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Divisibility Rules Display
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Elementary Education
Content Standard: Number and Operations
Process Standard: Communication, Representation, Connections
Overhead

Divisibility Rules Display

Terminology – Mathematics – Patterns – and Art

This is a paper presentation of a project assignment for Elementary Teachers Preparation. The topic is Divisibility Rules.

I have found that the elementary teacher candidate frequently does not remember whatever rules for divisibility that may have been part of that individual's elementary education. In addition, the concept of divisibility is not well understood.

In an effort to have the student practice the rules enough the fully understand and memorize those for 2, 3, 4, 5, 6, 8, 9, 10, and 11, I have a project that is meant to produce an excel file that can be enlarged and laminated for use as a teaching tool wall hanging in the elementary classroom.

On this divisibility chart we display the short and succinct but carefully written word statements for divisibility by each of the numbers listed above. These statements must be carefully worded and punctuated for clarity. It is not an option to state that a number is divisible by three if the sum of its digits is divisible by three. This would not be acceptable in an English definition and hence is not acceptable in a Math definition.

The tests are displayed by step wise mathematical calculations for each number on the chart. Numbers and digits of interest are highlighted or underlined to help the student have a visual path between the written statements for the rules and the corresponding mathematical work to complete the rule test. Results for the test are color-coded.

The chart serves the prospective teacher as a requirement that forces consideration of the meaning of the rule statements and practice in making use of those statements. It serves as a tutor for accuracy in sentence structure and punctuation as well as in step-by-step mathematical solutions. The coloration problem helps to gain understanding of patterns and to consider the artistic side of presentation of material. If the chart is visually attractive, the student is more interested in learning what it means.

Divisibility as a topic in elementary education is presented in the 3 – 5 grade level block. From the viewpoint of addressing objectives in the NCTM Standards, grade level 3-5, we can see from the list below that the study of divisibility presented by the finished wall hanging Divisibility Chart will encompass understanding of the base number system as the student addresses rules for 2, 5, and 10 dealing with the units' digit. In the rules for 4 and 8, which are often misunderstood as written in many texts, by stating our interest in the number represented by the tens' and units' digits and for divisibility by 8, the number represented by the hundreds', tens', and units' digits.

Although the given test numbers are not written in factored form on the chart, the mathematical step-by-step work contains many multiple representations of numbers. We represent the sum of digits in horizontal sum form with an equal sign and the sum in base ten. We write an improper fraction equal to its quotient and remainder form. We write the absolute value of a difference equal to the appropriate number in base ten.

We use colors to group or classify subsets of the set of test numbers divisible by the same factor. We try to use color to point out the common factor properties when testing for divisibility by 6 and coloration of the column for divisibility by 10 using the colors for divisibility by 2 and for divisibility by 5.
The computation steps shown on the chart are not written in algorithmic orientation as if they were to be completed with pencil and paper. Thus, the presentation requires and implies the desire to compute mentally in this divisibility test work.
Repetition of the steps in completing tests produces a place to develop fluency with sums and differences and requires division by 4, 8, and 11.

Number and Operations Standard
Understand numbers, ways of representing numbers, relationships among numbers, and number systems - - Expectations:

- Understand the place-value structure of the base-ten number system
- Recognize equivalent representations for the same number and generate them by decomposing and composing numbers
- Describe classes of numbers according to their characteristics such as the nature of their factors

Compute fluently and make reasonable estimates

- Develop fluency with basic number combinations and use these combinations to compute mentally
- Develop fluency in adding and dividing whole numbers

My intention in this paper is to provide a lesson for the prospective Elementary or Middle School Educator. As such, the questions in the presentation are directed to an adult audience the members of which may or may not remember the rules but are not likely to have no prior work with the concept. If I were to use the material for an elementary class presentation, my methods would be very different. I want to think that the chart could be displayed piecemeal. That is, I would put up columns for 2, 5, and 10 with no teaching and hope students would ask curious questions to get us started on the divisibility concept. I would then add columns to the chart grouping them with common patterns and let class discussion drive the lessons. The presentation here is for Teacher training.
Each prospective teacher would be given a black and white copy of the Excel file Divisibility Chart. This chart has nine rows and ten columns. The label for the first row (entered in the upper left corner of the chart) is "Factor". Across the top row of the chart after this tile we see the numbers 2, 3, 4, 5, 6, 8, 9, 10, and 11. The second row on the chart has in the left most tile "Test number". The apparent title for the chart appears centered in the remaining row two columns: "Divisibility Rules and Tests for some factors". Reading down the remaining tiles of the first column we find: 746988, 81342, 15810, 4201012, 1001, 10001, 3944.

My question: We are going to make a large copy of this to place as a wall hanging in the classroom. What is it for?
The lesson from this point is quite dependent upon the responses, but in this way I can discern what the students already know about divisibility - - - what rules and whether they understand the basic concept:
I will be looking for:

In the second column we will answer the divisibility question for the test numbers and the factor 2, in the third column for the test numbers and the factor 3, in the fourth column for the test numbers and the factor 4, and so on.

I will ask how we might show the answers…and move to clarify that a wall chart having "yes" or "no" in each tile would be rather uninteresting and not very educational. So, I will need suggestions as to how I might communicate the answers to my test question without words.

I will be hoping for a suggestion to use color for this communication. We can then discuss how the color can also emphasize the chart meaning by coloring the top label tiles in a color and the tiles under

them when the test produces "yes" in the same color or a lighter shade of the same color. We will also discuss the use of colors in general. We will discuss colors that colorblind students find problematic to discern (most often red next to green and pastels for either). We will discuss colors that are too bright or active if words are written over them (neon colors). We will discuss color groupings that are not sufficiently different for effective communication and groupings that are just too uninteresting to look at. We will discuss that the image of the color on the screen when setting up the file may look different than that of the printed copy.

My next direction will be to work on the full entries for the top row of the chart. These entries are to be the stated rules. I will not accept the rules stated in my students' current text because I find rules like "A number is divisible by three if the sum of its digits is divisible by three." This definition is unsatisfactory in English and certainly no more useful in mathematics.

My suggestion: "A test number is divisible by the factor 3 if, and only if, the sum of its digits divided by three has remainder 0."

In my definition, I have used the "if, and only if" format necessary for these definitions as for many in mathematics. We can clarify the meaning of this usage with young students by examples such as: "If a toy is called a tricycle, it has three wheels." Is it then true that a three wheeled toy is a tricycle? The young student can probably provide examples to show that three wheeled "if, and only if" tricycle does not work. Is there a ready example to use in the classroom for a true if, and only if statement outside of mathematical examples? Help my Teachers in training, please!

I want the rules to clearly demonstrate the concept of divisibility (involving division with remainder 0) and I want them to be succinct and state mathematical procedures that must be performed to use the test.

My students are sent home to write the rules statements. When I return their attempts at writing the statements, I provide the following rule list by writing it on the board. I want them to write them by hand, and later to type them into the appropriate tile in the Excel file. I want them to experience the need to carefully edit their work for a wall hanging to appear in the classroom. Teachers need to understand that their written work and classroom decorations are a reflection of their professionalism.

- A number is divisible by 2 if, and only if, its units' digit is 0, 2, 4, 6, or 8.
- A number is divisible by 3 if, and only if, the sum of its digits divided by 3 has remainder 0.
- A number is divisible by 4 if, and only if, the number represented by its tens' and units' digits divided by 4 has remainder 0.
- A number is divisible by 5 if, and only if, its units' digit is 0 or 5.
- A number is divisible by 6 if, and only if, the number is divisible by both 2 and 3.
- A number is divisible by 8 if, and only if, the number represented by its hundreds', tens', and units' digits divided by 8 has remainder 0.
- A number is divisible by 9 if, and only if, the sum of its digits divided by 9 has remainder 0.
- A number is divisible by 10 if, and only if, its units' digit is 0.
- A number is divisible by 11 if, and only if, the absolute value of the difference of the sum of the first digit and every other digit and the sum of the skipped digits divided by 11 has remainder 0.
- We want the mathematical entries in the cells to clarify and explain the rules, if this is possible. We want the mathematical steps to be easy to see and methodical in their presentation in the cells of the chart.

In each tile of the column beneath a rule, we want to show the mathematical steps for determining whether the test number is divisible by that factor.

In the columns for 2, 5, and 10, the test is a visual one. So, we want to type the test number in the tile and highlight the units' digit to clarify which digit is to be examined. We then color or not color the cell as we can complete the test without any calculations.

In the columns for 3 and 9, we must start with the sum of the digits. We enter the test number in the tile and on the next line of the tile we enter the sum of the digits written as a horizontal (mental) calculation with equal sign and result. On a third line of the tile, we show the division of this sum by 3 (or 9, depending on the column) with equal sign and result in quotient with remainder form. (It is hoped that the repeating zeros from these calculations will remind the students of the basic remainder concept behind divisibility.)

For divisibility by 4 and 8, the test number is written in the tile and the appropriate digits highlighted. We then show the division on the next line equal to result in quotient with remainder form.

The factor six is determined with results from columns for 2 and 3. To communicate this test, we color the appropriate tile in the 6 column (this column should be made up of two columns to most readily get a nice coloration) with the results from the 2 and 3 columns.

The test for 11 is on the chart to show the students why we don't have a bigger chart. The rules get so complicated that it is easier to simply divide the test number by the factor and find out what the remainder is than to remember or use the test. For the 11 test, the test number needs entered with every other digit highlighted. Using the highlighted entries we write the sum of the first and every other digit. Then using non highlighted digits, we write the sum of these digits. We then write the absolute value symbol and the difference of the highlighted sum and the non-highlighted sum, closing absolute value symbol, equal sign, and result. Then on the next line we write the appropriate fraction with the equal sign and quotient and remainder form.

There is some problem solving for the Teachers in training in the highlighting step. The digits must be spaced apart to deal with them individually.
Questions from the Teachers in training arise when the numerator is smaller than the denominator in our division steps, so discussions about the meaning of the quotient and remainder form can be discussed.

The finished chart can then be used for discussion as to its completeness and questions one might pose for the elementary students once the chart was fully displayed. Also, one might question the Prospective Teachers about the completeness of the chart in terms of the selected test numbers. The two inquiries are very much related. I wonder about some below:

What is the least number divisible by all of the numbers on the chart?

Is there a number we tested for divisibility for which we never have a positive result? If so, can we provide a number that does pass this divisibility test? (Not the number itself, please!)

Check the columns to be sure that each has an uncolored tile. Describe this situation in words. If there is a column that does not have an uncolored tile, provide a number greater than 400 that would produce an uncolored tile in that column.

The rule for 6 is dependent on other rules. Do we have all possible ways that test can pass or fail shown on the chart? If not, provide a number greater than 400 that illustrates any missing possibility to fail the test for divisibility by 6.

I then assign my Prospective Teachers: "What questions might we ask the elementary students to answer by reading the finished chart?

Can my Prospective Teachers gain concepts and skills by completing this project?
Is the project too tedious for the Prospective Teacher?
Can I incorporate group learning in the presentation design to a greater degree?
Is there a way to guarantee each student complete the practice himself or herself?

I have a starter divisibility Excel file. Sheet 1 has no color. Sheets 2 & 3 have some color questions addressed. Contact me by email if you would like the Excel file. crumb@jaguar1.usouthal.edu