A Specialized Course for Mathematics Education Majors

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This paper describes the creation of a mathematics education course that focuses on pre-calculus mathematics. The course has been incorporated as a requirement for the mathematics education major at Humboldt State University.

Undergraduate Environment

At Humboldt State University, we have approximately 35 mathematics education majors in a department with around 100 mathematics majors. The mathematics education major consists of the calculus series, a transitional course on mathematical thought and proof, a sophomore-level linear algebra course, the new course focusing on an advanced study of pre-calculus mathematics, and upper-division courses in number theory, mathematics history, abstract algebra, geometry, and a capstone course. The capstone course is a 3-semester-unit course that is taken at the end of the senior year and considers broad themes that permeate mathematics, both in secondary schools and throughout undergraduate study. Additional units are required in upper division mathematics electives, a computer programming course, and a two-course sequence in which applications of mathematics are studied. Prior to our instituting the new mathematics education course, the linear algebra requirement consisted of two 3-semester unit courses, one at the sophomore level and one at the junior/senior level. The linear algebra requirement has been reduced, and the new mathematics education course replaces the junior/senior-level course.

Motivation for the Creation of a New Course

Several factors motivated the creation of the new mathematics education course. First, the Conference Board of the Mathematical Sciences' *The Mathematical Education of Teachers* report calls for programs that prepare high school teachers by including a six-hour capstone course "connecting their college mathematics courses with high school mathematics" [2], and the department wanted to comply with this recommendation. A second motivating factor was the accreditation of our mathematics education program by the California Commission on Teacher Credentialing: its guidelines call for a renewed emphasis at the undergraduate level on the content and standards for California secondary mathematics classrooms. Finally, the mathematics education faculty felt that the existing mathematics education program did not entirely address the needs of the future teachers, in particular, in attention to precalculus-level topics.

The new course, entitled *School Mathematics from an Advanced Viewpoint I*, is intended for those who have completed Calculus II and the transitional course on mathematical thought and proof. While the content of the new course focuses on precalculus content and the material on proofs could be incorporated into the course, the calculus and mathematical thought prerequisites ensure a certain mathematical sophistication and maturity will be common to class members. Students are usually prepared with these prerequisites by the end of the sophomore or beginning of the junior year.

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The existing mathematics education capstone course is organized thematically, around the ideas of multiple representation, algorithmic thinking, mathematical applications, reasoning and proof, and variation. The course is open to any interested mathematics major, but is only required for those students in the mathematics education major. The course is designed to develop students' mathematical insight and facility through problems posed from contexts related to teaching (drawing undergraduate level mathematics out of grades 7–12 math activities), problems in mathematical modeling (tying together areas of undergraduate mathematics through applications), and reflective writing on grades 7–12 curricular materials or content areas (viewing 7–12 math curriculum from an advanced viewpoint). The goals of the course are to enable students to build connections both among the mathematical areas they have studied and between undergraduate mathematics and school mathematics, to develop their understanding of mathematics as an integrated discipline, and to strengthen their oral and written communication skills in mathematics.

Description of the New Course

The newly developed course differs from the existing capstone course in that it is focused on specific content in the secondary schools and on using tools acquired in undergraduate courses to analyze that content from an advanced perspective. The two courses are named *School Mathematics from an Advanced Viewpoint I and II*, but it is not intended (or discouraged) that they be taken consecutively. Rather, the new course can be viewed as a bookend that starts the students' study of upper-division mathematics, and the capstone course provides closure at the end of their formal undergraduate study.

The three mathematical content areas at the focus of the new course are functions and equations, encompassing the study of rate problems, mixture problems, averaging problems, quadratic equations and functions, and harmonic, arithmetic, and geometric means; complex numbers, including the study of real numbers, complex number geometry and the Reimann sphere; and trigonometry, encompassing the ideas of injectivity, surjectivity, inverse, composite functions, limits, including epsilon-delta definitions, solving inequalities, domain, range, and operations on functions.

The problems students study in the new course are motivated by those typical in high school mathematics, and the problems are analyzed using the machinery of undergraduate-level mathematics. The result of this study is a deeper understanding of mathematical content, a sense of how to use more sophisticated mathematical tools (like proof, algebraic structures and number theory) to analyze simple problems; and a connection between types of problems, between material in undergraduate mathematics and school mathematics, and between mathematics and its applications. For each topic, students are asked to analyze problems, extend problems, and make connections.

Evaluation of student work in the course is accomplished primarily through weekly homework sets; also required are one group project with a written paper and accompanying mathematical poster, and several take-home examinations. Because of the in-depth analysis that problems require, problems not successfully completed at the "B" level are returned to the student with comments, and the student is required to redo the problem in order to earn credit. Problems successfully redone earn B credit (see [4] for a description of grading philosophy). The textbook used for the course has been Usiskin et al, [5].

Examples from the Three Areas of Focus

To give the reader a sense of the kinds of problems and activities that students in the new course study, the following three examples are provided.

Functions and Equations

One type of problem that allows students to investigate ideas related to functions and equations is the rate problem. Students are familiar with standard distance, rate and time problems from the high school algebra curriculum, but the experience from this course showed that undergraduate students did not have a deep understanding of the nature of these problems, or of the connection between rate and work problems, for example. The textbook provides many useful activities and problems dealing with rate, and an article in the *Mathematics Teacher,* "Averaging Rates: Deciding When to Use the Harmonic or Arithmetic Mean" [1], provides the opportunity to explore harmonic and arithmetic means using distance-oriented speed (measured in distance per unit time) and time-oriented speeds (measured in time per unit distance).

The study of complex analysis

Most undergraduate mathematics education majors do not have a course in complex analysis, and many do not have a course in real analysis. Their understanding of complex numbers comes mainly from finding roots by using the quadratic formula. This course provides an opportunity to investigate the complex numbers and their connection to geometry. The textbook provides problems in roots of unity, polar coordinates, and the Riemann sphere. The MAA publication *Complex Numbers and Geometry* [3] is used to supplement this unit, and provides problems that connect complex numbers and geometry as well as problem solving opportunities and cultural perspective.

Trigonometry

Most mathematics education majors did not take the precalculus course at our institution; many came to our university with precalculus as a high school course. Consequently, other than some limited experience with trigonometric functions in calculus, most have not had an in-depth study of trigonometric ideas beyond the high school level. In particular, the difficult areas for students come from considering the inverse trigonometric functions and the domain and range of the composition of trigonometric functions and their inverses. For this topic, the main resource used is the text.

Lessons Learned

This course has been taught once, and results from that pilot teaching have influenced the design of the course as presented here. In the pilot teaching of the course, the instructor found several weaknesses. First, students were not prepared for the significant collaboration that was required. Second, they were excited to be taking a course that impacted their chosen careers, and were dismayed when they found the course difficult, and sometimes questioned whether they had chosen the right career path. Finally, the initial list of topics was too ambitious.

As a result of these issues, several adjustments have been made. To encourage and facilitate student collaboration, the course is now organized as a one-hour block one day a week, followed by a two-hour block on a second day. Generally, these blocks are used so that the one-hour block is used to introduce topics, and the two-hour block is used for student group work and preparation and presentation of their results to the entire class. The second adjustment is in the attitude in presenting this difficult material. The instructor found it necessary to give students "permission" to be confused, and remind them that learning happens when an individual works to become unconfused. This approach seemed to alleviate many of their early concerns. Finally, the list of topics has been pared down to those presented here.

Other Considerations

In designing this course, the Mathematics Department created a mathematics education advisory board. Five local secondary mathematics teachers, along with two mathematics department faculty, were invited to participate by attending several board meetings to discuss the needs of student teachers and new teachers. One item that the advisory board identified was to have undergraduate students in the course consider problems from the focal topics in the light of work from school students on similar topics. The advisory board is currently collecting samples of school student work to use in the course.

The creation of this specialized mathematics education course was part of a larger project to focus our department on the mathematical needs of mathematics teachers. Prof. Elena Marchisotto, one of the authors of the course textbook, and Prof. Deborah Ball, each visited our department and participated in our colloquium series. The result was an on-going dialog about what mathematics teachers need to know. Two tangible results of this dialog have been the collaboration of faculty whose specialty is not mathematics education and those whose specialty is mathematics education, resulting in joint publications on issues in the undergraduate education of mathematics education majors.

References

[1] Brown, S. L. and M. A. Rizzardi, "Averaging Rates: Deciding When to Use the Harmonic or Arithmetic Mean," *Mathematics Teacher* 98(9), pg 626-636.

[2] Conference Board of the Mathematical Sciences (CBMS). *The Mathematical Education of Teachers Part I.* Washington, D.C.: Mathematical Association of America (MAA), 2001, pg 8.

[3] Hahn, Liang-shin. *Complex Numbers and Geometry*. Washington, D.C.: MAA, 1994.

[4] Stutzman, Rodney Y. and Kimberly H. Race, Everyday Rubric Grading," *Mathematics Teacher* 97(1), pg 34.

[5] Usiskin, Peressini, Marchisotto, & Stanley, *Mathematics for High School Teachers: An Advanced Perspective*, Pearson: 2003.