

A Mathematics Capstone for Preservice Secondary Teachers:
A Cornerstone for Career-long Teaching and Learning

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Abstract

This paper discusses the design of a mathematics capstone course for preservice secondary teachers with the goal of developing the profound understanding of mathematics needed for teaching. The goals, implementation, and structure of the course are described. In addition, we describe assignments that aid preservice teachers in making connections needed for improving their pedagogical content knowledge.

The *Mathematical Education of Teachers (MET)* publication charges mathematics departments with supporting “the design, development, and offering of a capstone course sequence for teachers in which conceptual difficulties, fundamental ideas and techniques of high school mathematics are examined from an advanced standpoint.” (Conference Board of Mathematical Sciences (CBMS), 2001, p. 39) In addition, the report recommends that the capstone course be taught through a collaboration of mathematics and mathematics education faculty. In support of these recommendations, this article discusses a capstone course that studies mathematics content and pedagogy.

The Knowledge Needed By Effective Mathematics Teachers

The National Council of Teachers of Mathematics (NCTM, 2000) has set guidelines for all secondary students to engage in learning mathematics with understanding. Teaching that promotes students' mathematical understanding as set forth by the NCTM requires a strong conceptual knowledge of high school mathematics as well as the ability to provide powerful representations and promote meaningful classroom discussion (Brown and Borko, 1992; Lloyd and Wilson 1998). Ma (1999), in her book *Knowing and Teaching Elementary Mathematics*, provides a description of the type of mathematical knowledge a teacher needs to teach

mathematics conceptually which she calls a "profound understanding of fundamental mathematics (PUFM)" (p. 120). This knowledge of mathematics has depth, breadth, and thoroughness, and is associated with teachers that demonstrate connectedness, multiple perspectives, reinforcement of basic ideas, and longitudinal coherence. To begin to develop PUFM, preservice teachers need to explore the mathematics behind K-12 mathematics problems as well as make connections with their college mathematics courses. They need to develop techniques for gaining PUFM, including skills in reading math, understanding definitions, mathematical reasoning including writing proofs, problem solving along with developing models, and communicating mathematics effectively (Ma, 1999).

It is well documented that the current preparation for secondary math teachers is not sufficient as evidenced by the studies showing that many preservice teachers do not currently have the necessary subject matter knowledge to be successful in the classroom (Ball 1990; Bolte, 1993; Fi, 2003; Thompson, 1992; Van Dooren, Verschaffel, & Onghena, 2002). Research shows that simply increasing the number of math classes taken often does not improve a person's ability to teach concepts effectively (National Center for Research in Teacher Education, 1991). Furthermore, Borko et al. (1992) note that there is little focus on secondary school mathematics content topics in undergraduate mathematics courses.

The Capstone Course

One way to address low subject matter knowledge in preservice teachers, lack of preparation to teach mathematics conceptually, and lack of connections between college and K-12 mathematics is to establish a capstone course with the goal of building a profound understanding of fundamental mathematics. Two of the three authors developed and taught a capstone course meeting these objectives. We provide a description of the course, a brief

description of the sources of mathematical material, and finally, research-based recommendations for assignments that can be used for capstone courses.

The capstone was taught at a small (approximately 1250 students) national liberal arts university that has approximately 10 math majors per year of which 2 or 3 plan to teach math and because of their small numbers they had no content specific methods course. Two faculty members, a mathematician and a math and science educator, collaborated to create a capstone course for the preservice mathematics teachers. The goal of the course was accomplished by emphasizing problem solving and mathematical reasoning, making connections between undergraduate and secondary math, in addition to familiarizing students with state and national standards. The content explored in the course was the advanced study of mathematical content taught at the secondary level, including real and complex number system, concept of function, and concept of equation. In addition to modeling effective teaching practices throughout the course, we investigated pedagogy by reading and discussing literature on research based issues involving teaching and learning mathematics.

Each class started with a student presentation of a mini-lesson aimed at helping their classmates build deeper understanding of secondary mathematics content and pedagogies to support the learning of this content, with particular emphasis on integrating a variety of manipulatives and technologies. This was followed by discussion of the assigned reading on pedagogical issues such the nature of mathematics, mathematical definitions and proofs, how students learn mathematics, and components of high quality mathematics teaching. We then spent time discussing the textbook reading and having student presentations of the assigned homework. Due to the complexity of the mathematics problems, class discussions on one section of the book often lasted more than one class meeting.

The evaluations from the students of this course have been overwhelmingly positive and they expressed appreciation for the mathematical content that they saw as relevant. Before the course one student described her notion of ideal mathematics teaching as “direct teach” with student input, after the class she changed her description to a teacher that “values classroom community, focuses on problem solving, advocates cooperative learning groups and mathematical dialogue, and stresses importance of questioning to guide students rather than giving information.” She was able to implement this in her student teaching the following semester.

Sources of Mathematical Material

The textbook used was *Mathematics for High School Teachers: An Advanced Perspective* by Z. Usiskin, A. Peressini, E. Marchisotto, and D. Stanley (2003). With the goal of building profound understanding of high school mathematics, the text includes an account of the historical development of key mathematical concepts as well as probing, non-routine problems that require the sophistication of an advanced mathematics student. The text is divided into two sections, one that addresses algebra and one that addresses geometry and has sufficient material for two semesters.

Some additional resources of problems used in the capstone courses include professional development materials created to help teachers implement national or state standards (e.g. NCTM e-examples; Texas Essential Knowledge and Skills (TEKS) with Clarifying Activities available online from the Mathematics TEKS Toolkit), high school text books, state assessments of teacher content (e.g. PRAXIS available from the ETS website), along with articles and problems from the *Mathematics Teacher* Journal or other journals for high school teachers.

Recommended Assignments

With the emphasis on building PUFM, we suggest two assignments that utilize writing and encourage students to make connections across the curriculum. The first is a lesson planning activity in which the same assignment is given twice to allow for evaluation of increased conceptual and pedagogical knowledge. The second is a writing assignment designed to encourage students to seek out and make connections between topics in the capstone course as well as the college and secondary curricula.

Lesson Planning Activity

In the lesson planning activity proposed in the research of Winsor (2003), before the preservice teachers have covered functions in their capstone course, they are asked to plan an introductory lesson on the topic of functions for a class of high school freshmen. They are allowed to use and cite any resource they wish. They are instructed to list any prerequisite knowledge that students needed to be successful in the lesson as well as to include all overhead transparencies, handouts, activities, or homework that they might use in the lesson. In addition to the introductory lesson, preservice teachers are asked to sketch an outline of the lesson that would follow. After studying functions and discussing mathematics pedagogy, the preservice teachers are asked to reexamine their lesson plans and resubmit them. They can re-plan their lesson, make minor changes to the lesson, or leave it as it is.

The purpose of the lesson planning activity is to explore the effect of content knowledge on pedagogical content knowledge. Furthermore, it gives students the opportunity to experience constructing a lesson plan as well as the opportunity to apply the mathematical knowledge they have acquired in college to a high school mathematics lesson. This activity helps to demonstrate

to the students the connection between subject matter knowledge and pedagogical content knowledge.

Connections Writing Activity

Carpenter and Lehrer (1999) argue that students construct knowledge by constructing relationships, reflecting about experiences, articulating what they know. Hiebert and Carpenter (1992) define mathematical understanding in terms of a network of connected representations. “The degree of understanding is determined by the number and strength of the connections” (p. 67). To encourage and assess this deeper understanding, Fisher (2004) designed a writing assignment to encourage students to seek and make connections between topics in a mathematics course for preservice elementary teachers. Fisher found that the writing assignment was effective in helping the students construct mathematical connections. The assignment has been modified for the capstone course in order to help students explore the mathematics behind high school problems as well as make connections with their college mathematics courses.

The connections writing assignment asked students to select nine problems, three from the capstone course, three from other collegiate math courses, and three from the high school mathematics curricula. The problems students chose were to be somehow connected and students were instructed to explore and explain the connections between all the problems as well as between any subset of the problems. Their papers were to include the question and the solution to each problem, an explanation of the ways in which the questions are related, and an explanation of the ways in which the solutions to the problems are related. In this way students were provided with an opportunity to construct knowledge by reflecting about experiences and by articulating what they know.

Conclusions

The capstone course aims to be a transformational experience for preservice teachers with lasting impact on their future classroom pedagogy. Our findings indicate that the capstone course has the potential to help address the problem of low subject matter knowledge in preservice teachers as well aid them in developing a profound understanding of fundamental mathematics. The authors assert that allowing preservice teachers to explore the mathematical content taught at secondary level using conceptually based pedagogies is a crucial aspect of this outcome. Additionally, many recommendations make it clear that the capstone should not be the end of the preservice teachers' learning, instead it should be viewed as the cornerstone of career-long learning of content and pedagogy (Committee on Science and Mathematics Teacher Preparation, 2001; Graham & Fennell, 2001; NCTM, 2000).

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