Assessing the Mathematics Major with a Bottom-Up Approach: First Step

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Abstract. This paper looks at the developing stage of an assessment plan at a small liberal arts university. This plan assesses the calculus sequence through the use of pre and post-tests. In this paper, we discuss the selection of the questions used for the exams, the implementation of the assessment tool, and results obtained.

What did we hope to accomplish?

Washburn University is a municipally supported, state assisted university located in Kansas's capital city of Topeka. The university is comprised of six major academic units; the College of Arts and Sciences, the School of Law, the School of Business, the School of Nursing, the School of Applied Studies and the Division of Continuing Education and has an undergraduate population of around 7000 students. The number of mathematics majors is small, with approximately eight majors graduating per year. These majors are spread out over three tracks: pure mathematics, secondary education, and actuarial science.

Although the three tracks have a common set of courses, there is enough variance amongst the degrees to make assessing the major quite a challenge. With such a small number of overall majors, there are not enough students in any one track to justify implementing three completely different assessment plans. Our challenge has been to find an assessment plan that assesses the core knowledge all majors have, yet also addresses some of the specific skills inherent to the individual tracks.

The department decided it would be easiest to start assessing our majors from the bottom, where courses for the three tracks are the same. With this in mind, we decided to focus our initial assessment activities on the calculus sequence.

What did we do?

In March 2003 two members of our department attended the first part of the SAUM Workshop #3 in Phoenix where they developed a pre and post-test assessment system for the calculus sequence. The hope was that a pre and post-test system would provide information on student learning while also giving valuable feedback on student retention of knowledge. Ideally, pre-test questions would test specific prerequisite skills that would be needed for completing post-test problems for the same course. Further, some post-test questions from Calculus I and II would be used as pre-test questions in Calculus II and III respectively. Thus, the pre-test would not only give us a basis of comparison for knowledge gained in the current course, but also provide us with information on retention of knowledge from the prior course.

The pre-tests were implemented in the form of a review quiz given within the first week of the calculus course. This review quiz was counted in the student's overall quiz score for the semester. It should be noted that the instructors who teach the calculus sequence were already spending part of

the first week reviewing prerequisite skills and giving some type of review assignment. Hence, the implementation of a review quiz fell in quite naturally with the instructors' plans. Post-test questions were simply embedded in the final examination.

After conferring with several department members, it was decided that pre-tests would consist of 6–8 questions. Most, if not all, of these prerequisite skills would be required to solve a post-test problem. Additionally post-tests for Calculus I and II would contain some problems from the Calculus II and III pre-tests. Pre and post-test questions were scored on a scale of 0–4 as follows:

0 – completely wrong/incorrect approach taken
1 – some correct work, but mostly incorrect
2 – general idea with some mistakes
3 – correct approach with minor error(s)
4 – completely correct

One member of our department wrote 10–12 pre-test questions for each of the calculus courses. These questions were then shared with other faculty members. From this initial set of problems, the department agreed on eight questions for each of the pre-tests in Calculus I and II and seven questions for Calculus III. These pre-test questions were administered in the Fall 2003 semester. For all pre-test quizzes and post-test questions see the SAUM website.)

To illustrate the pre/post-test system, let us begin with Calculus I. Pre-test questions were written to test specific algebraic and trigonometric skills that would be needed to solve standard calculus problems, which were then administered on the post-test. As examples, consider the following questions given on the Calculus I pre-test:

- Find the equation of the line through the point (5, –7) and having slope $m = 11/3$.
- Rationalize the denominator: $x-6/\left(\sqrt{x+3}-3\right)$.
- Solve the equation for all x in $[0, 2\pi]$: $1 - 2\cos x = 0$.

The corresponding post-test questions are as follows:

- Find an equation of the line tangent to $f(x) = x^2 + 3x - 13$ at $x = 2$.
- Find the limit, if it exists. $\lim_{x\to 2} x-6/\left(\sqrt{x+3}-3\right)$.
- Find the critical numbers of the function $h(x) = x - 2\sin x$ on the interval $[0, 2\pi]$.

As demonstrated with these three examples, the algebraic and trigonometric skills necessary to correctly answer the post-test calculus problems depends on the students' ability to correctly solve the pre-test problems. Similar to these examples, all pre/post-test problems for Calculus I directly connect calculus knowledge to prerequisite skills.

Our initial attempt at obtaining such a clear connection of pre and post-test questions in Calculus II and III was not as clean as in Calculus I. There are several reasons for this.

First, our attempt at using pre-tests questions in Calculus II and III for the dual purpose of testing prerequisite skills needed to solve post-test problems and of testing retention of prerequisite knowledge caused instructors to feel limited when writing final examinations. For example, the Calculus I final contained eight post-test questions, which were directly connected to that course's pre-test problems. Separate from these problems, the final examination also had four questions that appeared on the Calculus II pre-test. Thus twelve problems on the Calculus I final exam were some form of an assessment question which left the instructor very little freedom in writing the exam.

Second, only the pre-tests were written before the semester began with the understanding that instructors of the calculus courses would write post-test questions which used the same skills as the prerequisite problems. The instructor who wrote the review quizzes is the same instructor who taught Calculus I in Fall 2003. This instructor planned the types of problems that would be on the Calculus I final examination before writing the pre-tests and then geared Calculus I pre-test questions toward the algebraic and trigonometric skills needed to solve these final examination problems. When writing the pre-test questions for Calculus II and III, the instructor tried to anticipate final exam problems and wrote pre-test questions with these skills in mind. However, these final exam problems were not necessarily the same types of questions the faculty who taught these courses wanted to use. For example, a Calculus II pre-test question was to evaluate the integral

$$\int_2^3 \frac{dx}{(7-4x)^2}$$

The instructor who wrote the pre-test anticipated that the Calculus II final would include a question which asked the student to determine if the integral

$$\int_0^2 \frac{dx}{(7-4x)^2}$$

converged. Due to the length of the final examination, the Calculus II instructor opted not to use such a problem on the final.

The largest problem we had in mimicking pre-test questions on a post-test was in Calculus II. Core topics in Calculus II such as integration techniques, series convergence tests and polar function graphing are difficult to compare with a pre-test question. The only Calculus I prerequisite for these types of problems is basic integration skills. While there are several algebraic and trigonometric skills necessary to solve these problems, it is common for students to learn these skills at the same time they learn the calculus. Because of this, the department did not feel we could ade-

quately compare these types of standard Calculus II problems to a pre-test question.

The problems mentioned above have led us to make the following changes to the pre/post-test structure. The Calculus I assessment will remain largely as is, except that the post-test will mimic at least four, and not necessarily all eight, of the pre-test problems. These four questions will be up to the instructor's choosing and may rotate on a yearly basis. The post-test will also include four questions that appear on the Calculus II pre-test.

Calculus II will no longer be directly assessed with its own pre and post-test. Instead, the Calculus II pre-test will be used as a retention indicator for Calculus I. Three to four problems on the Calculus II post-test will be identical to problems on the Calculus III pre-test to test retention of Calculus II knowledge.

In addition to serving as a retention indicator for Calculus II, the Calculus III pre-test will contain four problems on prerequisite skills needed to correctly solve Calculus III post-test questions. As with Calculus I, these four problems may vary depending on the instructor.

What did we learn?

With only limited data and the problems we have faced in the first administering of the exams, it is difficult to comment on the effectiveness of our assessment tool. It is our hope that through time the pre and post-test system will give us a clearer understanding of whether or not our students are retaining essential skills from one course to the next and whether or not students are able to expand upon prerequisite information and combine it with newly acquired skills to solve a new set of problems.

Data regarding the means of student responses to pre and post-test questions can be found in Appendix A. Since the department feels that the best pre/post-test system occurred in Calculus I, comments on insights will refer to this course.

At this time it is appropriate to mention that the grading of post-test questions was done in two methods. One score was given for how the student performed overall on the problem, considering both the calculus and algebraic/trigonometric skills required. Another score was given on how the student performed based solely on the algebra and trigonometry and disregarding the calculus portion.

Of the students in Fall 2003 who took both the pre and post test ($n = 37$), the mean scores considering only algebraic and trigonometric skills improved on six of the eight problems and fell slightly on the other two. When comparing pre-test algebraic and trigonometric means to post-test means on calculus skills, there was improvement in only four of the eight questions.

For students who passed the course with a D or better ($n = 32$), the mean scores considering only algebraic and trigonometric skills improved on seven of the eight pre-test questions. The post-test means of the calculus skills still showed improvement on only four of the eight questions.

Only 17 students who took the Calculus I pre and post-test continued with Calculus II in the Spring 2004 semester. The Calculus II pre-test contained seven questions that tested similar concepts on the Calculus I final exam. Of these seven questions, the means of the calculus skills improved on four questions. The means on the remaining three questions dropped from 3.24, 3.00, and 3.71 to 2.71, 1.82, and 1.59 respectively. It was somewhat alarming to see such a drop in nearly half the questions. However, these were the last three questions on the review quiz. The students were allotted 20 minutes to complete the quiz and many students did not finish the last problems.

At this time, the department sees no need to make changes in the way the calculus sequence is taught. However, we do plan to continue to use the pre/post-test structure to track student learning and retention. If we notice problematic trends, we will address those problems at that time.

Along with the changes already mentioned in the previous section, we intend to more closely involve all calculus instructors in the selection of both pre and post-test questions. Also, we will allow ourselves flexibility in that post-test questions need not reduce to the identical problems on the pre-test. It is enough to have problems similar in nature that test the same skills. Finally, a logistical change is that the post-test questions will be the beginning questions on the final exams and given in the same order as the pre-test question they are associated with. This will aid in the grading and recording of scores.

Appendix A. Pre- and Post- Assessment Results for Calculus

Calculus I

The table below gives the results of students who took both the Calculus I pre- and post-test in Fall 2003 (n = 37).

Problem	Pre-Test Mean	Post-Test Mean Alg/Trig skills	Post-Test Mean Calculus skills
Line/tangent	3.65	3.32	2.89
Inequality/increasing	2.76	3.51	3.43
Equation/extrema (rational exponents)	2.73	3.32	2.49
Rationalize/limit	2.38	3.49	3.46
Complex fraction/limit	1.86	2.59	2.49
Simplify/integrate (divide)	1.92	3.11	2.49
Trig Equation/critical numbers	2.54	2.84	2.16
Ladder/ Rate of Change	3.76	3.62	3.11

The following table give the results of students who took both the Calculus I pre- and post-test in Fall 2003 and passed Calculus I in Fall 2003 with a grade of D or better (n = 32).

Problem	Pre-Test Mean	Post-Test Mean Alg/Trig skills	Post-Test Mean Calculus skills
Line/tangent	3.69	3.50	3.09
Inequality/increasing	2.91	3.78	3.69
Equation/extrema (rational exponents)	2.84	3.50	2.78
Rationalize/limit	2.22	3.91	3.88
Complex fraction/limit	2.06	2.84	2.72
Simplify/integrate (divide)	2.13	3.41	2.75
Trig Equation/critical numbers	2.56	2.94	2.44
Ladder/ Rate of Change	3.72	3.84	3.25

Calculus II

The table below gives the results of students who took both the Calculus II pre- and post-test in Fall 2003 (n = 12). All of these students passed the course.

Problem	Pre-Test Mean	Post-Test Mean Alg/Trig skills	Post-Test Mean Calculus skills
Product/chain rule	3.67	3.58	3.67
Nested chain rules	3	—	—
Definite integral/u-sub	2.33	—	—
Critical numbers (Trig)	2.17	1.42	2.5
Intersection points	4.00	4.00	4.00
Integrate after dividing	1.58	—	—
Inverse trig integration	1.92	3.17	2.67
Ladder/ Rate of Change	1.00	—	—

Calculus III

The table below gives the results of students who took both the Calculus III pre- and post-test in Fall 2003 (n = 12). All of these students passed the course.

Problem	Pre-Test Mean	Post-Test Mean Calc II skills	Post-Test Mean Calc III skills
Integration by parts	3.75	3.58	3.08
u-sub integration	3.42	—	—
Partial Fractions	2.25	—	—
Area between graphs	3.08	3.67	2.67
L'Hopitals	3.17	—	—
Equation of a plane	2.25	3.17	3.08
Parametric Equations	2.50	—	—

Combined Results (Fall 2003 & Spring 2004)

The following table gives mean scores on the Calculus I post-test and corresponding questions on the Calculus II pre-test for those who took Calculus I in Fall 2003 and are taking Calculus II in Spring 2004 (n=17).

Problem on Calculus II pre-test	Calc I post-test mean	Calc II pre-test mean
Product/chain rule	2.82	3.82
Nested chain rules	3.12	3.65
Definite integral/u-sub	3.29	3.11
Critical numbers (Trig)	2.41	3.18
Intersection points		3.76
Integrate after dividing	3.24	2.71
Inverse trig integration	3.00	1.82
Ladder/ Rate of Change	3.71	1.59

The following table gives mean scores on the Calculus II post-test and corresponding questions on the Calculus III pre-test for those who took Calculus II in Fall 2003 and are taking Calculus III in Spring 2004 (n = 6). (*Note*: There was an error made in Spring 2004 and the wrong version of the Calculus III pre-test was given. Because of this, the questions regarding L'Hopital's rule and graphing a curve from its parametric equations were not given on the pre-test.)

Problem on Calculus III pre-test	Calc II post-test mean	Calc III pre-test mean
Integration by parts	3.00	2.83
u-sub integration	2.83	0.83
Partial Fractions	2.83	2.50
Area between graphs	4.00	3.00
L'Hopital's Rule	2.83	
Equation of a plane	2.50	1.67
Parametric Equations	3.67	