Linear and Abstract Algebra for Teachers – A Course for Non-Mathematics Majors

By Marie Vanisko and Viji Sundar, Department of Mathematics, California State University Stanislaus

Background for the Course

In December 2002, the California Commission on Teacher Credentialing came up with a new set of guidelines to teach mathematics in secondary schools. The shortage of mathematics teachers qualified to teach secondary and middle school mathematics had reached alarming proportions. According to a 2002 NCES (National Center for Education Statistics) report on qualifications of public school teachers, 37% of high school math teachers lack a major or certification in their field and at the middle school level 69% of math teachers lack a major or certification in their field. Shortage of qualified mathematics teachers comes just when the expectations for what students should know in these subjects are rising.

In order to mitigate this problem, CCTC created a bifurcation credential for subject matter competence – the full credential and a foundation credential. Consequently, it provided two paths to become a secondary or middle school mathematics teacher. The mathematics content knowledge to enter the credential program can be met by coursework or by passing the CSET - California Subject Examination (in mathematics) for Teachers (www.cset.nesinc.com/CS_testguide_Mathopener.asp). The level of competence to teach is based on the coursework completed (if coursework path is chosen) or the examinations passed (if examination path is chosen).

Subject Matter Competence

For the Full Credential the subject matter competence can be met in two ways.

(i) obtain a major in mathematics from an accredited post secondary institution or

(ii) obtain a Bachelor's degree in a subject other than mathematics and pass CSET 1, 2, and 3.

For the Foundation Credential the subject matter competence can be met in two ways.

(i) obtain a Bachelor's degree in any subject from an accredited post secondary institution and complete 32 units of baccalaureate level mathematics covering specified topics in Algebra, Number Theory, Geometry, Probability and Statistics or

(ii) obtain a Bachelor's degree in any subject and pass CSET 1 and 2.

Rationale for the Course

The topics for the subject matter competence –also known as Subject Matter Requirement (SMR) – are clearly spelled out by CCTC (www.ctc.ca.gov/educator-standards/)**.** The Subject Matter Requirement for Algebra includes concepts involving matrix algebra, groups, rings, and fields – topics that are not usually

covered in mathematics courses taken by non-mathematics majors seeking Foundation credential to teach mathematics at the middle school level. As such, for those who are not majoring in mathematics, finding a lower division course on topics involving linear and especially abstract algebra is extremely difficult, if not impossible. This is what prompted us to apply for a PMET – Preparing Mathematicians to Educate Teachers - mini grant.

How the Course Was Developed.

It is not surprising to see the linear and abstract algebra requirements for middle school and high school mathematics teachers. Beyond simple arithmetic, linear algebra is one of the most widely used mathematical subjects today. It has much in common with high school algebra and forms a bridge between high school algebra and abstract algebra. In terms of abstract algebra, classroom textbooks even at the elementary level reinforce the axioms (properties) of the whole numbers and the real number system. It is important for middle school and high school mathematics teachers to have an understanding and working knowledge of the mathematical structures that encompass these properties.

Our original plan was to have two courses, one in linear algebra and one in abstract algebra. However, because we wanted to use concepts in linear algebra as examples in abstract algebra and it would be difficult to guarantee that teachers would take the classes in succession, we decided to develop a single four-semester-credit course rather than two separate courses. We consulted with our colleagues in the mathematics department to assure the mathematical integrity of the course and to begin the process of making the course part of the regular departmental offerings. Next we engaged in a dialogue with teachers to get their input as the course being designed was specifically for teachers. In summer 2004, we invited future teachers and current classroom teachers for two sessions to get feedback on their perspectives on such a course and to explore the readability of materials we considered using for the course. We wanted to ensure that our course was geared at the appropriate level. The teachers were given a stipend for their participation. These sessions were very beneficial in fine tuning the course topics and the structure of the course offerings.

We realized that this course would not only meet the needs of future middle/junior and high school teachers; it would also be a mathematical content course for current elementary teachers wanting to teach mathematics at the 6 – 8 level. After we researched and discussed the topics to be included in the course, we arrived at the 'Outline of the Course' listed in what is to follow. As this course is less theoretical and focuses more on contemporary applications of the topics covered, it is listed in the catalog as a lower division mathematics class with pre-calculus as the pre-requisite.

Outline of the Course (meeting for 4 hours per week for 14 weeks)

Part I – Vectors (Week 1)
- Review of Sine and Cosine Functions
- Vectors and Coordinate Systems
- Vector Algebra and the Dot Product

Part II – Linear Algebra (Weeks 2 – 8)
- Linear Systems as Models
- Basic Operations on Vectors and Matrices
- Matrix Multiplication
- Gaussian Elimination
- Matrix Inverses
- Determinants and Vector Cross Product
- Introduction to Eigenvectors and Eigenvalues
- Angles, Orthogonality and Projections

Part III – Abstract Algebra (Weeks 9 – 12)
- Symmetry Groups
- Abstract Groups
- Coding Theory
- Introduction to Permutation Groups
- Rings and Integral Domains (Integers and Integers Modulo n and Matrices)
- Fields of Rational, Real, and Complex Numbers and Polynomials Over a Field

Part IV -- Course Wrap-up and Final Presentations (Weeks 13-14)

Sample Lessons

Linear algebra begins with vectors, both from algebraic and geometric perspectives. In geometric projections, the role of the dot or scalar product in linear regression and correlation are used as examples. Students are amazed to learn that the correlation coefficient for two sets of data is the cosine of the angle between the vectors. Throughout the course, we ground examples in real life applications and tap into previous knowledge to form a bridge to the new concepts introduced.

The following example demonstrates the manner in which we give students a rationale for vector operations. We select an example associated with ordering food from a fast food restaurant. Each entry under a given name is a one-dimensional array representing the quantity of each specific item to be ordered, so these must be kept separate from one another, but the one-dimensional arrays taken together can form a matrix. Using this, students learn to distinguish between rows and columns. Taking the product of the appropriate "place an order" matrix with the column vector representing the costs of each

item, respectively, results in a vector representing the costs for each individual. This allows students to see multiplication of a vector by a matrix as successive applications of the dot product.

Example: Fast Food Orders
Motivate

- Multiplication by a scalar
  Double a person's order; each entry in that column is doubled.
- Add vectors
  Add together the orders of the three people to determine how many of each item must be prepared.
- Dot or scalar product
  For each person, multiply the number of each item by the price of the item and add up the results to determine the cost of each person's order.
- Multiplication of a vector by a matrix
  Switch the array of person's orders to rows instead of columns and take the product of the matrix with the column vector representing the costs. Each entry and the overall dimensions make sense.

Items	Kim	Tom	Sal	Cost
hamburger	8	5	2	$3.60
fries	6	5	1	$1.50
soda	4	5	0	$.90
milk shake	2	0	1	$2.40

$$\begin{bmatrix} 8 & 6 & 4 & 2 \\ 5 & 5 & 5 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \$3.60 \\ \$1.50 \\ \$.90 \\ \$2.40 \end{bmatrix} = \begin{bmatrix} \$46.20 \\ \$30.00 \\ \$11.10 \end{bmatrix}$$

Linear systems of equations are used to investigate Gaussian elimination, matrix inverses, and determinants. These linear systems provide rich examples of the role played by matrices that do not possess inverses and have zero determinants. Additionally, they lend themselves to an introduction to sensitivity analysis using the inverse of the coefficient matrix. With Markov processes, even eigenvalues and eigenvectors are introduced.

Abstract algebra begins with symmetry groups, and, in particular, the symmetries of a square. This visual perspective relates transformational geometry with matrix multiplication and sets the stage for the concept of a group. Applications in coding theory using modular arithmetic further extend the concept of a group and lead naturally into rings and fields. Permutation groups are studied briefly. The development of the real number system from counting numbers through real numbers is seen in the context of groups, rings, integral domains, and fields.

Course Materials and Technology

Finding appropriate materials for teaching this course was difficult, especially for the abstract algebra portion. We selected a very special book, *Principles and Practice of Mathematics* (ISBN: 0-387-94612-8), that was developed through the Consortium for Mathematics and its Applications (COMAP). In particular, we chose two chapters from this text, Chapter 3 on Linear Algebra, authored by Alan Tucker, and Chapter 9 on Abstract Algebra, authored by Joseph Gallian. These two chapters provide exceptionally

clear explanations for a person with a pre-calculus background, and they contain valuable examples and exercises. The abstract algebra portion was supplemented with materials on rings and fields.

In the linear algebra portion of the course, the use of technological tools is virtually essential. Although *Mathematica* or Maple would be ideal, such software is not readily available to middle school and high school teachers. Graphing calculators work well, but we opted for spreadsheet technology, since nearly every teacher and classroom has access to a spreadsheet program such as Excel. Many of the commands used in linear algebra are functions named in Excel; there are just a few variations in the manner in which some of these are executed. In our pilot offering of the course, even students who were unfamiliar with Excel at the start were working well with the program in a short period of time.

Assessment

Assessment has three components: take-home exams, shorter in-class exams, and a final project. The culminating project requires the students to develop a teaching module involving topics covered in this course that would be appropriate for the grade level they teach or expect to teach. The lesson should comply with both the NCTM standards and the California mathematics content standards for the grade level targeted. Students have the option of working alone or in pairs, and the final report has both an oral component and a written component. In the pilot offering, participating teachers chose a wide variety of applications, including having young children determine the day of the week they were born using modular arithmetic and having high school students investigate the connection between scores on the California High School Exit Exam and school course grades.

Conclusion

It is not easy to get mathematics department approval for courses for classroom teachers. The first offering of Linear and Abstract Algebra for Teachers was as a special session class through University Extended Education. However, this course received full approval as a catalog offering from the University-wide Education Policy Committee. It is a lower division course (Math 2670) and is intended for both pre-service and in-service teachers who are not mathematics majors but who want to have the proper authorization to teach mathematics at either the middle school or high school level in California. In the pilot course offering for teachers, students enrolled found the course challenging, interesting, and yet not overwhelming. At the start, few felt that the topics covered would relate to what they taught, but, by the end, each person found an application that was associated with his or her grade level.