The Power of Intersecting Circles:
A Model for Integrating Mathematics Teacher Knowledge

by

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"Being able to teach has been a huge step for learning about teaching." Another pre-service teacher in our new course claimed it was an "Excellent idea to get us in the classrooms and teaching. It helps make what we learn in the classroom have a practical, weekly context. We always need more classroom experience!"

Building on experiences at recent PMET conferences, we developed a new course for the spring 2006 semester to help prepare pre-service teachers to teach middle-grades mathematics (grades 5-9). We created an integrated content-methods course that featured a field experience in which students planned and taught a math lesson weekly in teams. Based on our observations and the assessment of our students in the course, we believe this combination of mathematics content, mathematics methods, and a carefully-constructed weekly field experience created a most effective structure for preparing university students to teach mathematics.

Background

Taylor University is a small (enrollment of 1875) midwestern liberal arts college with six full-time and two part-time mathematics faculty. Of the approximately fifty math majors, about half are enrolled in the mathematics education program. Our secondary mathematics education degree is a 43-semester hour program which includes courses in calculus, linear and abstract algebra, college geometry, mathematical statistics, and electives in applied math and modeling, as well as a research seminar and a senior capstone experience.

A few years ago, the math department determined to re-create a middle school math licensure program, which had earlier been discontinued by the teacher education program (due to state licensing requirement changes). Because our math department lacks sufficient student enrollment and available faculty to offer a separate program exclusively for the middle grades, we needed to build the new middle grades curriculum around current course offerings. However, we wanted to add at least one new math course that would be focused specifically for middle grades teachers.

Our goal was to create a program, and within it a new course, which would prepare our pre-service teachers to more effectively teach mathematics, given the limits posed by the size of our institution and department. This led to our interest in the PMET conferences as a means to strengthen and improve the teacher education component within our department. Upon participation in the conferences, we were introduced to many resources

for better aligning our newly-created course with professional standards of best practice and pedagogical theory.

Framework and Rationale for Designing the Course

Recent research has shown that teacher knowledge is not monolithic, but is a large integrated dynamic system consisting of multiple components (Fennema and Franke, 1992). Teacher educators are recognizing that effective teachers possess several types of knowledge and are exploring the questions relating to what kinds of knowledge teachers actually need to successfully teach (Shulman, 1987, 1992, Peterson, 1988), comparisons of teacher knowledge across cultures (Ma, 1999; Stigler and Hiebert, 1999), and the effects of teachers' knowledge on student achievement (Hill, Rowan, and Ball, 2005). Specifically in math education, Battista (2006) talks about 1) knowledge of mathematics which includes general mathematics knowledge and mathematics knowledge particularized for teaching (pedagogical content knowledge), 2) knowledge of how students learn mathematics, and 3) knowledge of teaching mathematics. Although many have speculated on the components of teacher knowledge, those components most often discussed are content knowledge, knowledge of learning, and pedagogical knowledge.

In proposing a model for developing mathematics teachers' knowledge in context, Fennema and Franke (1992) discuss the components of knowledge of the content of mathematics, knowledge of pedagogy, knowledge of students' cognitions, and teachers' beliefs. They, as well as other researchers (Orton, 1989; Shulman, 1987) conclude that some components of teacher knowledge evolve through teaching. Thus, teaching is a process in which new knowledge is generated. Shulman (1987) states:

> The key to distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students (p. 15).

This assertion is consistent with the work of other educators who have emphasized the role of "reflective teaching" in the process of developing teacher knowledge via field experiences (Posner, 1989; Grant and Zeichner, 1984). As a result of her studies, Moore (2003) has argued for increased practica experiences prior to full-time student teaching in which all stakeholders routinely reflect on and examine concrete classroom situations as a means to integrate theory with practice and vice versa. Thus, field experiences can provide opportunities for beginning teachers to explore relationships between what is taught, how it is taught, and what students learn.

Many teacher education programs are based on the assumption that pre-service teachers must be well prepared with 1) the necessary content knowledge, 2) the appropriate pedagogical knowledge, and 3) the sufficient amount of field experience necessary to teach effectively. However, the traditional model has been to have pre-service teachers work through these three modes of learning in a basically linear format, where teacher candidates usually take a series of content courses first, then a methods course or courses, and then a subsequent (or accompanying) field experience component. While teacher candidates may have limited field experiences earlier, they often do not have regular and

systematic opportunities to develop and teach lessons, and reflect on their instructional practice, until student teaching.

In light of these research results on teacher knowledge, current recommended best practices, and our own previous successful experiences with an integrated content/methods course and accompanying field experience for pre-service elementary teachers (Benbow, 1993, 1996), we wanted to utilize these same principles in the design of our new middle grades math course.

Development of a New Course

Because the PMET conferences provided ample opportunities to examine a variety of standards and curricula for middle grades math, we used the following documents as a guide for designing our own middle grades program:

- MET standards for Middle Grades Teachers (CBMS, AMA, MAA, 2001),
- Principles and Standards for School Mathematics (NCTM, 2000),
- PRAXIS Middle School Mathematics examination,
- Indiana Middle School Mathematics content standards (for students), and
- NCATE Mathematics Teacher Education standards.

The goal behind our analyses of these standards was primarily to determine the areas of greatest need (gaps) in the content of our current courses, and then to make recommendations for revisions to those courses and to determine the content for a new course specifically designed for pre-service middle grades teachers.

Developing a new middle-school course was an exciting and challenging opportunity to put into practice what we had been learning from PMET conferences and from other sources. Several common content strands emerged from the documents listed above and we chose four around which to structure the course: Algebra, Rational Numbers, Geometry and Measurement, and Data Analysis and Probability. We also chose to emphasize several key elements crucial to pedagogy as well as content: conceptual understanding, communication, verification, multiple representations and solutions of problems, and analysis of students' work as a means to better understand students' thinking and learning. Once the outline for the new course was envisioned (MAT 280), we added it to the required menu for all candidates seeking a middle school math licensure.

Perhaps the greatest contribution of the PMET conferences was the opportunity for participants to discover new available resources for the preparation of mathematics teachers. One of the most important resources we found at the conference was the wonderful new textbook by Rheta Rubenstein, Charlene Beckmann, and Denisse Thompson: Teaching and Learning Middle Grades Mathematics, published by Key College Publishing, 2004. After reviewing this text and comparing it to other possible resources, we concluded that this book would be a great fit for implementing our vision of an integrated content/methods course.

We began by creating the following general course description for our new course, Mathematics in the Junior High/Middle School (MAT 280):

> An integrated content-methods course for middle school and introductory high school preparation. This course is a study of the mathematics taught in the middle grades (grades 6-8 with extensions to grades 5 and 9) with special reference to teaching aids, laboratory methods, and pedagogy, including classroom use of manipulatives and technology.

We also formulated five main objectives for the course and these are listed later in the "Assessment of the Course" section of this paper. These objectives reflect our intention to engage students in activities that would impact their teacher knowledge in multiple domains.

Course requirements included two exams, weekly quizzes, homework exercises, weekly field experiences, and a resource file project. Homework exercises consisted of a variety of mathematical problems, readings, pedagogical activities, and short reflective writing assignments.

Implementation of the Course

The course met three days per week for 50 minutes in our mathematics lab, and class sessions were spent involving students in a variety of activities and discussions, including discussions of their field experiences. Activities were designed to both help pre-service teachers gain a deeper conceptual knowledge of middle-grades topics as well as formulate ideas for how to help students learn appropriate content. One pre-service teacher commented, "It was very helpful to actually do the activities that the students would do." Then the pre-service teachers had the opportunity to try some of the activities they were learning in class in the field experience to see how they really worked with middle-grades students. Another commented, "It was a great way to practice what we were learning in class and to gain more experience."

For example, we studied the seven cooperative learning strategies presented in our textbook (Think-Pair-Share, Round Table, Co-op, Learning Stations, Random Reporting, Pairs Check, and Jigsaw) by having the pre-service teachers involved in an activity that fit each strategy. While they were experiencing various cooperative learning strategies in class, they were also thinking more deeply about patterns, proof, solving equations, linear and quadratic relationships, functions, and other algebraic topics. This combination of simultaneously studying the relevant mathematics content and appropriate teaching methods proved to be a very effective approach for helping the candidates develop and connect various types of knowledge.

Each week pre-service teachers also met in groups of three to plan a lesson, and the lesson was often based on topics we had been studying in the course. Then, as a team they taught their lesson in a local school classroom and wrote a reflection paper on the effectiveness of the lesson and what they had learned. We organized the teaching groups so that there was a mix of males and females, older and younger students, more experienced and less experienced students (the class included freshmen, sophomores,

juniors, and seniors). One student commented, "Working in groups of three was another thing that I enjoyed about this experience. I would have felt very overwhelmed if I had needed to write lesson plans and teach all by myself."

During class time, we also did a variety of activities to help students experience multiple representations of concepts, particularly with rational numbers (candy bars, fraction strips or bars, pattern blocks, Cuisenaire Rods, tangrams, decimal squares, Base-10 blocks, etc.) For example, from pattern blocks, we used two hexagons to represent the whole and then represented the trapezoid as one fourth, the rhombus as one sixth, and the triangle as one twelfth. A candy bar divided into 12 sections was also used for similar representations. Several of the pre-service teachers then tried similar activities with their students at the local school.

We also did a variety of activities to help students go beyond simple procedural knowledge of a middle grades topic to more conceptual knowledge, so they could understand the fundamental concepts more deeply. For example, we studied ways to represent 2/3 x 3/4 with diagrams, why the rule for multiplying numerators and denominators works, and how to write word problems that model given symbolic expressions.

The structure of the field experience was also a key component of the Middle School math course. Students were assigned to a team of 3 people (there was one group of 4) to work cooperatively in planning and teaching each week's lesson. Students arranged time each week to plan, discuss, and critique their math lessons. The middle-grades classroom teachers were already well-acquainted with the goals and structure of the field experience, having previously participated in the similar program for pre-service elementary teachers. One of us (Mark) visited the school and observed the groups teaching on several occasions. Periodically, time was allocated during the MAT 280 class to discuss issues and questions that arose from students' field experiences. Each course member submitted a weekly reflective report in which they described and analyzed (by using a series of 8 given questions) the instruction and learning results for their lesson. At the end of the semester, we provided each supervising classroom teacher with an evaluation form to complete for each of the university students assigned to their classroom. Overall, we would characterize the field experience for the MAT 280 students as being highly-structured, challenging yet supportive, and possessing a significant emphasis on student "reflection."

Assessment of the Course

At the conclusion of the spring 2006 semester, we asked the pre-service teachers for feedback on the effectiveness of the course. Responses to whether the course objectives had been met were provided by students using a Likert scale, in which 1 represented Strongly Disagree and 5 represented Strongly Agree. Nearly all 16 students who completed the assessment either agreed (A) or strongly agreed (SA) that we had met the five course objectives, as shown in the table below. We have included a few short comments from the students as well.

1. To help prospective teachers develop an adult-level understanding and conceptual knowledge of the mathematics content taught in the middle grades, particularly algebra, rational numbers, geometry and measurement, and data analysis and probability. Average: 4.5

This is the methods class I have been waiting for!

It was very helpful to actually do the activities that the students would do.

Yes, it was done very well. I am disappointed there aren't classes like this for the high school level.

2. To involve prospective teachers in mathematical activities that will equip them with the skills and resources needed to effectively teach mathematics to a wide range of diverse learners, including experiences with a variety of manipulative materials. Average: 4.6

Showing how to use manipulatives effectively is great because mostly we just hear about them.

I really liked that we used the manipulatives and did not just talk about them.

Our use of multiple manipulatives and activities introduced numerous methods of teaching various concepts.

3. To give prospective teachers opportunities to experience working with middle-grades students so they can appreciate the joys and challenges of working with these students. Average: 4.6

I loved the field experience.

Being able to teach has been a huge step for learning about teaching.

The lab was frustrating to me at times because of the time it took and trying to do it in a group. However, I did learn some valuable lessons from it.

4. To provide opportunities to develop, analyze, reflect on, and teach mathematics lessons appropriate for middle-grades students. Average: 4.6

TEMP (lab) reports were a great way to encourage reflection.

We had so much discussion. So I was always analyzing and thinking. This was great!

Yes, because I never really wrote a lesson until this class.

5. To involve prospective teachers in reflection and discussion of current issues in mathematics education so they can further develop their philosophy of teaching mathematics, particularly as it relates to active learning, fun, technology, connections, communication, and teaching from a problem-solving perspective. Average: 4.3

This class has really got me thinking about my overall teaching philosophy a lot.

We could have done more of this.

Class discussions of lessons from previous weeks were helpful to see what the rest of the class was doing.

General comments from several students indicate that they grew in their thinking and their teaching abilities throughout the course. One said, "I really enjoyed the group activities. I am learning to more appreciate the non-traditional way of teaching math." Another commented, "It is important to grow and mature, but I do not think it is possible without reflecting on the past. This class has really shown me a lot about who I am as a math teacher and as a person." Another said, "I learned so much about proper organization and thorough planning because of these situations; it is only by experience I can learn some of teaching's hardest lessons."

Conclusions

Based on our formal and informal results, we believe the convergence of mathematics content, mathematics methods, and a carefully-designed field experience provides an ideal environment in which to prepare university students to teach mathematics. Traditionally, most teacher education programs have offered mathematics content courses, mathematics methods courses, and then field experiences, in a somewhat linear model (see figure 1). This might involve a series of mathematics courses, followed by a general or content-specific methods course(s), followed by student teaching. In some cases, a field experience component is attached to the methods course. We believe, however, an integrated model which includes the concurrence of all three components (see figure 2), can be more effective in promoting professional growth, because pre-service teachers, as learners, can readily examine their actions, as teachers, within the context of a realistic classroom environment. The university mathematics course activities take on more meaning to the pre-service teachers when they can immediately implement those same activities with "real" students and assess the results. As the preservice teachers have opportunities to learn "deeper" mathematics, to teach mathematics in a classroom setting, and then participate in public and private reflection, they construct a multi-faceted knowledge base for teaching mathematics. As a result of this study, we strongly believe our pre-service teachers acquired more integrated "mathematical teacher knowledge" (of content, of learning, and of teaching) by means of the teacher education structure modeled in figure 2.

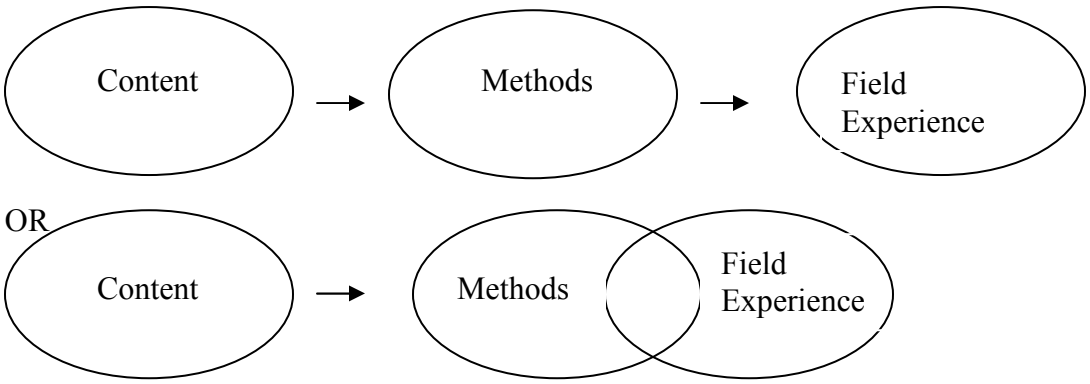

Figure 1: Traditional Teacher Education Structure (primarily linear)

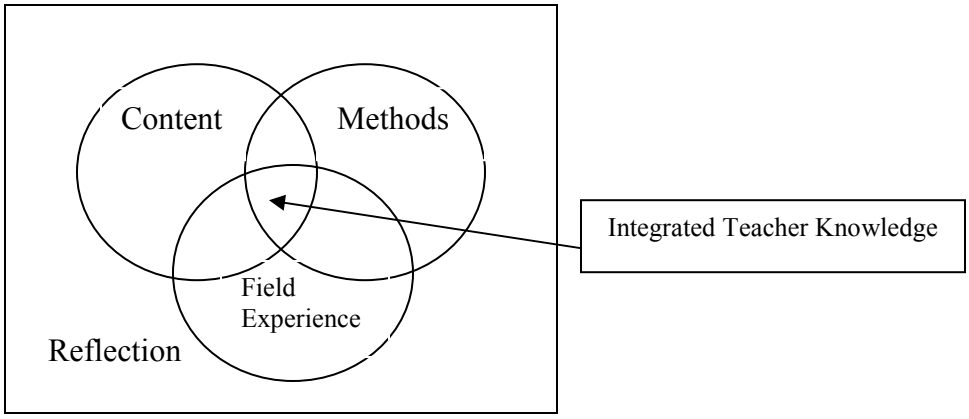

Figure 2: The Power of Convergence (concurrent experiences)

The equation suggested by Posner (1989), **Experience + Reflection = Growth,** is also an important underpinning to our model of teacher education. Reflective thinking means "turning a subject over in the mind and giving it serious and consecutive consideration" (Dewey, 1933). In the model represented by figure 2, which we have tried to implement, an emphasis on reflective thought (using such tools as reports, questions, discussions, etc.), encourages the pre-service teachers to carefully examine all of their experiences and beliefs, to make connections between educational theory and practice, and to form relationships among the various types of knowledge they are constructing.

While our example comes from a study conducted in a specific context utilizing a middle school mathematics course and practicum, we believe the essential results might as easily emerge from similarly structured environments at other levels and in other disciplines.

While some university programs may not be able to offer courses that combine content, methods, and field experience all at once, our experience suggests that whenever possible these three components should be integrated in some way. When taught separately, pre-service teachers may have more difficulty finding meaning in the individual components, but when combined, they can more readily apply what they are learning to other authentic teaching and learning situations. Just as we learn mathematics by 'doing,' we believe our pre-service teachers learn best by 'teaching' in the context of learning and reflecting. As one of them commented at the end of the course, "I saw myself grow as a teacher each time I taught, which was very encouraging. It is very true that the best way to learn is to just get out there and do it."

REFERENCES

Battista, M. "What Kinds of Mathematics-Related Knowledge Do Mathematics Teachers Need?" Paper presented at the PMET conference: Kent State, June, 2006.

Benbow, R. (1993). Tracing Mathematical Beliefs of Preservice Teachers Through Integrated Content-Methods Courses. ERIC document ED388638, 1993.

Benbow, R. (1996). The Relationship Between Preservice Elementary Teachers' Mathematics Beliefs and An Early Teaching Experience. Unpublished doctoral dissertation, Indiana University, 1996.

Conference Board of the Mathematical Sciences. The Mathematical Education of Teachers. Providence RI and Washington DC: American Mathematical Society and Mathematical Association of America, 2001.

Dewey, J. How We Think: A Restatement of the Relation of Reflective Thinking to the Educative Process. D.C. Heath, 1933, p. 3.

Fennema, E., and M. Franke. "Teachers' Knowledge and Its Impact", in D. Grouws (ed) Handbook of Research on Mathematics Teaching and Learning, MacMillan, 1992.

Hill, H., B. Rowan, and D.L. Ball. "Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement." *American Educational Research Journal*, v. 42, no. 2, Summer 2005.

Ma, L. Knowing and Teaching Elementary Mathematics. L. Erlbaum Associates, 1999.

Moore, R. "Reexamining the Field Experiences of Preservice Teachers." *Journal of Teacher Eduction*, 54(1), 31-42, 2003.

National Council of Teachers of Mathematics. Principles and Standards for School Mathematics, NCTM, Reston, VA, 2000.

Peterson, P.L. "Teachers' and Students' Cognitional Knowledge for Classroom Teaching and Learning." *Educational Researcher*, 17(5), 5-14, 1988.
Posner, G. Field Experience: Methods of Reflective Teaching. Longman, 1989

Rubenstein, R., C. Beckmann, and D. Thompson. Teaching and Learning Middle Grades Mathematics. Key College Publishing, 2004.

Shulman, L. "Knowledge and Teaching: Foundations of the New Reform." *Harvard Educational Review*, 57(1), 1-22, 1987.

Shulman, L. "Ways of Seeing, Ways of Knowing, Ways of Teaching, Ways of Learning about Teaching." *Journal of Curriculum Studies*, 28, 393-396, September-October, 1992.

Stigler, J. and J. Hiebert. The Teaching Gap. The Free Press, 1999.