

What is algebra and why do students find it so hard?

Many young children find arithmetic hard to learn, but most succeed, to varying degrees, after a lot of practice. What makes this possible is that the basic building blocks of arithmetic, numbers, are tied closely to concrete things in the world we live in—we count things, measure things, buy things, make things, use the telephone, go to the bank, check the baseball scores, etc.

Algebraic thinking is not just arithmetic with letters standing for numbers. It is a different kind of thinking.

Algebra is thinking logically about numbers rather than computing with numbers. In algebra you are a second step of abstraction removed from the everyday world: those x 's and y 's usually denote numbers in general, not particular numbers. In algebra you use analytic, qualitative reasoning about numbers, whereas in arithmetic you use numerical, quantitative reasoning with numbers.

For example, you need to use algebra if you want to write a macro to calculate the cells in a spreadsheet like Microsoft *Excel*. It doesn't matter whether the spreadsheet is for calculating scores in a sporting competition, keeping track of your finances, running a

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Comment: "Algebra" in the title becomes "algebraic thinking" in the first sentence without justification. Then algebraic thinking is contrasted with "arithmetic," which has not been presented as a kind of thinking at all, so the contrast is awkward. [1]

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business, or figuring out the best way to equip your character in *World of Warcraft*, you

need to think algebraically, rather than arithmetically, to set it up to do what you want—that means thinking about or across numbers, rather than in terms of numbers.

When students start to learn algebra, they inevitably try to solve problems by thinking arithmetically. That's a natural thing to do, given all the effort they have put into mastering arithmetic. And at first, when the algebra problems they meet are particularly simple (that's the teacher's classification), this approach works. In fact, the stronger a student is at arithmetic, the further they can progress in algebra using arithmetical

thinking. (Many students can solve the quadratic equation $x^2 = 2x + 15$ using basic arithmetic, using no algebra at all.) Paradoxically, or so it may seem, however, those better students may find it harder to learn algebra. Because to do algebra, for all but the most basic examples, you have to stop thinking arithmetically and learn to think algebraically.

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Comment: Full transition from "algebra" and "arithmetic" to "algebraic thinking" and "arithmetical thinking."

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Comment: Do we want to say this? The implication is that greater control over arithmetic thinking makes algebraic thinking increasingly obsolete.

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Comment: And, see, now we have closure between the thinking aspect and the whatisit aspect.