

# Teacher Preparation in a Research Mathematics Department

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A complete version of this paper is available at [www.math.ucla.edu/~twg](http://www.math.ucla.edu/~twg) .

## Summary

We describe how the UCLA Mathematics Department has modified its curriculum in order to strengthen its undergraduate program for the training of future teachers and teacher leaders. The main modifications are the introduction and development of a yearlong “applied capstone course” for seniors interested in teaching, and the recent introduction of a Mathematics for Teaching major. These curricular innovations mesh well with recent recommendations of professional organizations for the mathematical preparation of teachers, though they are attuned to the special circumstances of a department whose primary focus is research and the training of graduate students.

## Introduction

In the past decade, several reports have appeared with recommendations for the undergraduate preparation of mathematics teachers. A principal recommendation of both the 2001 report of the Conference Board of Mathematical Sciences [CBMS] and the 2004 report of the MAA Committee on the Undergraduate Program in Mathematics [CUPM] is for a senior level experience that connects the traditional undergraduate mathematics coursework to the high school mathematics curriculum. Certification of the mathematical competency of secondary teachers in California requires observations of and reflections on teaching in local schools. This type of coursework and fieldwork involves issues that are outside the expertise of the ordinary research mathematician. Appropriate treatment of these issues requires the participation of mathematics educators whose expertise lies outside the domain of the traditional research mathematics department. How then should such a department mount a mathematics program for undergraduates interested in a career in teaching? What should be the teacher-preparation goals of a mathematics department whose priorities are research and the training of graduate students, and how should it go about attaining those goals?

In this paper, we describe how an undergraduate program for the preparation of mathematics teachers evolved over the past several years at UCLA. The primary goal of this program is to build a solid foundation for the formation of future *teacher leaders*. We believe, along with David Sánchez [Sá, p. 82], that “undergraduate training will be a cornerstone” of efforts to produce teachers “who will become master teachers ... exemplifying the best in content mastery and pedagogy.”

## The Case for Modifications in our Undergraduate Preparation of Mathematics Teachers

High school mathematics departments across the country mount the traditional algebra, geometry, trigonometry, analytic geometry, and calculus courses as well as statistics, computer science, discrete math, and more recently, even multivariable calculus courses. Second year algebra and analytic geometry courses incorporate topics from linear algebra, complex numbers, plane curves and probability. Secondary mathematics teachers need to understand this breadth of content *from beneath and above* in order to anticipate students' ways of understanding particular mathematical ideas, to anticipate the topics that are of particular importance in students' mathematics training, to field sophisticated questions from mathematically curious students, and to choose representations for mathematical concepts that build accurate foundations for students' future learning. Teachers also need to *understand an expert's ways of thinking* about mathematics in order to teach students how mathematicians pose questions, solve problems, prove statements, and communicate mathematics, and to sequence instruction in a mathematically sound progression. Secondary mathematics teachers need a *deep and connected understanding of the mathematics they are to teach* in order to provide sound justification of statements in the classroom, to develop multiple representations of mathematics concepts, to connect the appearances of a mathematics concept in different fields of mathematics and other fields of study, and to recognize the "mathematical capital" in students' nontraditional solutions or ways of understanding mathematics.

Secondary mathematics teachers also need to understand what current research says about *how students learn mathematics* and *best practices in mathematics instruction, assessment, and curriculum*. These practices include choosing developmentally appropriate yet mathematically sound representations, accurately assessing students' mathematical knowledge, appropriately using technology for instruction, addressing the literacy issues unique to mathematics and the needs of all students in today's diverse mathematics classrooms.

To illustrate a few of these demands, consider the simple arithmetic  $9^2 - 3^2 = 72$ . In addition to the computational knowledge a teacher uses to perform the arithmetic, consider the additional knowledge the teacher uses in determining the error of a student who obtains  $9^2 - 3^2 = 36$  (a common student error), in identifying the way in which the student understands the mathematics involved, and in identifying an appropriate model or representation that will address the student's misunderstanding. This suite of knowledge is both mathematical and pedagogical in nature [BHB, p. 17-21], is not addressed in the traditional courses for the mathematics major, and requires instruction from both a mathematician and a math educator.

### The Applied Capstone Course

As illustrated above, and further delineated by Deborah Ball and Hyman Bass in [BB, p. 3-14], the mathematical and pedagogical knowledge required for teaching overlaps. This understanding has informed work that began some five years ago on converting an existing math methods course into a hybrid course integrating mathematics and pedagogical topics. The course is co-taught by a mathematician and a math educator

with extensive secondary teaching experience. The course aims to connect undergraduate coursework to the secondary mathematics curriculum. Such connections, if made in upper division coursework, are most likely made in passing. The course aims to deepen student understanding of the mathematics they will teach, as many mathematics majors have never considered or been asked to consider the justification of many mathematics statements at the secondary level. (See [Wu, p. 7].) The course aims to teach new mathematics content using various research-based instructional strategies, as students develop a more complete understanding of an instructional strategy when they experience learning under its employment.

The course also emphasizes problem solving, student presentation of solutions, and multiple solutions. This provides an opportunity for students to practice accurate verbal presentation of mathematics and to learn to recognize the validity of multiple approaches to the solution of a problem. The course aims to teach a variety of research based instructional strategies, skill with the technology and software used in schools [CUPM, 56] and skill with various models for secondary mathematics topics such as Lenart spheres or algebra tiles [CFP, 150]. The course includes readings of current math education research as well as state and national content standards for the teaching of secondary mathematics. It also requires observation in local secondary schools.

One vehicle the course often uses towards some of these aims is the consideration of an actual question from a secondary student, such as “Why is the product of two negative integers a positive integer?” After guided mathematical activity in a group setting, students develop a response appropriate at the undergraduate level and a response appropriate at the secondary level. Students then participate in and evaluate instructional strategies for teaching related content to secondary students. For more detail see the web paper [CGR], which provides a snapshot of the status of the course in 2004.

#### The Mathematics for Teaching Major

Recently, the department took a significant step in mounting a Mathematics for Teaching major, which effectively serves to ensure the subject matter competency of future mathematics teachers. Its philosophy is not simply to provide for the mathematical education of future teachers, but further to provide a solid foundation for future teacher *leaders*. In the first years in the classroom, teachers must sort out logistical and discipline issues. We do not focus strongly on these issues. Our goal is that, once these issues are under control, our graduates will be in a strong position to provide leadership for their department and for the profession.

Preparation for the major includes the usual two-year calculus sequence together with science and programming courses. A course in mechanics is required in response to the frequent appearance of mechanics in applications throughout the high school curriculum. A programming course is required in response to the incorporation of computer programming courses in the offerings of secondary mathematics departments. Thirteen upper division courses are required for the major. Three of these constitute the applied capstone course described above. A math history course is required in response to the appearance of math history on the CSET exam, and to the desire for future teachers to

present mathematics in its human context and to realize the struggle of civilizations to arrive at modern concepts of mathematics. Two courses form a probability and statistics sequence, the latter of which is offered by the Statistics Department. The requirement of a course in statistics responds to the recent incorporation of statistics topics and courses into the high school curriculum. The remaining seven courses are “standard” upper division mathematics courses, selected from targeted menus, and including linear algebra, basic analysis, a further course in algebra, a further course in analysis, a geometry course, a course in applied mathematics, and an elective course. As a result, the Mathematics of Teaching major is more prescriptive than the traditional mathematics major, as it is more akin to a professional degree program, like engineering. Additionally, the Mathematics of Teaching major requires a portfolio project, which is incorporated into the third quarter of the applied capstone course, and thirty hours of observation in local middle and high school classrooms.

*How does the Mathematics for Teaching major compare with other mathematics majors?*

The pure math major requires 12 quarters of upper division math courses. The Mathematics of Teaching major requires 9 standard upper division math courses plus another four courses that incorporate about a course and a half of standard mathematics material. With a total of 13 courses and supplementary requirements, the Mathematics of Teaching major carries a more challenging workload than the standard math major. The courses in the Mathematics for Teaching major do not compromise on rigor. According to Sanchez, future mathematics teachers should “have the best possible training and strong, conceptual understanding of mathematics.” [Sá, 82]. As H. Wu argues in “The Education of Mathematics Teachers”, future teachers need different, but not watered down coursework [Wu, 8].

*How will we attract students to a major that requires more effort than the standard major?* First, we will assist students in the Mathematics for Teaching major in passing the CCTC requirements for certification of a teacher’s mathematical competency. If the major is not certified by the CCTC we will prepare students to take the CSET exam and we will cover their exam expenses. Second, we will use support funds to offer stipends to students in the major.

#### Fitting into a Research Mathematics Department

There are a number of issues and challenges that we have confronted in mounting a program for the undergraduate preparation of mathematics teachers in the Mathematics Department. The basic difficulty is the paucity of resources to the department as a whole. In an environment where pre-calculus courses have been cut back, and honors calculus courses have been eliminated, it is difficult to argue for resources for innovative teacher preparation courses.

This squeeze on resources affects the support for faculty involved in the math education aspect of the department. Administration of the math education activities of the Mathematics Department is time intensive, if well done. While providing moral support and encouragement, the department has never offered monied resources for program

administration or course development. Currently, the department is aware of the problem and considering options for increasing support.

One challenge to generating support for teacher training is that math education issues are not well understood by many department members. Often top-flight research mathematicians, to whom secondary mathematics came very easily, have difficulty understanding that the challenges in teaching secondary students exceed knowing the underlying mathematics. Further, “the assumption that the traditional curriculum for a mathematics major is adequate preparation for students preparing to teach secondary school is simply incorrect [CUPM, p.53]

Our strategies for dealing with this situation are multifold. One is to keep faculty members posted on developments in the departmental math education operation. They are inclined to support an operation they feel is well run, even if they might not want to participate themselves. Occasionally one can take advantage of opportunities to provide information to colleagues about math education issues, such as, the fascinating research question, “What mathematics should teachers know to teach mathematics and how should they know it?” (See [BB].)

Another strategy is to persuade the department of the usefulness of teacher preparation activities for the purposes of advertising for the department. A 1999 American Mathematical Society Report makes the case that a strong commitment to issues of societal concern should elevate the status of the department within the university and generate support for its other goals ([E, p. 151-155]: See also [CBMS, p.9].) To this end, the department should communicate the success of teacher preparation activities to the rest of the university and to the community beyond. In fact, a departmental newsletter will soon appear with extensive coverage of these activities.

Still another strategy is to make changes in the syllabi of certain of the math courses in order to make them more relevant to future schoolteachers. This is related to the first of the two principal recommendations of the CBMS report [CBMS, 39], and it can be done at no cost. The changes might simply mean incorporating one or two lectures into the syllabi on topics in the textbook that are more relevant to the state’s mathematical certification exam for teachers. Changes can also amount to a more detailed set of specifications for an entire course. For instance, the math history course description was rewritten to specify coverage of the four topic areas emphasized on the CSET exam: the development of the place value number system, the idea of an axiom system and proof, the evolution of algebraic thinking, and the foundations of calculus. Each instructor has ideas about how to give the course, but this specificity in the course description provides a useful checklist.

When seeking monied resources, we have attempted insofar as possible to avoid a direct competition at ground level with other constituencies in the department. The department dynamics inevitably favor the other constituencies. One casualty of tight resources is the Visiting High School Teacher Program, which was suspended several years ago. Such a program has a difficult time surviving without direct support from a higher administrative

source or an endowed fund specifically for the position. We are actively seeking such funds.

### Concluding Comments

Of the two major curriculum changes we have described, the applied capstone course has produced some record of success in the form of favorable instructor evaluations and student comments. Students say, "I thought [it] was sad that math majors could not explain why math works. I feel I learned a lot about where the mathematics comes from and the class helped me remember some concepts I forgot." They say develop a desire for "students to believe that they can understand and derive mathematical truths." They say they learn how "students can construct an understanding of mathematics through using physical models, student prior knowledge, technology, etc." Finally, they state that "understanding the content and creating good pedagogy are heavily dependent upon each other."

We hope that the Mathematics for Teaching major will attract a reasonable number of students. We will evaluate its success over the next several years. We hope that our current program might serve as a model for what a research mathematics department can do for the undergraduate preparation of future mathematics teachers and teacher leaders.

### Bibliography

[BB] D. Ball, H. Bass. *Toward a practice-based theory of mathematical knowledge for teaching*. In B. Davis, E. Simmt (Eds.), *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group*.

[BHB] D. Ball, H. Hill, H. Bass. "Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide?". *American Educator*, 2005.

[CBMS] Conference Board of Mathematical Sciences. *The Mathematical Education of Teachers*. , Issues in Mathematics Education, Vol. 11, American Mathematical Society, 2001.

[CFP] *Curriculum Foundation Project: Views of the Partner Disciplines*.  
[www.maa.org/cupm/crafty](http://www.maa.org/cupm/crafty)

[CGR] H. Calahan, T. Gamelin, and B. Rothschild. *Symbiosis of mathematician and math educator in teaching math teaching*. [www.math.ucla.edu/~twg](http://www.math.ucla.edu/~twg) , 2005.

[E] J. Ewing (Ed.). *Towards Excellence Leading a Mathematics Department in the 21<sup>st</sup> Century*. American Mathematical Society, 1999.

[Ha] G. Harel. "What is Mathematics? A Pedagogical Answer to a Philosophical Question." In R. B. Gold & R. Simons (Eds.), *Current Issues in the Philosophy of Mathematics From the Perspective of Mathematicians*, Mathematical American Association, In Press.

[MSEB] Mathematical Sciences Education Board. *Knowing and Learning Mathematics for Teaching*. National Academies Press, 2001.

[Sá] D. Sánchez. "The Mathematics Major Overview." In *CUPM Discussion Papers on Mathematics and the Mathematical Sciences in 2010: What Should Students Know?*, MAA Reports, 2001.

[SH] J. Stigler and J. Heibert. *The Teaching Gap*. The Free Press, 1999.

[Wu] "On the Education of Mathematics Teachers". [www.math.berkeley.edu/~wu](http://www.math.berkeley.edu/~wu) , 1997.

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