Developing Problem Solving Skills of Prospective Elementary Teachers

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While observing students' approaches to solving problems at a previous PMET workshop, I was not so subtly reminded of how different the process used by most students is from those employed by mathematicians. This is one case in particular: a group of students was trying to calculate the volume of a tower constructed of increasingly larger squares made out of unit blocks. The group came to the consensus that they had previously learned a formula for this type of problem, but since none of them could recall the formula, they could not do the problem.

Much research has been done to indicate that with typical mathematics instruction students learn to solve problems without a significant understanding of the underlying mathematical concepts and without being able to transfer their knowledge to situations outside of the context in which it was learned The students being observed in this case were unable to make use of ideas previously presented to them in school, and the students did not appear to know how to approach the given problem once this transfer of knowledge failed

Clearly, these students are an example of the many who view mathematics as a set of formulas to recall and match to the problem at hand. What seems to be missing is the critical component that mathematical problem solving is a process that originates from within oneself and makes use of computational tools along the way. The act of doing mathematics is a messy, exciting, non-linear process. Errors are made and corrected in the process of observing, looking for patterns, making conjectures, formulating findings symbolically and proving deductively. Students need mathematical experiences that help them develop thought processes similar to those belonging to mathematicians. This is especially important for prospective teachers who will impact their students in so many ways.

To address this, I have compiled ideas from PMET presentations and discussions into four focus areas to improve my instruction so that I provide my students with a strong foundation of understanding from which to develop stronger problem solving skills.

Viewing Presentation of Student Problem Solving Skills

Watching students live or on video serves two purposes. Primarily, it helps prospective teachers see that many young students creatively find a solution to a problem when they have not yet been exposed to the standard algorithm for solving that problem. This is an opportunity for them to experience students "doing mathematics" firsthand and for many it is their first opportunity to see mathematics as more than memorized formulas. Additionally, pre-service teachers become aware of a need to understand mathematics at a higher level in order to rationalize or object to alternative algorithms presented by students.

Exploring Topics that Span the Curriculum

Illustrating to students topics that stretch across grade levels helps them to see the inter-connectiveness of mathematics. Methods that were used to solve a problem at one level can be used to solve a problem at another level. For example, calculating the area of a rectangle learned in

elementary school can be used to understand operations with fractions and decimals, represent multiplication and factoring of polynomials, and is used in Calculus to find areas bounded by curves.

Introducing Intellectually Interesting Problems

The DNR system, presented by Guershon Harel at the 2005 SUNY, Oswego PMET workshop, offers a conceptual framework for improving instruction. Based on theories of Piaget and Vygotsky, the system views student learning as "a continuum of disequilibrium-equilibrium phases together with (a) the ways of understanding and ways of thinking that the learner utilizes or newly constructs during the various phases and (b) the cognitive, social, and affective stimuli that result from or instigate these phases." The three foundational instructional principals (DNR) of the system are the duality principle (developmental interdependency between what the student produces, "ways of understanding," and the character of their mental acts, "ways of thinking"), the necessity principle (students must have an intellectual need for what we intend to teach them) and repeated reasoning (reinforces desirable ways of understanding and ways of thinking). (Harel, In Press) Harel notes in our every day life and in school, emphasis is put on empirical proof. In order for students to develop the reasoning skills of mathematicians and to understand the nature of mathematics, they must be provided with intellectually interesting problems that provide opportunities for the learner to move beyond empirical justifications, and develop deductive ways of thinking.

I carefully select problems that will capture human nature's desire to figure things out and I use this opportunity to lead them through the process of problem solving. As an example, two sets of objects are on opposite ends of a line and must switch places. The challenge is to determine the least number of moves required to switch the two groups. This problem becomes overwhelming when tried in its entirety. Breaking the problem down into simpler smaller problems works well and is suggested. It is fun to try to get the sets switched, while at the same time data is being generated. Looking for a pattern leads to representation in a recursive form and an explicit form. This knowledge provides an opportunity to introduce ideas of mathematical induction. Problems such as these move beyond just identifying patterns to deduction.

Encouraging Logical Deduction

To assess where the learner is in relationship to a given problem, I have them complete my *Show Me What You Know/Show Me How You Know* worksheet. In the "*Show Me How You Know*" portion, students must provide logical justifications for what they know. ("Because I know the formula" does not qualify as a logical justification.) This encourages the student to analyze what they know and practice documenting it, while offering me an opportunity to assess the learner's understanding and provide necessary scaffolding.

Participation in PMET workshops and conferences has been instrumental in improving the education of my students in Mathematics for Elementary Educators classes. Improved courses have resulted in students with comprehensive views of problem solving that they will pass on to future generations.

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