Creating Mathlets with Open Source Tools

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Abstract

Today's Internet provides a large number of freely available interactive materials for mathematics learning and teaching. Most of these *mathlets* are ready to use but not ready to be modified or extended. In this article we present a collaborative environment of open source tools around the dynamic mathematics software GeoGebra that gives educators the freedom to create new and modify existing materials in an online community. We describe how to do this based on design principles for multimedia learning and experiences from our work with teachers.

See http://www.joma.org for an interactive version of this article.

Technologies Used in This Article

This article uses Java (1.4.2 or later) and JavaScript for several interactive figures created with GeoGebra. Please install Java from www.java.comand activate JavaScript in your browser if necessary.

Contents

Introduction

Today's Internet provides a large number of freely available interactive materials and environments for mathematics learning and teaching. Many of them are small self-contained learning objects, so called *mathlets*, that focus on a specific mathematical topic or problem and are ready-to-use for demonstrations by teachers or self-directed learning by students. No matter whether they are scattered on personal web pages or organized in libraries like Math Forum, Library of Virtual Manipulatives or the Digital Classroom Resources, most of these mathlets have one characteristic in common: they cannot be changed at all or only with a lot of effort. Let us now distinguish three types of mathematics educators using mathlets:

1. *Consumers* use existing mathlets as they are.
2. *Customizers* adapt mathlets to their needs; e.g. they modify questions or tasks that explain what should be done with the mathlet.
3. *Authors* create their own mathlets from scratch.

Most web pages providing mathlets focus on the first and last group whereby only a small number of authors creates materials that can be used by others. Those educators who want to change existing materials are often neglected. First of all, there is a technological barrier as many mathlets cannot be changed at all or only with some special commercial software. Secondly, changing a mathlet may constitute a violation of its author's copyright. However, there are good reasons why it seems desirable to be able to change existing interactive teaching materials. On the one hand, the quality of mathlets is quite diverse as only a small portion of them is reviewed or edited before getting published on the Internet (see for example MERLOT). By being able to change such materials, educators can work together to improve their quality. On the other hand, someone may want to add missing features or remove unneeded parts for their specific purposes. Basing this work on existing mathlets can save a lot of time and effort.

In this article we present an environment of open source tools around the dynamic mathematics software GeoGebra (Hohenwarter & Preiner 2007) where educators can join an online community for creating and modifying mathlets. By using cross-platform open source tools and collaborative software we overcome many of the technological and financial barriers of creating and editing mathlets. All materials in this environment are subject to a Creative Commons license that allows everyone to make customized works for noncommercial purposes.

Collaborative Environment and Discovery Learning

We are currently using the following environment of open source tools and collaborative software with mathematics teachers in the NSF math and science partnership project Standards Mapped Graduate Education and Mentoring of Florida Atlantic University and The Broward County School District. Our environment consists of the following parts that are used by the teachers in this project to create and edit interactive online materials.

- GeoGebra: an open source software for dynamic geometry, algebra and calculus
- NVU: an open source software for creating and editing web pages
- GeoGebraWiki: an open pool of materials for online collaboration of teachers (based on MediaWiki)
- GeoGebra User Forum: an online forum for users of GeoGebra (based on phpBB)

GeoGebra is an easy-to-use tool for learning and teaching mathematics that lets you export interactive web pages, so called *dynamic worksheets*. These mathlets can then be changed and extended using GeoGebra, a text editor or html editor (e.g. NVU). The GeoGebraWiki is a pool of materials that allows everyone to contribute their own creations or take an existing worksheet and produce a customized version. In the user forum educators can help each other with specific questions. By using collaborative software like a wiki and a forum, we want to support cooperation among teachers by giving them the opportunity to share their ideas and materials as well as discuss problems and solutions.

Our dynamic worksheets usually consist of an interactive applet together with a short description of the mathematical situation and several questions. Students should then work on these questions by using the applet for mathematical experiments.

Discovery learning is a type of learning where learners construct their own knowledge by experimenting with a domain, and inferring rules from the results of these experiments. The basic idea of this kind of learning is that because learners can design their own experiments in the domain and infer the rules of the domain themselves, they are actually constructing their knowledge. Because of these constructive activities, it is assumed they will understand the domain at a higher level than when the necessary information is just presented by a teacher or an expository learning environment. (van Joolingen 1999, p. 386)

Although this open approach of discovery learning is desirable from an educational point of view, we have to make sure that students don't get lost due to a lack of guidance. Therefore, we are using a *guided discovery learning* approach where we provide a series of specific questions or tasks to guide the students' experiments. In this way, they can make step by step discoveries towards a predetermined goal with a minimum risk of failure.

In the following sections we will explain how to create and modify dynamic worksheets by looking at a series of examples. We will also discuss design principles for multimedia learning and present several guidelines for the creation of effective materials based on our experiences with the teachers in our NSF project.

GeoGebra Mathlets

We are going to use the open source tools GeoGebra and NVU to create and modify mathlets. GeoGebra is a dynamic mathematics software that combines features of dynamic geometry with some capabilities of computer algebra systems to support mathematics learning from middle school up to college level (Hohenwarter & Preiner 2007). The software allows you to export any construction as an interactive web page in a very easy way. In the following section we will show how to create such *dynamic worksheets* with GeoGebra. After that we are going to use GeoGebra and the html-editor NVU to modify these worksheets. Both programs are freely available for all major operating systems (i.e. Windows, Mac OS X and Linux).

Other examples for cross-platform and open source mathematics software that allows exporting interactive web pages are C.a.R. and Geonext. An interesting freeware to create interactive web pages with tests and quizzes is Hot Potatoes.

Creating Dynamic Worksheets with GeoGebra

A GeoGebra mathlet or *dynamic worksheet* is an interactive web page (html file) that consists of a dynamic figure (interactive applet) with corresponding explanations, questions and tasks for students. The dynamic figure is constructed in GeoGebra and can easily be exported to an html file. Students can use this dynamic worksheet both on local computers or via the Internet to work on the given tasks by modifying the dynamic figure. In this way you can foster mathematical experiments as well as guided discovery learning for students from secondary schools up to college level.

Example: The worksheet Shape of a Slope Function allows students to find the shape of the slope function of a quadratic polynomial by interactively modifying a dynamic figure. It is part of a series of three dynamic worksheets that guides students towards discovering the slope function of a quadratic polynomial.

Creating the Dynamic Figure

We will now show you how to create the worksheet Shape of a Slope Function step by step. First, you need to construct the dynamic figure including the quadratic polynomial `f`, point `P` and the tangent line `t` as well as a special point `M` whose `y`-coordinate equals the slope of the tangent line `t`. To do this, you can use the interactive applet below that represents the user interface of GeoGebra but includes a customized tool bar to simplify

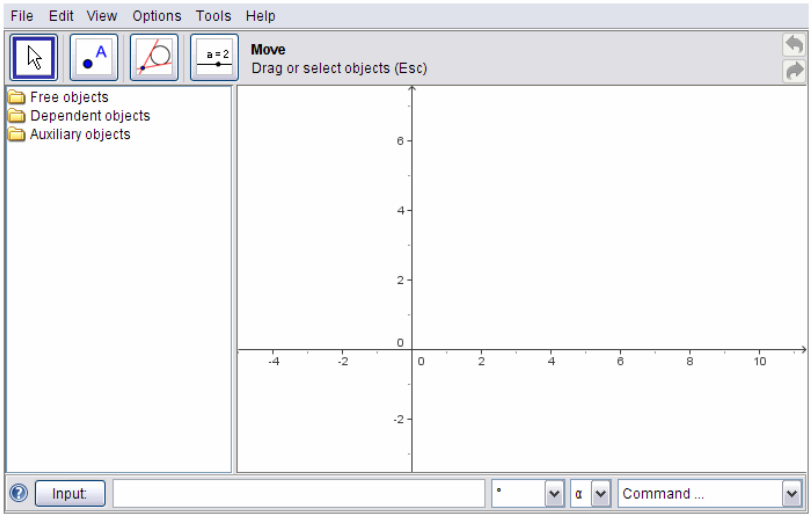
the construction. It enables you to do the construction and export it as a web page. Since this setting should only provide an environment to gain first experiences with the use of GeoGebra while reading this article, we recommend to install GeoGebra and to have a look at the GeoGebra Quickstart document for additional information.

General Hints for the Use of GeoGebra

- In order to activate a tool you need to click on the corresponding toolbar icon. *Tip*: Note the toolbar help next to the toolbar icons in order to find out how to use the selected tool.
- If you type an algebraic expression into the input field at the bottom of the GeoGebra window you need to hit the Enter-key in order to process this information.
- Right click (Mac OS: ctrl-click) an object in order to
  - Show / hide the object
  - Rename the object
  - Delete the object
  - Change properties of the object
- A slider is the graphical representation of a number. If you create a number using the input field you need to show the number in order to get the corresponding slider.

Step-by-step Instructions

1. Create the parameters `a = -0.5`, `b = 2`, and `c = 4` using the input field or the "Slider" tool.
2. Define function `f(x) = a x^2 + b x + c` using the input field.
3. Activate the tool "New point". Move the mouse over the graph of function `f` and click on it when it is highlighted to get a point on the graph. Rename this point to `P`. *Tip*: Check if point `P` really lies on the graph of function `f` by selecting the tool "Move" and dragging point `P` along the graph.

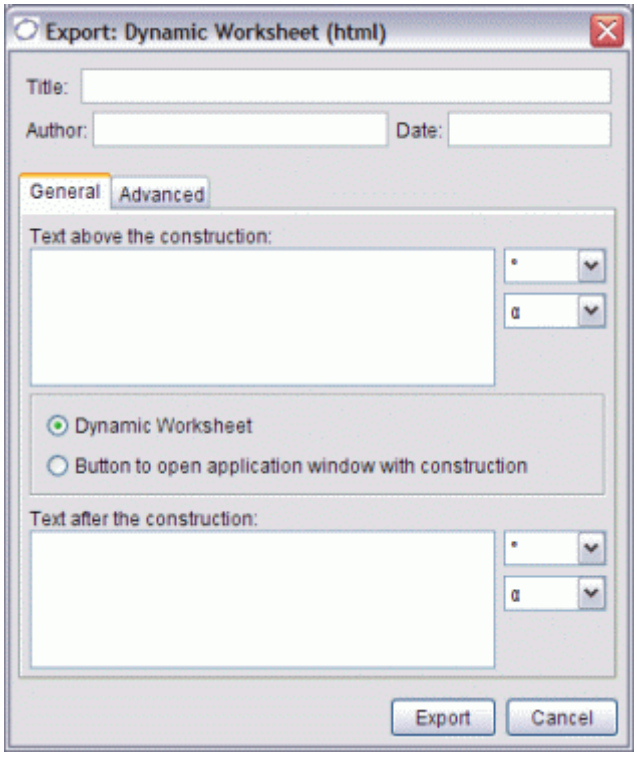


4. Select the tool "Tangents" and successively click on point `P` and the graph of function `f`. Rename the tangent to `t`.
5. Use the command `m = Slope[t]` in order to get the slope triangle of the tangent line.
6. Create point `M` by typing `M = (x(P), m)` into the input line. *Tip*: `x(P)` gives you the *x*-coordinate of point *P*.
7. Export your dynamic figure as a dynamic worksheet by selecting "File" - "Export" - "Dynamic Worksheet as Web page (html)..." in the menu bar.

Exporting the Dynamic Worksheet

After creating the dynamic figure you are now ready to export it as an html-page. When you export a construction from GeoGebra both the graphics window and the algebra window will be shown on the dynamic worksheet. However, you can hide the algebra window using the "View" menu if you don't need it. To save space for text on screen you should decrease the size of GeoGebra's application window by dragging its lower right corner before you proceed. If you are using the applet provided before, you don't need to worry about this.

To export your dynamic figure as a dynamic worksheet select "File" - "Export" - "Dynamic Worksheet as Web page (html)..." in the menu bar.



Tab "General"

- Fill in the text fields of the appearing export dialog (Title: "Shape of a Slope Function"; Author; Date).
- Add some text that should be displayed above the construction in the corresponding text field, for example: Below you can see the graph of a quadratic polynomial `f(x) = -0.5 x^2 + 2 x + 4`.
- In the last text field you can provide tasks for your students, for example
    1. Try to find out how the coordinates of point M are connected with point P and the slope m of tangent t by dragging point P with the mouse.
    2. Right click (Mac OS: ctrl-click) point M and select "Trace on". When you move point P now you get the graph of the slope function of f(x). What does the shape of the slope function look like?
    3. Click the little icon in the right upper corner to reset the construction. Change the values of the sliders a, b, and c and turn on the trace of point M again. What does the shape of the slope function look like when you move point P now?

Tab "Advanced"

You might want to check the applet option "Show icon to reset construction" in order to include a reset icon into your interactive applet.

Export

Click the "Export"-button and save your dynamic worksheet as "ShapeSlopeFunction.html".

Tip: We recommend to export your worksheet into a new folder as GeoGebra will create several files during the export. Do not use any space or special characters in your filename to prevent problems when using the dynamic worksheet.

Access

To access your dynamic worksheet open the html-file in a web browser.

Note: Internet Explorer may restrict interactive html pages opened from your local computer. In this case you need to click on a yellow bar at the top of your browser window and approve a security question to use the dynamic worksheet. However, this problem does not occur when you upload your dynamic worksheet to the Internet.

Deploying your Dynamic Worksheet

GeoGebra creates several files when you export a dynamic worksheet:

- The files with the extension ".jar" contain the interactive GeoGebra applet.
- The file with the extension ".html" represents the actual dynamic worksheet and can be opened with any web browser.
- The file with the extension "_worksheet.ggb" contains the dynamic figure which is displayed in the interactive GeoGebra applet.

To give your dynamic worksheet to your students you need to keep all of these files together in one folder (e.g. copy them to a CD, USB drive or web server). If you don't have web space, you can use the GeoGebra Upload Manager to publish your dynamic worksheets on the Internet. If you do so, we would also like to invite you to provide a link to your worksheet in the GeoGebraWiki. In order to use dynamic worksheets, your students need a web browser and Java 1.4.2 or later (free download from www.java.com) on their computers. Note, that they don't need to have GeoGebra installed to work with your dynamic worksheets.

Modifying Dynamic Worksheets

Dynamic worksheets created with GeoGebra can easily be modified at a later date. Recall that an html-file and a ggb-file were created when you exported your worksheet. The html file is the textual part of your worksheet, the ggb file is your dynamic figure.

Modifying the Dynamic Figure with GeoGebra

To edit the interactive figure of your dynamic worksheet you need to open the ggb file (e.g. "ShapeSlopeFunction_worksheet.ggb") in GeoGebra. For example, you may want to change certain properties of objects (e.g. color, line thickness,...) to enhance the appearance of an interactive applet or add new elements.

- Right click (Mac OS: ctrl-click) on one of the objects and select "Properties".
- In the dialog window that appears, modify the appearance of the selected object.
- In order to modify another object, select it in the object list of the properties dialog.
- Close the properties dialog when you are done.

After saving the ggb file, you should open your worksheet again in a web browser to see your updated version of the dynamic figure. Note that you should not change the name of your ggb file when saving; otherwise the dynamic figure won't be displayed in your html page any more.

Modifying the Text

In order to edit the text of a dynamic worksheet, you need to open its html file (e.g. "ShapeSlopeFunction.html") with an editor like for example NVU. There, you can modify the text of the dynamic worksheet, for example to set the color of certain expressions so that they match their representation in the dynamic figure. This increases the readability of the tasks and makes it easier for your students to find corresponding objects in the text and the applet. If you have a series of worksheets, you may also want to create hyperlinks to navigate between them. After saving the html file you should open the updated version with your web browser to check your changes.

Example Worksheets

The following example worksheets show how GeoGebra can be used to foster mathematical experiments and discovery learning. We will give you detailed information on how to create these worksheets using GeoGebra and a text or html editor (e.g. notepad, vi, NVU).

- Example worksheets and how to create them
  - Slope Function and Derivative
  - Tangents to a Circle
- You can download all files of these example worksheets in one zip package.

Slope Function and Derivative

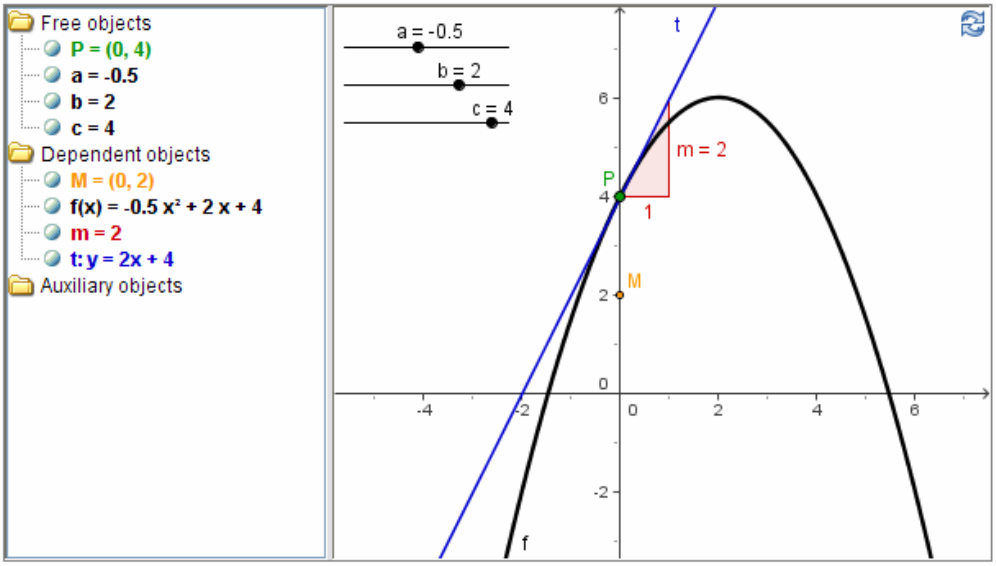
The following series of worksheets guides students towards findig the equation of the slope function of a quadratic polynomial on their own.

Dynamic Worksheets

1. Shape of a Slope Function: Guides students to discover the shape of the slope function of a quadratic polynomial by modifying the interactive applet.
2. Equation of a Slope Function: Encourages students to find the equation of a slope function and to check their answer using the interactive applet.
3. Calculating Slope Functions: Inspires students to come up with a rule to calculate the equation of a slope function using the coefficients and exponents of the initial quadratic polynomial.

Shape of a Slope Function

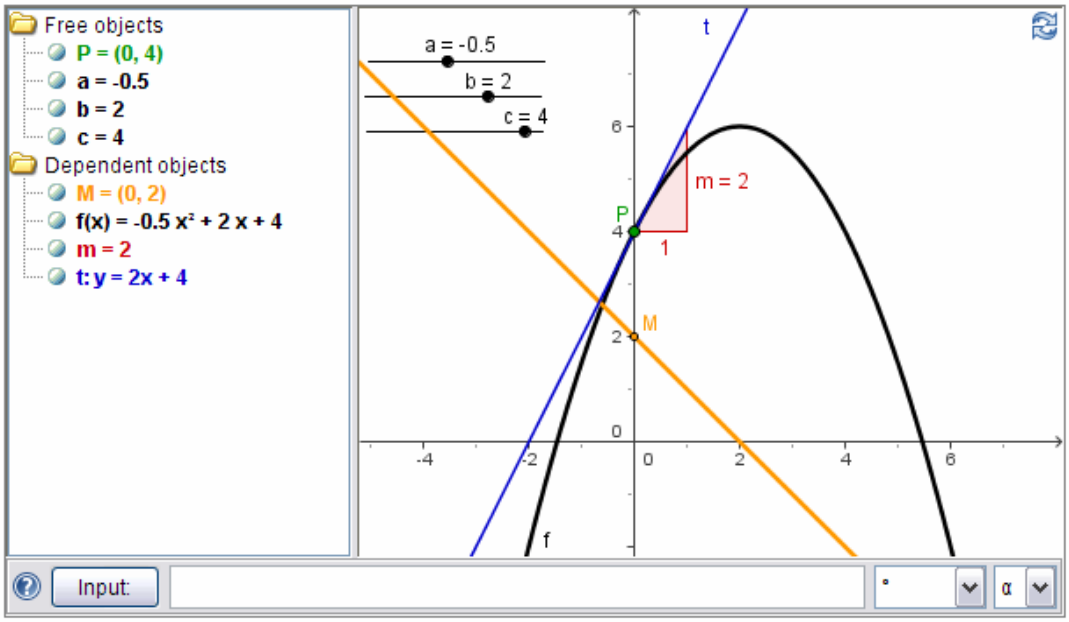
Below you can see the graph of $f(x) = a\,x^2 + b\,x + c$ and its *tangent t* through *point P*.



1. Try to find out how the coordinates of *point M* are connected with *point P* and the *slope m* of *tangent t* by dragging *point P* with the mouse.
2. Right click (Mac OS: ctrl-click) *point M* and select "Trace on". When you move *point P* now you get the graph of the **slope function of *f*(*x*)**. What does the shape of the slope function look like?
3. Click the little icon in the right upper corner to reset the construction. Change the values of the sliders *a*, *b*, and *c* and turn on the trace of *point M* again. What does the shape of the slope function look like when you move *point P* now?

Equation of a Slope Function

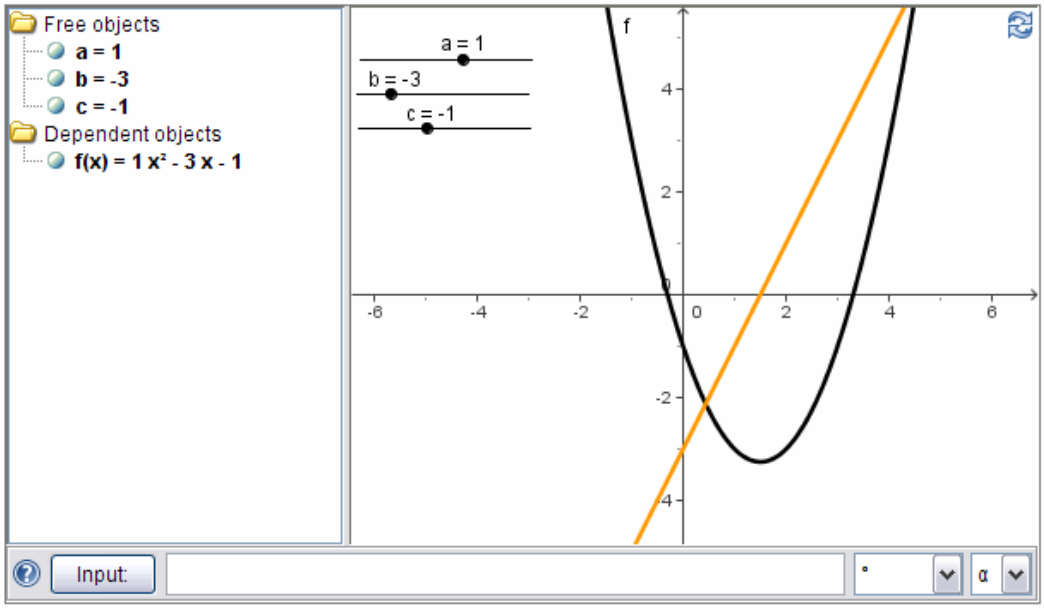
Below you can see the graphs of a quadratic polynomial $f(x) = a\,x^2 + b\,x + c$ and its *slope function*.



1. Find the equation of the *slope function*. Write down the quadratic function $f(x)$ as well as your slope function $g(x)$.
2. Check your answer by typing your slope function's equation $g(x) = ...$ into the input field. Hit the Enter-key. *Tip*: If your answer is not correct you can change it by double clicking on $g(x)$.
3. Click on the little ellipse-icon in the upper right corner to reset the construction. Move the sliders to match the following values: $a = -1$, $b = 4$, and $c = 2$. Repeat steps (1) and (2).

Calculating Slope Functions

Below you can see the graphs of a quadratic polynomial $f(x) = a\,x^2 + b\,x + c$ and its *slope function*.



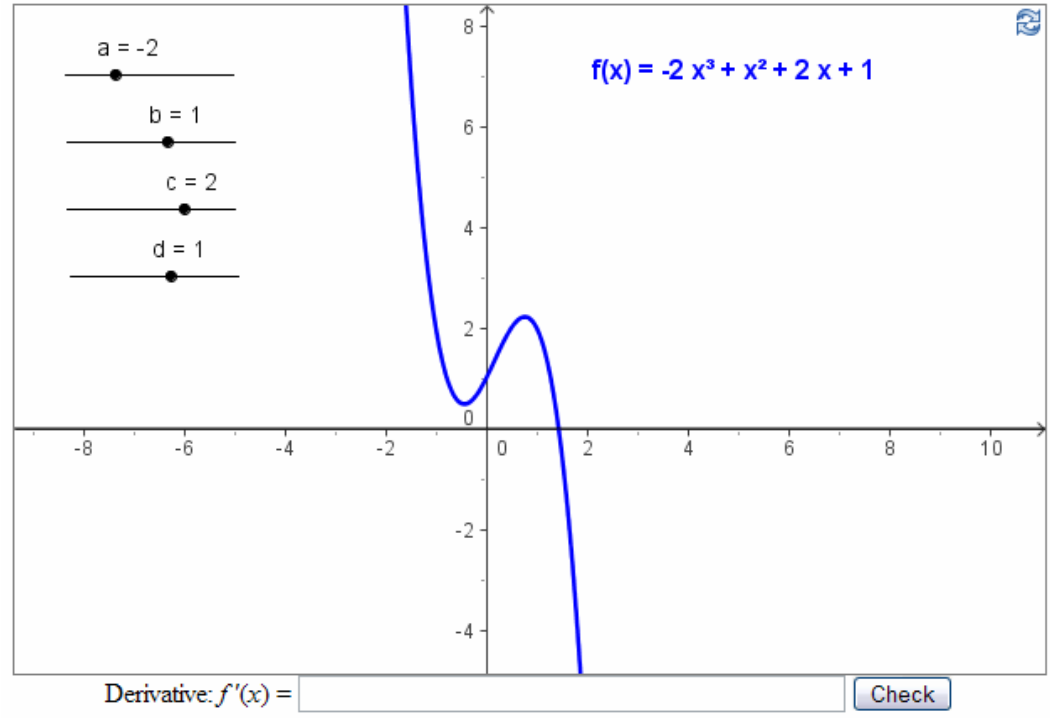
1. Find the equation of the *slope function* and check it using the input field. Write down both functions.
2. Compare the equations of the quadratic function f and the slope function g. Try to find out how the equation of the slope function can be calculated using the equation of $f(x)$. Can you come up with a rule?
3. Check your rule for at least three different functions by changing the sliders for a, b, and c. Write down your examples.

Interactive Exercise

These three worksheets provide a basis for deeper and more formal discussions about the concepts of *derivative* and *tangent to a function*. The additional interactive exercise Derivative of a Polynomial allows students to practice calculating the derivative of cubic polynomial functions.

Derivative of a Polynomial

Use the sliders to change the cubic function $f(x)$. Then type the equation of its derivative into the textfield below and check your answer by clicking the button.



Creating these Mathlets

This section explains how to create the three example worksheets about slope function and derivative as well as the interactive exercise mentioned before. You may want to download all files of the example worksheets in one zip package.

- Creating "Slope Function"
  - Worksheet 1: Shape of a Slope Function
  - Worksheet 2: Equation of a Slope Function
  - Worksheet 3: Calculating Slope Functions
- Creating "Derivative of a Polynomial" (interactive exercise)

Creating "Slope Function"

Worksheet 1: Shape of a Slope Function

Please find information on how to create the first dynamic worksheet Shape of a Slope Function in the section Creating Dynamic Worksheets with GeoGebra. Additionally, the interactive applet of this dynamic worksheet includes a reset button in the upper right corner. It is created by setting the GeoGebra Applet Option "Show icon to reset construction".

Worksheet 2: Equation of a Slope Function

Let's create the second dynamic worksheet called Equation of a Slope Function. Its interactive applet includes the graph of the slope function without showing its equation. Besides this, the dynamic figure is similar to the one in the previous worksheet which allows us to use the corresponding construction file as a basis for our modifications.

1. Open the file "ShapeSlopeFunction_worksheet.ggb" in GeoGebra.
2. Type `Derivative[f]` into the input field and hit the Enter-key.
3. Right click (Mac OS: ctrl-click) the equation of the derivative `f '` in the algebra window and check "Auxiliary object".
4. In the menu click on "View" and uncheck "Auxiliary objects". In this way, you can display the derivative `f '` of the quadratic function *f* without showing its equation in the algebra window.
5. *Tip*: You can change the properties of the derivative to match the color of point *M*.
6. Export the construction as a dynamic worksheet called "EquationSlopeFunction.html". Change the title of the worksheet and include corresponding tasks. The interactive applet of this dynamic worksheet should provide a reset button and an input field for the students. To do this, you need to set the coresponding applet options.

Worksheet 3: Calculating Slope Functions

The third dynamic worksheet was Calculating Slope Functions. Again, you can use the dynamic construction prepared for the previous worksheet as a basis for modifications.

1. Open the file "EquationSlopeFunction_worksheet.ggb" with GeoGebra.
2. Delete point *P* by right clicking (Mac OS: ctrl-click) it and selecting "Delete". This action will also remove tangent *t* as well as point *M* from the construction since these objects are dependent on point *P*.
3. Change the values of the sliders to provide a new exercise for your students.
4. Export the construction as a dynamic worksheet called "CalculatingSlopeFunctions_worksheet.ggb". Change the title of the worksheet and add new tasks for your students that correspond with the new dynamic figure you just created. Like with the previous worksheet, you need to set applet options to include the reset button and show the input field at the bottom of the interactive applet.

Tip: To enhance this series of dynamic worksheets you could create a title page with an index and links to all dynamic worksheets (e.g. with a text editor or NVU). Additionally, you could insert hyperlinks on every dynamic worksheet to make it easier for your students to navigate between them.

Creating "Derivative of a Polynomial"

In order to create the interactive exercise Derivative of a Polynomial you first need to create the following dynamic construction in GeoGebra. Although only the sliders *a*, *b*, *c*, and *d* as well as the polynomial function *f* are visible on the dynamic worksheet, you need to create some auxiliary objects for the exercise logic in JavaScript.

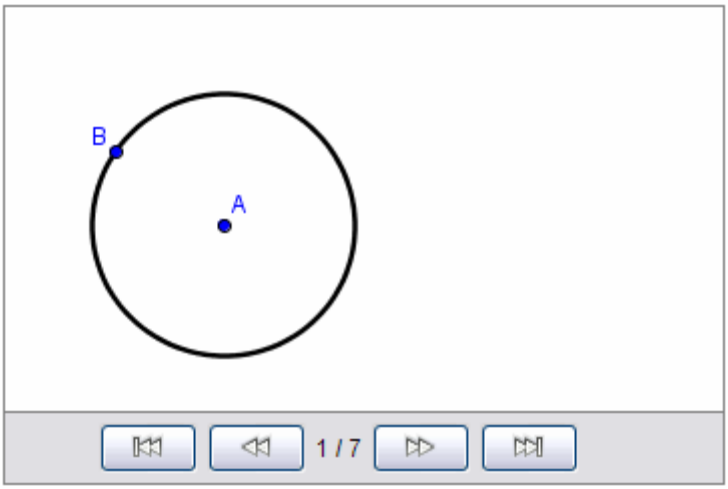
1. Open a new GeoGebra window and hide the algebra window by unchecking "Algebra Window" in the "View" menu.
2. Activate the tool "Slider". Create slider *a* by clicking on the drawing pad and clicking "Apply" in the appearing dialog window. Repeat this process to create sliders *b*, *c*, and *d*.
3. Type `f(x) = Polynomial[a x^3 + b x^2 + c x + d]` into the input field and hit the Enter-key.
   Tips: The command `Polynomial[]` simplifies an expression to get a polynomial function in standard form, i.e. here this removes the monomials with zero coefficients. You may change the properties of function *f* by right clicking the object (Mac OS: ctrl-click) and selecting "Properties", e.g. change the color and line thickness.
4. Type `h(x) = Polynomial[Derivative[f]]` into the input field an hit the Enter-key. Right click (Mac OS: ctrl-click) its graph and uncheck "Show object".
   Tip: You may change the color of function *h* in the properties dialog before hiding its graph.
5. Type `g(x) = 0` into the input field and hit the Enter-key.
6. Insert a dynamic text next to the graph of function *f*. Activate the tool "Insert text" and click on the drawing pad. Type `"f(x) = " + f` into the input field and click "Apply".
   Tip: This text consists of the following three parts: The quotation marks enclose the static part of the text. The plus sign connects the static part of the text with the dynamic part. The last letter *f* refers to the function *f(x)* and gives us its equation, which is the dynamic part of the text.
7. Type `diff(x) = Polynomial[h(x) - g(x)]` into the input field and hit the Enter-key.
   Tip: Function *diff* gives you the difference between function *h* and *g*.
8. Reduce the size of the GeoGebra window if necessary and export the construction as a dynamic worksheet.

Please find information on how to extend this dynamic worksheet into an interactive exercise Derivative of a Polynomial in the section GeoGebra and JavaScript.

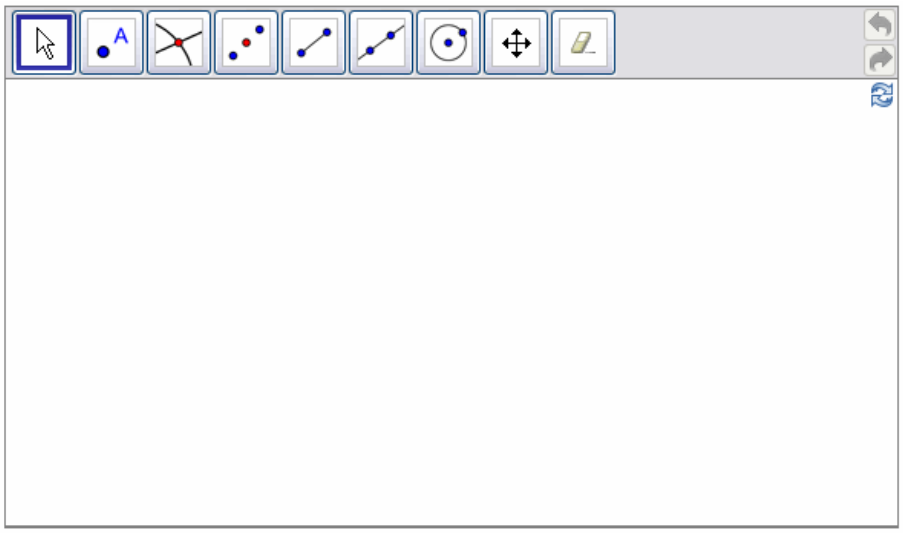
Tangents to a Circle

The dynamic worksheet Tangents to a Circle provides two applets that let students learn how to construct the tangents to a circle through a given point. The first applet allows them to go through the construction step-by-step using a navigation bar. The second applet lets them create this construction on their own. To simplify the construction process the second applet includes a customized toolbar that only shows those tools needed for this specific construction.

Use the arrow buttons below to go through the construction of tangents to a circle.



Try to do this construction on your own using the following tools.



Creating this Mathlet

First Applet

In order to create the first of the two interactive applets, you need to change the settings of the GeoGebra window. Therefore, open GeoGebra and hide the algebra window by unchecking "Algebra window" in the menu "View". Then, hide the coordinate axes by unchecking "Axes" in the menu "View".

Now, you are ready to prepare the construction of tangents through a point in respect to a circle that will guide your students through the construction steps.

1. Choose the tool "Circle with center through point" and click on the drawing pad twice to create the center *A* and a second point *B* that specifies the radius of the circle. *Tip*: Using the tool "Move" you are able to change the positions of both points later on.
2. Choose the tool "New point" and create a point *C* outside the circle by clicking on the drawing pad.
3. Choose the tool "Segment between two points" and connect points *A* and *C*.
4. Choose the tool "Midpoint or center" and click on the segment *AC* to get its midpoint *D*.
5. Choose the tool "Circle with center through point" and create a circle with center *D* through point *C*.
6. Choose the tool "Intersect two objects" and click successively on both circles to get the intersection points *E* and *F*.
7. Choose the tool "Line through two points" and connect each of the intersection points with point *C*.
8. Change the properties of the constructed objects to enhance the layout of your construction.
   Tip: Right click (Mac OS: ctrl-click) one of the objects and select "Properties".
9. Display the Navigation Bar for Construction Steps

Displaying the Navigation Bar for Construction Steps

After preparing the construction, you are now ready to display a navigation bar for the construction steps at the bottom of the graphics window.

1. In the "View" menu select "Navigation bar for construction steps". In the same menu uncheck "Play button" as well as "Button to open construction protocol".
2. Open the construction protocol of your file by selecting "Construction protocol" in the menu "View".
3. In the appearing construction protocol window click on "View" and select "Breakpoint" to get an additional column that allows you to specify groups of objects that should be displayed at the same time when navigating through the construction steps.

4. Check those construction steps that should be used as breakpoints when navigating through the construction. For example, to display the circle with center *A* through point *B* together with these two points, only check the box next to the last one of these objects - in this case the circle.
5. In the construction protocol window's "View" check "Show only breakpoints" before you close the construction protocol window.

To save space for the second applet on the dynamic worksheet, the interactive applet showing the construction steps needs to be relatively small. Therefore, we need to decrease the size of the GeoGebra window as much as possible before exporting the construction as a dynamic worksheet called "TangentsCircle.html". Include a title and short explanations on what your students need to do when working with the interactive applets before saving the file.

Tip: You may want to use the tool  "Move drawing pad" to drag your construction to the upper left corner of the graphics window. By using the tools  "Zoom out" and  "Zoom in" you are able to decrease or increase the size of your construction.

Second Applet

Now, we need to prepare the second interactive applet of this dynamic worksheet.

1. Open a new GeoGebra window.
2. Hide the algebra window and the axes by unchecking "Algebra window" as well as "Axes" in the menu "View".
3. Adjust the size of the GeoGebra window to give your students enough space to do their construction. Keep in mind, that the other applet needs to fit on screen too and that your students should be able to see both applets on one screen without scrolling.
4. Save the GeoGebra file as "GeoGebra_empty.ggb".
   Tip: You don't need to export this "empty" construction as a dynamic worksheet.

After saving the ggb file you are ready to include the second interactive applet into your dynamic worksheet "TangentsCircle.html". Finally, you could use a customized tool bar in the second applet to simplify the construction process by limiting the number of available tools.

Quality Guidelines for Mathlets

In this section we discuss some basic guidelines for the design of online materials. They are both useful when you create your own materials and when you evaluate existing materials on the Internet. First, we present several design principles of multimedia learning (Clark & Mayer 2003) relevant for mathlets. Then we provide specific guidelines concerning the creation of dynamic worksheets with GeoGebra that are based on experiences from our work with teachers in the NSF math and science partnership project of Florida Atlantic University and Broward County Schools.

Design Principles of Multimedia Learning

The following design principles by Clark & Mayer (2003) are based on empirical research.

- **Multimedia Principle**: Use words and graphics rather than words alone.
  Using text in combination with graphics leads to better learning success than using text alone. So, the visualization of mathematical content should be a key element of all mathlets.
- **Contiguity Principle**: Place corresponding words and graphics near each other.
  You can either integrate text directly into an applet (e.g. when using dynamic text) or place corresponding text close to it.
- **Coherence Principle**: Adding interesting material can hurt learning.
  This principle means that you should avoid using unnecessary texts, graphics or sound only for decoration or to make things more interesting.
  "When things have to be made interesting, it is because interest itself is wanting. Moreover, the phrase is a misnomer. The thing, the object, is no more interesting than it was before." (Dewey, 1913). Unrelated, decorative material distracts the students from the actual learning objective.
- **Personalization Principle**: Use conversational style.
  Your material is easier to understand when you address the students directly. So, you should use the word "you" to address your students in explanations and tasks.

Guidelines for Creating Dynamic Worksheets

In our NSF project, middle school teachers can earn a master's degree in mathematics teaching by taking evening classes at Florida Atlantic University. Part of their assignments in these classes is to create multimedia lessons where they should also follow the design principles mentioned before. The following guidelines for the creation of dynamic worksheets with GeoGebra are based on our experiences from the work with teachers.

Dynamic Worksheet Layout

- **Avoid scrolling**: Your entire worksheet should fit on one screen. Students should not need to scroll between the tasks and the interactive applet (see Contiguity Principle). We consider 1024x768 or 1280x1024 pixels as today's usual screen size. With NVU you can use tables to arrange your texts, images, and interactive applets so they fit on one screen. If this is not possible, consider breaking your worksheet into several pages.
- **Short explanation**: At the beginning of your worksheet, give a short explanation of what the worksheet is about. This should be no more than one or two sentences.
- **Few tasks**: You will usually have questions or tasks to make sure that your students use the worksheet actively. Place these tasks close to the interactive applet (e.g. directly below it). Don't use more than three or four questions / tasks to avoid scrolling (see Contiguity Principle). If you have more tasks, consider breaking your worksheet into several pages.
- **Avoid distractions**: Neither use background or purely decorative images, nor background music on the web page (see Coherence Principle).

Interactive Applet

- **Interactivity**: Allow as much interactivity as possible in your applet. As a rule of thumb, all visible objects should be movable or changeable in some way. Your applet should provide plenty of freedom to explore the relations of its mathematical objects.
- **Easy-to-use**: Try to make your applet as easy to use as possible. If an object can be moved or changed, try to make this obvious, e.g. all movable points could be red. If you don't want objects to be changed, fix them (e.g. texts, functions or slider positions so they cannot be moved accidentally).
- **Size matters**: Your applet should be large enough to allow all intended manipulations, but small enough to fit on one screen and still leave sufficient space for explanations and questions on the surrounding web page.
- **Use dynamic text**: Dynamic text like the length of a changeable segment should be placed close to the corresponding object in your applet (see Contiguity Principle).
- **Avoid static text**: Too much text can easily clutter your interactive applet. Instead, place static text like explanations or questions on the web page that includes your applet.
- **First appearance**: When a dynamic worksheet is opened you should be able to read all labels (e.g. a point label should not be crossed by a line).

Texts

- **Short, clear and personal style**: Try to write your explanations and questions in a short, clear and personal style (see Personalization Principle).
- **Small number of questions**: Limit your number of questions or tasks per worksheet to three or four to avoid scrolling. If you want to ask more questions, create a new worksheet.
- **Use specific questions**: Avoid general questions like "What is always true about *X*?" and make clear what the students should do, e.g. "What happens to *X* when you move *Y*?". We recommend that your students should take notes while they work with a dynamic worksheet. If you want them to write down their answers on paper, say so on the worksheet.
- **Refer to your applet**: Your text should support the use of your interactive applet. For example, try to explain a new term by referring to your applet instead of using an isolated textual definition.
- **Your audience are learners**: If you want to provide information for other educators (e.g. lesson plan, solutions), do so in a separate document (e.g. web page, pdf). Your students should not be distracted or confused by such information.
- **Demonstration applet**: If your interactive applet is meant for presentation only it might be better to have no tasks or questions on the web page. If you include text, it should be understandable for students as well.

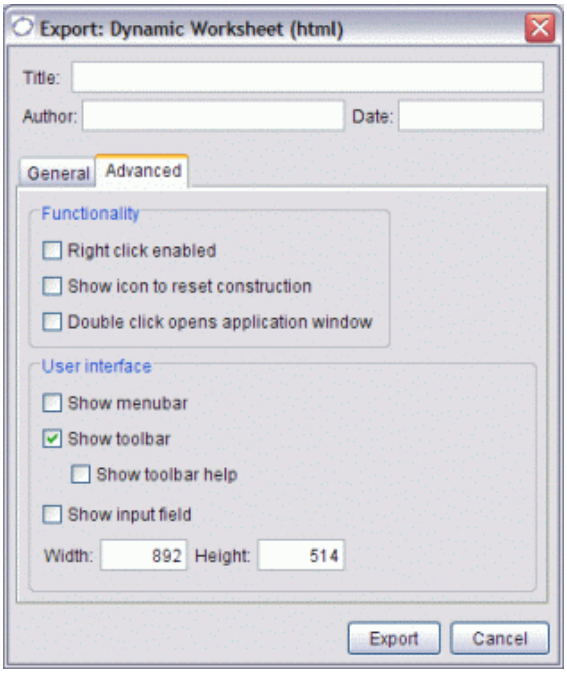
Enhancing GeoGebra Mathlets

With GeoGebra's applet enhancements you can increase the interactivity of your worksheets. For example, you may want to modify your applet to show the toolbar, input field, or a reset icon. You are also able to customize GeoGebra's toolbar to only show a certain set of tools needed for a specific construction (like the applet in Creating the Dynamic Figure) or to include two applets on one dynamic worksheet.

- GeoGebra Applet Options
- Customizing the Toolbar
- Including two Applets on one Dynamic Worksheet

GeoGebra Applet Options

The GeoGebra applet options are accessible in the export dialog for dynamic worksheets (see Exporting the Dynamic Worksheet). By selecting the tag "Advanced" you can check which of the following options the dynamic figure on your dynamic worksheet should include.



Reset Icon

By checking "Show icon to reset construction" you can include a reset icon into the dynamic figure of your worksheet. The icon will be located in the upper right corner of the interactive applet. By clicking on this icon students can reset the applet to its initial state. *Example*: See the first example worksheet Shape of a Slope Function

Toolbar

The applet option "Show toolbar" allows you to display the GeoGebra toolbar within the applet of your dynamic worksheet. Thus, the tools can be used to modify or extend a given construction. *Tip*: You can also display a customized toolbar in order to constrain the number of tools your students are allowed to use. *Example*: See the the second applet of our example worksheet Tangents to a Circle

Input Field

In order to display the input field at the bottom of the interactive applet you need to check "Show input field". This option allows your students to modify the dynamic figure by using the keyboardby to type in algebraic expressions.

Example: See the second example worksheet Equation of a Slope Function

Tip: You might need to increase the height of the interactive applet to provide additional space for the input field.

Customizing the Toolbar

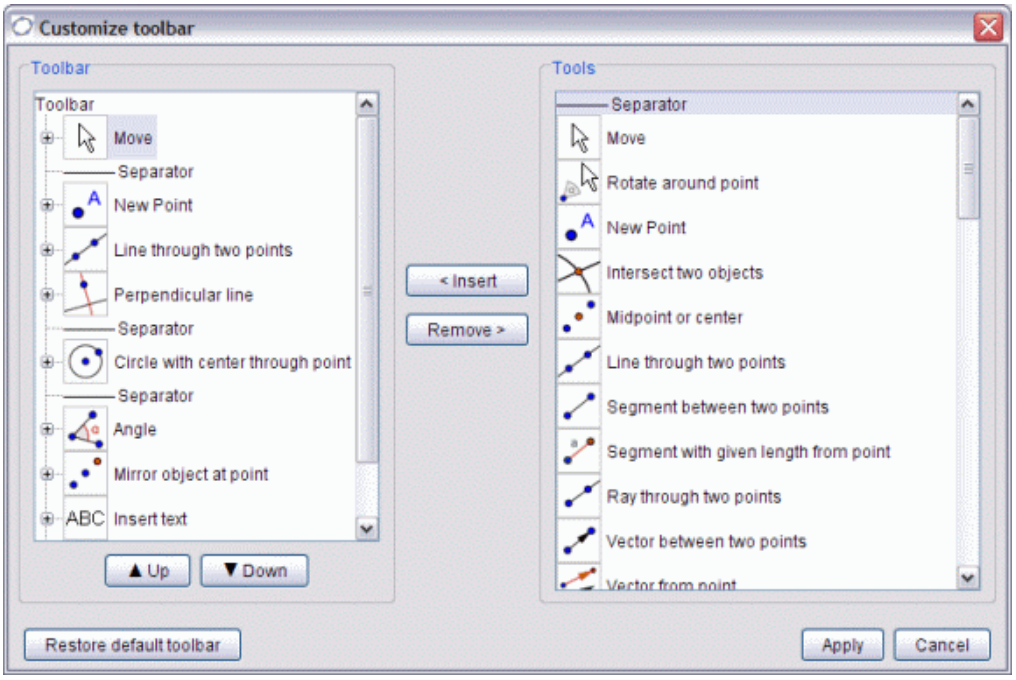
GeoGebra allows you to customize the toolbar by restricting the number of available tools. You can open the corresponding dialog by selecting "Customize toolbar..." in the menu "Tools".

Using the buttons "Insert" and "Remove" you can customize the toolbar which is shown on the left side of the dialog. All available tools are listed on the right side.

Tip: If you export your construction after customizing the toolbar and check the applet option "Show toolbar" your interactive applet will display this customized toolbar.

Example: See the applet in the section Creating the Dynamic Figure

Tip: Your customized toolbar will be saved with your GeoGebra construction. If you want your customized toolbar to be your personal setting you can save it by selecting "Save settings" in the "Options" menu.



Including two Applets on one Dynamic Worksheet

If you want to include two interactive applets on one dynamic worksheet, you need to modify the the applet tag within the html-code of the dynamic worksheet. Therefore, we recommend this option for advanced users with basic knowledge of HTML. In order to include a second applet, you need to open the html-file of your dynamic worksheet with any text editor or html editor. Then, copy and paste the entire applet tag and change the filename value within the applet tag of the second applet. In the following example you can find step-by-step instructions for the creation of our example worksheet Tangents to a Circle.

Accessing the Applet Tag

We are using the html editor NVU in order to illustrate the modifications of the html-code. Of course, you can use any other text or html editor of your choice.

1. Open the html-file of your dynamic worksheet in NVU (e.g. "TangentsCircle.html").
2. Click on the large empty space in the web page to select the interactive applet.
3. Click on the tag "Source" at the bottom of the NVU window to access the source code of your html-page. Because of step (2) the code of the applet tag will be highlighted and should be easy to find.

The following example shows a typical applet tag that was created by GeoGebra during the export of a dynamic worksheet.

Applet tag	Explanation
`<applet code="geogebra.GeoGebraApplet" archive="geogebra.jar"`	Beginning of the applet tag
`width=600 height=300>`	Width and height of the interactive applet
`<param name="filename" value="TangentsCircle_worksheet.ggb" />`	Dynamic construction that is shown in the interactive applet
`Please <a href="http://java.sun.com/getjava">install Java 1.4</a> (or later) to use this page.`	Error message if Java is not installed
`</applet>`	End of the applet tag

Tip: The applet tag contains several applet parameters (e.g. `<param name="..." value="...">`) if you selected some of the applet options in the export dialog of GeoGebra.

Copying and Modifying the Applet Tag

Now, you are able to copy and paste the entire applet tag (in NVU's source window or your favorite editor). By modifying the filename value in the new applet tag the second applet will display a different dynamic figure. You may also need to adapt width and height of the second applet in order to match the dimensions of the corresponding dynamic figure.

```
<applet code="geogebra.GeoGebraApplet" archive="geogebra.jar"
    height="240" width="360">
    <param name="filename" value="TangentsCircle_worksheet.ggb">
    Please <a href="http://java.sun.com/getjava">
    install Java 1.4</a> (or later) to use this page.
</applet>

<applet code="geogebra.GeoGebraApplet" archive="geogebra.jar"
    height="340" width="580">
    <param name="filename" value="GeoGebra_empty.ggb">
    Please <a href="http://java.sun.com/getjava">
    install Java 1.4</a> (or later) to use this page.
</applet>
```

Creating Interactive Exercises using JavaScript

Advanced users who have a little experience with HTML and programming can use the JavaScript interface of GeoGebra applets to add interactive elements like text fields and buttons to their dynamic worksheets. GeoGebra's JavaScript interface provides methods both to observe and modify objects in a construction. You can use these methods for simple things like hiding or showing certain objects when a button is clicked, but also for using GeoGebra as a plug-in for other projects.

In the following example, you will learn how the JavaScript interface can be used to turn a dynamic worksheet into an interactive exercise. To illustrate this process we use our interactive exercise Derivative of a Polynomial.

Creating the Dynamic Figure

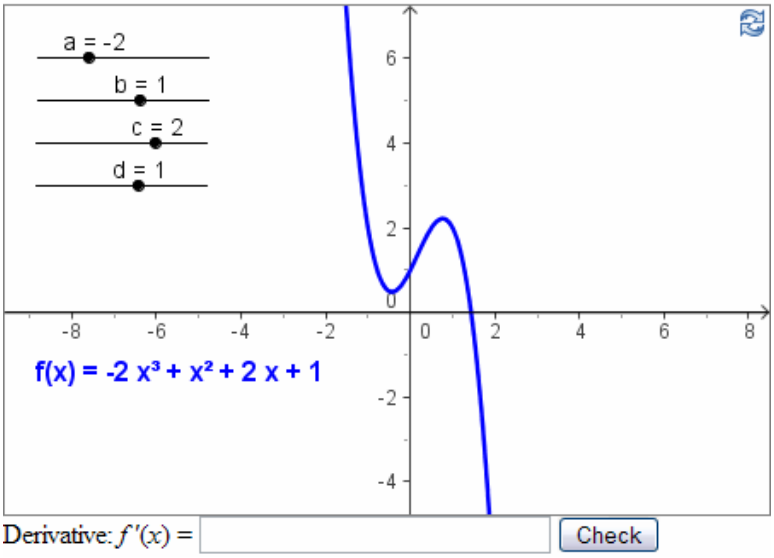
Please find information on how to create the dynamic figure for this example in the section Creating "Derivative of a Polynomial".

Note: Although only the sliders *a*, *b*, *c*, and *d* as well as the polynomial function *f* are visible on the dynamic worksheet, you need to create some hidden objects for the exercise logic in JavaScript.

Creating Interactive Exercises

In order to extend an existing dynamic worksheet to become an interactive exercise you need to...

1. Add interactive elements like text fields or buttons to your html page.
2. Add a short JavaScript program to your html page that implements the functionality for your textfields and/or buttons.



Please find a list of all available methods of GeoGebra's JavaScript interface at GeoGebra Applets and JavaScript. There are several example pages available that help you to get started: Basic JavaScript worksheet, Java to JavaScript communication and Applet to Applet communication.

GeoGebra Wiki and User Forum

In order to enable educators to share their interactive materials created with GeoGebra, we use the GeoGebraWiki, an open pool of materials where everyone can contribute their dynamic worksheets and accompanying documents (e.g. lesson plans). In principle, this pool is simply a collection of pages with links to dynamic worksheets on the Internet. However, in a wiki users can not only change the contents but also influence the structure of the pages. So, if one of the topic pages in the GeoGebraWiki gets too long, anyone can break the pages into smaller pieces by creating several pages for sub-topics.

We also urge educators to upload their materials to the GeoGebra web server (see GeoGebra Upload Manager) or provide downloadable packages of their materials to ensure that others will be able to modify existing materials (see Creative Commons license). In the GeoGebra User Forum you can get support from colleagues. While the GeoGebraWiki is mostly a place to publish your finished work, the discussions in the user forum help to get answers to specific questions or solve technological problems while creating materials.

The GeoGebraWiki and user forum can be seen as examples for the change from static web pages and libraries with unchangeable content towards a collaborative Internet that is driven by its users - in our case by a community of creative mathematics educators.

Conclusion

In this article we have shown you how to create new and change existing mathlets very easily with GeoGebra. One important idea of open source tools (like GeoGebra) and collaborative software (like wikis and forums) is to share knowledge and ideas freely. Free software does not only take away the burdens of license fees and proprietary systems, above all it promotes cooperation.

"Free software" is a matter of liberty, not price. To understand the concept, you should think of "free" as in "free speech," not as in "free beer." (Free Software Foundation)

When authors of mathlets follow this idea and allow others to change their work (e.g. with a Creative Commons license), both quality and quantity of interactive materials on the Internet can increase. Our free environment - GeoGebra with the GeoGebraWiki and user forum - can be seen as an example for the change from static web pages and libraries with unchangeable content towards a collaborative Internet that is driven by its users - in our case by a community of creative mathematics educators.

References

- Clark, R. C.; Mayer, R. E. (2003). E-learning and the science of instruction. San Francisco: Jossey-Bass.
- Dewey, J. (1913): Interest and effort in education. Cambridge, MA: Houghton Mifflin
- Hohenwarter, M.; Preiner, J. (2007): Dynamic Mathematics with GeoGebra. Journal for Online Mathematics and its Applications, Volume 7, March 2007, Article ID 1448
- van Joolingen, Wouter (1999): Cognitive tools for discovery learning. International Journal of Artificial Intelligence in Education, 10, p. 385-397.

Links

- Creative Commons, http://creativecommons.org/
- Digital Classroom Resources, http://mathdl.maa.org/mathDL/3
- Free Software Foundation, http://www.fsf.org
- GeoGebra, http://www.geogebra.org
- GeoGebra Forum, http://www.geogebra.org/forum
- GeoGebraWiki, http://www.geogebra.org/wiki
- Library of Virtual Manipulatives, http://nlvm.usu.edu/en/nav/vlibrary.html
- Math Forum, http://mathforum.org
- MERLOT, http://www.merlot.org
- NVU, http://www.nvu.com