

Experimental Mathematics

Summary: The Summer Experimental Mathematics (SEM) program at Michigan State University (MSU) provides research experience in mathematics for minority undergraduate students who have completed at least three sequences of college level calculus courses. Its objective is to encourage minority students pursue higher education in the mathematical sciences. The theme of this six weeks program is to explore major concepts of dynamical systems through computer experimentation. Using one dimensional maps and function iteration students will explore bifurcation and chaos theory. In the first two weeks, they learn these topics using the well known one dimensional map $F_c(x) = x^2 + c$. They will be introduced to topics such as hyperbolic and non-hyperbolic fixed points, saddle-node and period doubling bifurcation, the period doubling route to chaos and fractal sets.

Project1 Stability of one-dimensional tent-like maps: Participants will study the dynamics of tent-like maps defined by $f(x) = cx^b$, if $0 \leq x < \frac{1}{2}$ and $a(1 - x^b)$ if $\frac{1}{2} \leq x < 1$. Here a, b and c are parameters. The first part of the project is to find some relationships between a, b and c so that the problem can be reduced to a one parameter problem. Through computer experimentation and analytic work, students will classify fixed points and periodic points, investigate bifurcation theory and discuss chaotic and non-chaotic behaviors of these maps. They will give empirical (through orbit diagrams, histograms and density curves) as well as rigorous proofs, under special cases, for chaotic behaviors. They will also explore the connections between iterations of these maps and fractal sets.

Project2 Fractals through Iterated Function Systems (IFS) Students will study IFS and generate fractals as a fixed attractor set of IFS. Each IFS consists of affine transformations involving rotations, scalings by a constant ratio, and translations. First students get familiarity with IFS that generate classical fractals (such as the Sierpinski triangle). They will then investigate different IFS of their choice and write Mathematica programs to generate their own fractal. Among others they will find IFS that generate the classical fractals such as the Peano Curve, the Heighway-Harter Dragon and the Barnsley Fern. They will also generate modified versions of these and other fractals. For the fractals they create they will document a geometric description of how the fractal is constructed, the IFS transformations, the topological and fractal dimension of the fractal, and other facts (such as area, perimeter, etc).

Participants:

- 1. Zachary J. Flores**
- 2. Dalana Katherine Candler**
- 3. Scherrell Williams**
- 4. Oumarou Njoya**
- 5. Brittney Nicole McCullers**