Hands-On Explorations in Algebra and Combinatorics

A Short Course for Mathfest 2010
Mathematical Association of America
August 5–7, 2010
Pittsburgh, Pennsylvania

Course Rationale and Objectives

In recent years, a new piece of mathematical software has appeared on the scene: Sage (www.sagemath.org) is an open source package capable of doing high-powered symbolic and numerical computations. It features a web-based notebook interface, local or remote operation, and can interact with other packages, both open source and commercial (if available). In this short course we will introduce the package, giving multiple examples of how to use it for mathematical explorations, both elementary and advanced. We will focus on algebraic and combinatorial investigations.

Instructors

The course instructors are:

Patrick Bahls, Assistant Professor
Department of Mathematics
University of North Carolina at Asheville
CPO 2350, One University Heights
Asheville, NC 28804-8511
pbahls (at) unca dot edu
828-232-5190

Dr. Bahls received his Ph.D. in Mathematics from Vanderbilt University in 2002 and thereafter completed a three-year postdoctoral position at the University of Illinois. Initially trained in geometric and combinatorial group theory, his interests have shifted during the past several years to graph theory and combinatorics. Since 2007 he has been the director of the NSF-sponsored REU, "Groups, Graphs, and Geometry," run each summer at the University of North Carolina, Asheville. He has written several papers with student participants in this program, including "Ordering B(1,3) using the Magnus transformation" (with V. Collins and E. Heron) and "Utility and expandability of channel assignments" (with T. Mahoney). As the name of his program suggests, he is particularly interested in problems lying at the intersection of group theory, graph theory, and metric geometry.

Robert A. Beeler, Assistant Professor
Department of Mathematics
East Tennessee State University
Johnson City, Tennessee 37614
beelerr (at) etsu dot edu
423-439-4678

Dr. Beeler's interests are in enumerative combinatorics and graph theory, particular graph decompositions and representations. Some of his recent publications include: "The 2-Star Spectrum of Stars" with R.E. Jamison and E. Mendelsohn; "Valuations on Graphs and their Direct Products"; and "Decompositions of Mixed Graphs using Partial Orientations of P_4 and S_3" with A. Meadows. Beeler is a strong proponent of undergraduate research, co-authoring two papers with undergraduate authors. Currently, Dr. Beeler is preparing a manuscript on introductory combinatorics. He is also Co-PI (with J. Knisley, D. Stephens, and A. Govett) on the NSF supported Robert Noyce Scholarship for potential teachers.

Neil Calkin, Professor
Department of Mathematical Sciences
Clemson University
Clemson, South Carolina 29634-0975
calkin (at) clemson dot edu
864-656-3437

Dr. Calkin's interests are in combinatorial and probabilistic methods, particularly in number theory. In 1994 Herbert Wilf and Neil Calkin founded the Electronic Journal of Combinatorics. His recent publications include: “Euler Boole Summation Revisited” with J. M. Borwein and D. Mann; “Elliptic Curves, Modular Forms and Sums of Hurwitz Class Numbers” with B. Brown, T. B. Flowers, K. James, E. Smith and A. Stout; and “Computing the integer partition function” with J. Davis, K. James, E. Perez and C. Swannack.
Calkin is a strong proponent of undergraduate research. He is the coordinator of the Junior Honors Seminar and the leader of the Putnam preparation seminar. He is also the Co-PI with Kevin James of the NSF sponsored REU on Computational Number Theory and Combinatorics.

Dante Manna, Assistant Professor
Department of Mathematics and Computer Science
Virginia Wesleyan College
1584 Wesleyan Drive
Norfolk, VA 23502
dmanna (at) vwc dot edu
757.233.8750

Dr. Manna received an M.A. in Mathematics from Tulane University and a B.A. in Mathematics from Wesleyan University in Middletown, Conn. He has instructed at Dalhousie University where he taught Theory of Numbers, Discrete Structures and Mathematics for Liberal Arts. While there, he also served as coordinator of Dalhousie Analysis and Number Theory Seminar. He has also taught two years at Tulane University. Manna was recently granted the Atlantic Association for Research in the Mathematical Sciences Director's Postdoctoral Fellowship.
This is his second year at VWC.
His publications include: "Rational Landen Transformations on R, printed in Mathematics of Computation," and "A Simple Example of a New Class of Landen Transformations," published by the American Mathematical Monthly.

Dan Warner, Professor
Department of Mathematical Sciences
Clemson University
Clemson, South Carolina 29634-0975
warner (at) clemson dot edu
864-656-5244

Dr. Warner's interests range broadly over the interface between computers and mathematics. His dissertation, "Hermite Interpolation with Rational Functions" was awarded the Householder prize in 1976. Other significant publications include: "Maximal Chordal Subgraphs" with P. M. Dearing and D. R. Shier; "High-Order, Fast-Direct Methods for Separable Elliptic Equations" and "A Program for Solving Separable Elliptic Equations," with L. C. Kaufman. In 1994, he was a charter board member of the Shodor Education Foundation and has been actively involved with Shodor activities particularly those furthering the understanding and use of Computational Science and Engineering by undergraduate faculty.
Most recently he led the development of a new track, Computational and Experimental Mathematics, for the SuperComputing Education Program. Dr. Calkin and he presented the first three sessions in this track at the SC 2009 Conference on High Performance Networking and Computing.

Description of Presentations

The course will consist of 7 presentations and a final panel discussion. The first two sessions will focus on using Sage. The next five sessions will start with a problem or collection of problems in discrete mathematics and explore the topic with the assistance of the more advanced tools in Sage. Each of these sessions will end with a period of guided exploration by the participants. The closing session will focus on the questions: "What have we learned?" and "Where do we go from here?"

1. **Introduction to Sage**. Neil Calkin and Daniel Warner.
   This session will provide an overview of Sage and the Sage framework.

This session will focus on insuring that all participants will be able to use Sage on their own computers and also how to effectively use Sage in a classroom setting.

2. **Elementary Mathematics with Sage**. Neil Calkin and Daniel Warner.
   This session will provide a hands-on introduction to the basic capabilities of Sage. This will cover the bulk of the Maple and Mathematica capabilities that are used in Calculus and other undergraduate mathematics courses.
3. **Exploring Combinatorial Group Theory**. Patrick Bahls.
   This session will illustrate how to compute in a combinatorial group, highlighting some interesting conjectures.
4. **Generating Functions and Sage.** Robert A. Beeler.
   This session will demonstrate how to use generating functions in Sage to model problems in an introductory combinatorics course.
5. **Benoulli Convolutions.** Neil Calkin.
   This session will examine combinatorial questions arising from Bernoulli convolutions, illustrating how computation has improved upper bounds on the growth of such functions.
6. **Combinatorial Games and Symmetry**. Daniel Warner.
   This session will demonstrate how Sage's integration of different mathematical tools supports the analysis of old and new board games.
7. **Generalizing the Bernoulli and Euler polynomials.** Dante Manna
   We explore various generalizations of both the Euler and Bernoulli polynomials, both standard and q-versions.

Course Schedule

First Day

9:00 AM – 10:30 AM	Introduction to Sage
10:30 AM – 10:50 AM	Break
10:50 AM – 12:00 noon	Elementary Mathematics with Sage
12:00 AM – 1:30 PM	Lunch
1:30 PM – 3:00 PM	Exploring Combinatorial Group Theory
3:00 PM – 3:30 PM	Break
3:30 PM – 5:00 PM	Generating Functions and Sage
5:00 PM – 6:30 PM	Reception

Second Day

9:00 AM – 10:30 AM	Benoulli Convolutions
10:30 AM – 10:50 AM	Break
10:50 AM – 12:00 noon	Combinatorial Games and Symmetry
12:00 AM – 1:30 PM	Lunch
1:30 PM – 3:00 PM	Generalizing the Bernoulli and Euler polynomials
3:00 PM – 3:30 PM	Break
3:30 PM – 5:00 PM	Panel Discussion