisites for quantitative literacy. Although much of the standard curriculum certainly has the potential to enable students to gain number sense, practical skills, and symbol sense (three of the aspects of quantitative literacy discussed in the case statement), mathematics as it has been taught in most schools does not help individuals gain quantitative literacy. In fact, some aspects of the school mathematics curriculum have worked *against* the development of quantitative literacy. If we want the next generation to be quantitatively literate, we need to avoid these aspects by adopting at least the following five practices in school mathematics classrooms:

- *Spend less time on small whole numbers*.

Despite the perceptions of some education critics, the understanding of small whole numbers in the U.S. population is very high. National Assessment of Educational Progress (NAEP) studies show that virtually all 17-year-olds in school compute accurately with small whole numbers and can apply operations on them appropriately in simple situations (Campbell, Voelkl, and Donahue, 1997, 57).

The reason for such stellar performance is the intense amount of time spent on these numbers and very little else. Although research indicates that most children enter kindergarten knowing how to count well past 20, traditional kindergarten mathematics programs spend most of the time counting to 10. By first grade most children can count past 100 (clearly learned outside of school), but first-grade texts spend a majority of the time on numbers no larger than 20. This slow start cannot help children develop the ability to deal with larger numbers.

Furthermore, the lower numbers are dealt with concretely, with models for counting, but when the larger numbers appear, they are almost always presented symbolically in the context of computation. In landmark studies in the second quarter of the twentieth century, William Brownell showed that teaching with attention to properties that relate numbers to each other (what he called *meaningful learning*) leads to higher performance than treating number facts as isolated bits of knowledge (*rote learning*) (Brownell, 1935). Adults' lack of ability to deal with arithmetic, even when computation was virtually the entire arithmetic curriculum, indicates that rote paper-and-pencil computational practice and facility does not enhance understanding of "number."

Those of us who have spent time teaching the real-world uses of mathematics believe that if we want people to understand these real contexts, we must teach them specifically and not expect automatic transfer from theoretical properties and relationships to practical situations. Contexts that apply fractions, decimals, percentages, and large numbers can fulfill the role of concrete experience for many students and lead to greater understanding of both theory and practice.

- *Give meanings of arithmetic operations that apply to numbers other than whole numbers*.

What does subtraction *mean*? An adult is likely to say "take away." Ask an elementary school teacher what multiplication *is*. If any answer at all is given, it is likely to be "repeated addition," for that is the meaning given in most elementary school textbooks. In books, division means splitting up into equal portions or repeated subtraction. Taking powers means repeated multiplication.

Each of these meanings applies well to whole numbers but each fails for fractions, decimals, percentages, and negative numbers. “Take away” does not explain why a temperature of 6° is 10° higher than one of -4°. Repeated addition does not readily explain why we multiply 9.25 by 12.417 (or $9\frac{1}{4}$ by $12\frac{5}{12}$) to find the area of a rectangular room 9’ 3” by 12’ 5”. Nor can splitting up or repeated subtraction easily explain why we divide to determine the speed of a runner who has run 100 meters in 11.35 seconds. Similarly, repeated multiplication does not explain the calculation $\$500(1.06)^{2.5}$ that gives the amount to which \$500 will grow in two-and-a-half years at an annual percentage rate of 6%.

Because of the ubiquity of calculators, applications of arithmetic no longer require that students be able to obtain the answers to the above questions by paper and pencil. These and many other applications do require, however, that the user understand the fundamental connections of subtraction with comparison, multiplication with area (and volume), division with rate, and powers with growth.

- *Employ calculators and computers as a natural part of the curriculum.*

I am old enough to remember when the first hand-held calculators appeared. I recall my immediate reaction: a godsend that would finally free us from the shackles of paper-and-pencil arithmetic. Thirty years later my belief is even stronger because calculators do far more than replace tedious arithmetic. Current technology enables ordinary people to work with mathematics that hitherto was inaccessible because of the computational limitations of paper and pencil.

Furthermore, current technology has caused much of the increase in the need for quantitative literacy. Without this technology, newspapers, financial institutions, scientific endeavors, and everything else that uses mathematics would not be the same. Why can a sports section report changes in baseball batting averages along with the box score from the game? Why can the price/earnings ratios of thousands of stocks be reported daily? What gives online financial planning sites the ability to calculate expected returns from any number of retirement plans in just seconds? The answer is the ability to do *automatic calculation*, calculation programmed into a spreadsheet or computer.

Applying these and a myriad of other uses of number in quantitative situations does not require that the reader or recipient be able to duplicate the calculator or computer. But it does require the ability to understand what the results of the calculations mean and to verify whether the calculations are correct to within some bounds of correctness.

• *Combine measurement, probability, and statistics with arithmetic*.

The content of the current school mathematics curriculum is often split into five strands: number (including computation), measurement, geometry, algebra (including functions), and probability and statistics. These are the strands by which items on the mathematics portion of the National Assessment of Educational Progress are classified (National Mathematics Consensus Project, undated); they are also the five content strands of the *Principles and Standards for School Mathematics* developed by the National Council of Teachers of Mathematics (NCTM, 2000).

It could be argued that any split of this type is artificial because all these strands are related; however, the separation of number from measurement has particularly negative implications for the teaching of quantitative literacy. For example, perimeter and addition, area and multiplication, similarity and division, and measurement conversion, multiplication, and division are inextricably related in reality, but rarely in school.

The study of measurement in elementary mathematics textbooks is almost exclusively devoted to geometric measures. Money, the most familiar form of measure to students, which could be used as an example of unit conversion both within and between systems, is not treated as measurement. Units of time are not studied along with other units. Units other than of length or weight (or mass) are rarely seen. Even counting units are not treated as units. Students most often encounter numbers as decontextualized.

Likewise, textbooks treat probability as a subject separate from arithmetic, unrelated to division even though probability is typically defined as a ratio. In the sections of the textbooks devoted to probability, there are discussions of a 25% chance of an event occurring, or equivalently that there is a 1 in 4 chance of occurrence, but these sections are often skipped by teachers. Probability is seldom mentioned in the sections on fractions and percentages that are always covered by teachers.

These artificial separations also apply to statistics. Students are not introduced to statistics as measures of a sample or a population, but as numbers resulting from formulas that typically rise from out of the blue. For instance, students do not learn to relate averages to division. They do not study percentiles as relating to percentages. Also, little attention is given to the origins of the numbers used in the calculations. A sample value is viewed as the value of a population without thought given either to the nature of the sample or the population it does or does not represent.

Students seldom study scales or the normal distribution, so they have little idea of the distributions of numbers that can be pivotal in their school lives: grade-level scores on standardized tests or scores on the SAT and ACT college admission tests. The ignorance of previous generations in these areas leads to major errors in the interpretation of test outcomes, such as viewing test scores as fixed rather than as sample measures subject to variability. This results in students being included in or excluded from academic programs based on differences in test scores that are too small to be significant.

- *Do not treat word problems that are not applications as if they were applications.*

The many examples of the need for quantitative literacy offered in the case statement can easily lead us to wonder why so little has been accomplished. I believe the problem relates in part to a perception by the majority of mathematics teachers about the "word problems" or "story problems" they studied in high school (e.g., "Mary is half as old as her father was . . ." or "Two trains leave the station one hour apart . . ." or "I have 20 coins in my pocket, some dimes . . ."). These problems have little to do with real situations and they invoke fear and avoidance in many students. So it should come as no surprise that current teachers imagine that "applications" are as artificial as the word problems they encountered as students, and feel that mathematics beyond simple arithmetic has few real applications.

I do not mean that the standard word problems should not be taught, or that there is no room for fantasy in mathematics. Most of the typical word problems make for fine puzzles, and many students and teachers appreciate this kind of mathematical fantasy. The point is that these kinds of problems are not applications, nor should they substitute for them.

This historical argument suggests that quantitative literacy will not become mainstream in our schools until a generation of teachers has learned its mathematics with attention to quantitative literacy—a chicken-and-egg dilemma similar to that regarding the public apathy about quantitative literacy described in the case statement. Short of forcing teachers to teach in a particular way—a practice I oppose on democratic grounds—there are at least two ways this dilemma can be resolved. We can hope that tomorrow's teachers will have studied in or are teaching one of the recent curricula that pay attention to the various aspects of quantitative literacy. Or we can engage in massive teacher training in quantitative literacy.

We may be able to obtain public support for attention to quantitative literacy if we emphasize that quantitative literacy is an essential part of literacy itself. From the first page through the feature articles, advertisements, editorial pages, business, entertainment, and sports sections, newspapers are filled with numbers. The median number of numbers on a full-length newspaper page is almost always well over 100; the mean is over 500 (Usiskin 1994, 1996). Numbers permeate tabloids and magazines as well. The popular press is not known for its familiarity with mathematics, and its editors are unlikely to have majored in science or mathematics in college, but numbers cannot be avoided. Without quantitative literacy, people cannot fully understand what is in the everyday news, what is in everyday life.

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