

Assessment in a Middle School Mathematics Teacher Preparation Program

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Abstract. At a small private comprehensive university in New Jersey, the mathematics department is developing a program for middle-school teachers who do not have mathematics certification, and for adults changing careers to become mathematics teachers. The purpose is to give these teachers a deeper understanding of the mathematics they will teach. The assessment plan is being developed along with the program itself, in line with NCATE standards as well as New Jersey requirements for mathematics certification.

Background and goals

Middle school is often the weakest link in a student's mathematical education. When (as currently) there is a shortage of teachers with certification in mathematics, those with certification usually end up in the high schools. Middle schools often resort to moving an elementary school teacher up to middle school, or have a teacher certified in another subject begin teaching mathematics, often with no more than one college mathematics course as background. Monmouth has been working, for the last several years, on developing a program for these middle-school mathematics teachers. The audience we originally intended the program for was certified teachers who do not have certification in mathematics but who find themselves, at some stage in their career, teaching mathematics at the middle school level. However, our program can also accommodate people entering teaching as a second career (alternate route), and with small modifications, standard undergraduates specifically interested in middle school teaching.

The aim of this program, which will finally consist of six mathematics courses (Foundations of Number Systems, Geometry, Discrete Mathematics and Problem Solving, Probability and Statistics, Foundations of Algebra, and History of Mathematics) is to give teachers a deeper understanding of the middle-school mathematics through connections with mathematics normally taught at the undergraduate level. (The latter is partly because the state requires courses that count toward certification to be courses that are part of a major in the subject.) We chose the courses because together they cover, at a deeper level, the content of middle school mathematics courses. The Number Systems course gives teachers a deeper understanding of the natural numbers, integers, rational numbers, and to a lesser extent the real and complex numbers, which they need to teach middle school students effectively about decimals, percents, proportion, etc. Many middle school students take prealgebra in middle school, and some take first year algebra as well; the Foundations of Algebra course (which looks primarily at polynomial rings as the context in which middle school algebra takes place) makes the teachers better prepared to teach these courses. Current middle school texts include a substantial amount of geometry, a bit of probability, a fair amount of descriptive statistics, and a lot of problem-solving activities. The history of mathematics course gives teachers a sense of how all of this was developed and how the different subjects interrelate.

We try to keep the courses as independent of each other as possible, to allow teachers to enter the program any semester. On the other hand, we need to ensure that, by completing the six-course sequence, the teachers will be well-prepared, both

in content and skills, to teach middle school mathematics. Since we accept students in the courses as long as they completed whatever mathematics they were required to for their undergraduate degrees (which may be as little as one college algebra course), for some of the students in the program this sequence is quite a stretch. However, at least for the students we have had so far, many are quite mathematically talented, and all are willing to work considerably harder than many of our undergraduates. As of Spring 2004 we have only taught the Number Systems and Discrete Mathematics courses (both of which I developed and taught), but the Geometry and Probability and Statistics courses, which my colleagues Lynn Bodner and David Sze (respectively) are developing, will be offered during the 2004–2005 academic year. The final two courses will be offered the following year.

These courses are strictly content courses — students get their methods courses from our Education School — but we are trying, in the courses, to *model* both appropriate teaching methods and a range of assessment methods, both formative and summative. Although this program is aimed at students who already have bachelors degrees, it could easily be adapted to form a concentration at the undergraduate level.

Figure 1 provides a detailed statement of the general program goals. These are then detailed further in each individual course's Course Objectives and Expected Learner Outcomes. (The Course Objectives say what experiences the teacher will provide, the Expected Learner Outcomes, what the student should be able to do as a result.) One example of these (for Discrete Mathematics and Problem Solving) is given in detail in Appendix A.

Developing the assessment program

Because I am leading the development of this program and have considerable experience with assessment, we have been developing the assessment plan along with the program. To prepare for developing the program, I participated in a PMET (Preparing Mathematicians to Educate Teachers) workshop in June, 2002 and 2003. As we develop the courses, we examine carefully several series of middle school texts developed in the last ten years with National Science Foundation funding, to see what topics are covered. (These are *Connected Mathematics*, Prentice Hall; *Mathematics in Context*, Holt, Reinhart & Winston; *MathScape*, Glencoe/McGraw-Hill; and *MATHThematics*, McDougal Littell; information about all four can be found at the Show-Me Center's website.¹)

One difficulty in developing this program is the lack of appropriate textbooks for this audience. Most texts directed

- Help middle school teachers develop a deeper understanding of the mathematics they teach.
- Give middle school teachers an understanding of the relationship between the mathematics taught in middle school and undergraduate mathematics.
- Give middle school teachers experience with a range of pedagogical styles and assessment methods.
- Allow middle school teachers to reflect on their own experience as learners of mathematics.
- Help middle school teachers learn to develop lessons which increase their students' critical thinking skills.
- Introduce middle school teachers to appropriate technology available for middle school mathematics students.

Figure 1. Program Goals

at elementary school teachers are less mathematically sophisticated than we want for our students and cover topics in less depth. Texts directed at mathematics majors going through the usual four-year program assume background our students do not yet have.

Our university is planning to apply for NCATE (National Council for Accreditation of Teacher Education) accreditation within a year or two. Thus, our assessment plan must enable us to satisfy NCATE's requirements. NCATE is in the process of adopting new standards, which can be found in the document, *NCATE/NCTM Program Standards (2003)* under Mathematics Education². A description of how these are organized, and of the parts of the standards relevant to our Discrete Mathematics and Problem Solving course, can be found in Appendix B. To develop the program, we have to determine, for each indicator, in which course (or, for the process standards, which courses) students will gain the knowledge required, and how they will demonstrate that they have this knowledge.

Details of the assessment program

Since we are still in the process of developing the program, the primary assessment so far has been in individual courses. We have, of course, given weekly homework assignments and both hour-long tests and final examinations. However, the wide range of both mathematical experience and current mathematical ability in the class, and our desire that all the students (assuming they are doing the work) benefit from the assessment activities, led us to keep the emphasis on these traditional methods relatively small. We use the examinations to test that students have learned the essential mathematical skills and content that we feel every student completing the course must have. For the discrete

¹ www.showmecenter.missouri.edu/

² www.ncate.org/standard/programstds.htm

mathematics course, this includes being able to use truth tables to determine whether statements are tautologies, use Venn diagrams correctly, formalize an argument in symbolic logic, correctly state definitions and a few theorems, do a proof by induction, prove simple theorems about sets, solve fairly straightforward counting problems, determine whether a graph has assorted properties we had discussed (Euler cycles, planarity, etc.), and use Euler's formula.

However, because our goals are considerably broader than simply teaching certain mathematical topics—in particular, in each course we want to make progress on the NCATE process standards as well—we use quite a range of other assessment tools.

Summative assessment activities:

Portfolio of activities for a middle school class. We start each class (after we have gone over homework from the previous class; class meets once a week for 2 1/2 hours) with a problem taken from a middle-school text to introduce the topic of the day. We then cover the day's topic by a combination of interactive lecture and student activities. In principle, students spend the last fifteen minutes of class working in pairs, developing an activity for a class (middle school unless they are definitely planning to teach at the high school level), based on the topic of the lesson. (In practice, in many class meetings time runs out and this activity becomes an additional homework assignment.) The activity may be something quite brief, which might take their students only five or ten minutes, and does not have to be in finished form. It may be a description of what they would do with the class, or it may be a handout or worksheet, etc. We collect and comment on these portfolios at midterm and at the end of the semester. They are assessed holistically (and commented on extensively), based on how appropriate they are for the grade level they'd be used with, how well and correctly the mathematical language and concepts are used, and how well the activities would help students develop critical thinking skills.

Computer labs. In each course so far, we have done two computer labs. Our criteria for good software are that it be free or nearly so, that it work with current computers, and that it have activities that both the teachers and their students could benefit from. (One exception may be Geometer's Sketchpad, which is reasonable for schools to buy.) The labs for our first two courses can be found on my web pages for the courses. (See links to MA 500-level courses on my web page;³ the labs are linked to their day on the course syllabus.)

³ bluehawk.monmouth.edu/~bgold/

The first lab in the Discrete Mathematics course uses several pieces of software developed at the University of Arizona quite a few years ago, prior to Windows and even prior to computer mice being common. So the programs are rather awkward to a modern user.⁴ They are free, of course, and they are excellent in their conception and keep our students engaged. I have not tried them with middle school students, but I think they would work well at that level as well. We also used other software found in the software section of the Math Archives under Discrete Mathematics.⁵ For the Number Systems course, we used a number of NCTM's e-Illuminations,⁶ as well as Excel and Maple.

Curriculum project. A few weeks prior to the end of the semester, students hand in a curriculum project. This must be a 3–5 day unit for a middle- or high-school class, based on some of the ideas we have studied during the semester. It must include detailed learning goals for the activity, an overview of what they would do with the class, detailed lesson plans for each day, worksheets, handouts, overheads and/or computer labs, an assessment plan for the activity, and a description of how the activity is related to what we have done in class. They are allowed to include some of the activities from their portfolios as part of their projects. The grading rubric is handed out with the project assignment.

Formative assessment activities

Students are expected to write each week in reflective journals about their struggles with learning mathematics. Often as much as half of each class is spent on discussion of their difficulties with the concepts, how it relates to what they will be teaching, etc. We spend some time in class discussing school-level problems and the mathematical difficulties involved in teaching the material. For example, in the Number Systems course, when we were discussing the rational numbers, I took eight word problems from the text we use in our undergraduate course for future elementary school teachers, had the students make up a simpler problem and a more difficult problem of the same sort, and asked them to decide what properties of the rational numbers a student would need to understand in order to solve the problem.

Findings and success factors

The students in the two courses I have taught so far have probably been much stronger than those we will generally

⁴ www.math.arizona.edu/software/azmath.html

⁵ archives.math.utk.edu/

⁶ illuminations.nctm.org/index.asp

have once the program is actually attracting in-service middle-school teachers. In the Number Systems course, of the six students, two were sufficiently mathematically talented that I would be happy to recommend them for graduate work in mathematics, and two others were reasonably talented and worked harder than any undergraduate student I have ever had. If they could not solve a problem in a few hours, they would come back to it over and over again until they solved it! The informal peer pressure from these four—their level of discourse in class, the activities they shared—influenced the remaining two to work well above the level they would have in a less enthusiastic class. All felt that there was too much work, but they made enormous progress. Their reflective journals were astonishing for their depth of insight into their learning processes and the detail of what they tried. For example, from one: “The Euclidean algorithm is still fascinating to me as a way to determine the greatest common divisor. At first I did not realize that there could be more than one value of x and y for the Theorem: $by - ax = \gcd(a, b)$. (I had it confused with the fact that $b = aq + r$ is unique. It wasn’t until I discussed it with Sandy [another student in the class], that I realized that this must not be the case.) After discussing with Dr. Gold, I started to look at Lemma 1.1.10 and reached understanding by working it through. What Dr. Gold told me worked for my homework problems, but I wanted to understand why we must choose k such that ka will be $> y$ and kb will be $> x$.” (We were working with the natural numbers at that point. She then goes on to explain what she found.) In this course, students asked that I make the course available on our course management software (WebCT), because they wanted to be able to interact with the other members of class outside of class hours, and they had found this system efficient for this purpose in their education courses.

Some of my experiences to date may not be typical of what we will find when we teach the course to actual in-service teachers. The quality of the portfolio activities and the course projects varied considerably. In each class I had one student who was currently a teacher in a private school; most of the best activities came from these two students. However, most of the students in the classes were fairly early in their work on their MAT and had little teaching experience. I was surprised to find that several of the students who were themselves most creative mathematically tended to make up rather routine worksheets for use with classes, or to be overly rigid in their lesson plans. This suggests that these activities are very appropriate for the students we intended the courses for, but less appropriate for the actual audience we currently have.

Both classes enjoyed the computer labs, and the activities I had found on the internet stimulated them to look for more. The Number Systems class seemed to find the Maple project less worthwhile. The logic games from the University of Arizona website were particularly intriguing for our students, as were the NCTM activities which were more game-like in nature (the Product Game and Paper Pool). In the Discrete Mathematics course, I should have broken the second lab into two parts, since it was too long, but I had not expected to find so many appropriate activities for this material.

The Discrete Mathematics and Problem Solving course had no students with the mathematical sparkle of the Number Theory class—it was taken by students with a much weaker background—but again there were several students in the class who worked extremely hard and the class as a whole made excellent progress. In both courses, students learned significantly more than most of our undergraduates do in the corresponding courses. I am not sure whether this was due primarily to their maturity, or to their commitment to their future careers, but the classes were a joy to teach. Possibly my choice, in both cases, of textbooks that were a bit too hard for the class (because they were the only texts I could find that covered most of the subjects I wanted to include) was also a factor in the level of effort they put forth. Examinations were the assessment item they were most anxious about, but all showed, on examinations, a good understanding of correct mathematical definitions and an ability to solve problems of the kind we had practiced, although most had significant trouble developing proofs in a timed environment.

If the classes were substantially larger (say, over 15 students), it would be important to develop rubrics to enable more rapid evaluation of the portfolio items and journals. We would not be able to read every contribution of each student, nor would we grade every homework problem submitted. The curriculum projects could be done in groups of two or three. However, the range of assessment items shouldn’t prove overwhelming in classes of up to twenty-five or thirty students, as only the homework is collected weekly.

Use of the findings

The amount of time it has taken the very strong students in the Number Systems course to do the problems has made me realize that the course, as it stands, is too ambitious. The topics are about right, but the level of detail is too high for students who have little mathematical background. We will need to find an easier text or write our own materials. Fortunately, there are a few programs that are developing, with NSF support, materials for some of these courses. They were not yet ready when we first offered the courses, but

first drafts are now available. The students' relative lack of enthusiasm for the Maple computer lab I did in the Number Systems course, together with the likely unavailability of such software at the middle school level, led me to develop labs in the Discrete Mathematics course that use only software freely available on the web. We will replace Maple with graphing calculators in the future.

We also need to revise the courses, and somewhat the assessment plan, to reflect the likelihood that, at least unless we get funding, most of the students will not already be practicing teachers. We will therefore decrease the emphasis on projects for their students, as they have no active experience with what middle school students can be expected to do. On the other hand, we need to increase the attention paid to pedagogical issues such as typical student errors and confusions, how to recognize them, and how to respond to them.

Next steps and recommendations

One part of the assessment plan still needs to be developed, namely, to examine how the students in the program change

as teachers as a result of this program. So far, in each of the courses we have taught, only one of the six students in the class has been an actual classroom teacher, although some others have been working part-time as substitutes. Once the program is in full operation, however, we hope that at least half the students will be current teachers. For these students, we will collect information on their classes at the beginning of the program and again once they finish the program. The materials we will gather will be two assignments or projects they feel proud of using with their students, as well as a videotape of their class. The assignments will be examined for evidence of appropriateness of the problems for the level of student, development of students' critical thinking skills, and use of mathematical understanding in the choice of problems. The videotapes will be examined to see how their increased understanding of the mathematics improves their ability to respond to their students questions and ideas, and whether they use more interactive teaching styles, encourage students to be active learners, etc.

Appendix A. Discrete Mathematics Courses Objectives and Learner Outcomes⁷

(*Note:* This course will be designated MA 520 once the program is approved; currently we offer it as MA 598, our graduate Special Topics number.)

Course Objectives: This course will

- Give students a deeper understanding of topics in discrete mathematics taught at the K–12 level;
- Introduce students to heuristics for problem-solving;
- Give examples of how to use these concepts to develop classroom materials to enhance student learning;
- Introduce students to mathematical software available at the school level for investigating discrete mathematics;
- Involve students in problem solving activities via exploration and experimentation to allow students to construct (and reconstruct) mathematics understanding and knowledge;
- Encourage visual reasoning as well as symbolic deductive modes of thought (by incorporating models, concrete materials, diagrams and sketches);
- Introduce multiple strategies of approaching problems by discussing and listening to how others think about a concept, problem, or idea;
- Involve students in small group work and cooperative learning;
- Help students become aware of their own mathematical thought processes (and feelings about mathematics) and those of others;
- Introduce students to multiple methods of assessment in mathematics.

Expected Learner Outcomes: Students will develop the ability to:

- Approach problems from multiple perspectives and help their students become better problem solvers;
- Use mathematical language to correctly state mathematical definitions and theorems;
- Begin developing mathematical proofs;
- Use assorted counting techniques to solve problems;
- Work with the assorted concepts from graph theory and apply them to a range of problems, including map coloring, networks, traversing routes.

⁷ mathserv.monmouth.edu/coursenotes/gold/MA520hom.htm

Appendix B. An Introduction to NCATE Accreditation Requirements

NCATE (National Council for Accreditation of Teacher Education) is in the process of adopting new standards, which can be found in the document, *NCATE/NCTM Program Standards (2003)* under Mathematics Education.⁸ These standards are very detailed, and describe both content knowledge and skills that teacher candidates must demonstrate. Some of the skills will be learned in their education courses, but “process standards” 1–6 (knowledge of “mathematical problem solving,” “reasoning and proof,” “mathematical communication,” “mathematical connections,” “mathematical representation,” and “technology”) as well as the “content standards” (there are also “pedagogy” and “field-based experiences” standards) are primarily learned in the mathematics courses. For each standard, several indicators of what it would mean to meet that standard are listed. The process standard indicators are the same for all levels (Elementary Mathematics Specialists, Middle Grades, and Secondary Level), but the content standard indicators become more elaborate the higher the grade level.

As an example of Process Standard indicators, for “Knowledge of Mathematical Problem Solving” the indicators are

- 1.1 Apply and adapt a variety of appropriate strategies to solve problems.
- 1.2 Solve problems that arise in mathematics and those involving mathematics in other contexts.
- 1.3 Build new mathematical knowledge through problem solving.
- 1.4 Monitor and reflect on the process of mathematical problem solving.

For the middle level, the Content Standard “Knowledge of Discrete Mathematics: Candidates apply the fundamental ideas of discrete mathematics in the formulation and solution of problems,” has as indicators

- 13.1 Demonstrate a conceptual understanding of the fundamental ideas of discrete mathematics such as finite graphs, trees and combinatorics.
- 13.2 Use technological tools to apply the fundamental concepts of discrete mathematics.
- 13.3 Demonstrate knowledge of the historical development of discrete mathematics including contributions from diverse cultures.

NCATE’s matrix for each of these indicators asks “How do our candidates acquire and demonstrate the knowledge addressed to this standard?” “What evidence supports candidates’ knowledge acquisition and performance?” and “What are our findings?”

⁸ www.ncate.org/standard/programstds.htm