Using Mathematical Modeling in the Methods Class

Ronald A. Ward
Western Washington University

In the publication *Before It's Too Late*, the National Commission on Mathematics and Science Teaching made it clear that "The preparation our students receive in mathematics and science is, in a word, unacceptable." Among other problems cited with K-12 teacher preparation in science and mathematics that might result in such an unacceptable state, the NRC's *Committee on Science and Mathematics Teacher Preparation* pointed to the failure of professional development programs for continuing teachers to "enhance teachers' content knowledge." Fortunately, one of the ways both of these problems can at least be addressed is through the use of mathematical modeling. As the NCTM's *Principles and Standards* states: "One of the most powerful uses of mathematics is the mathematical modeling of phenomena. Students at all levels should have opportunities to model a wide variety of phenomena mathematically in ways that are appropriate to their level."

Mathematical modeling is one of the richest forms of representation in mathematics. It requires students to work with and apply a variety of mathematical concepts, processes and relationships. An emphasis on mathematical modeling provides future teachers with an approach to teaching mathematics that supports the approach taken by many contemporary curricular materials for mathematics instruction. These teachers will be able to promote the processes of problem solving, reasoning, communicating, representing, and connecting mathematical concepts and principles using modeling as a medium. As John Dossey, former NCTM President, has pointed out: "Mathematical modeling is central to understanding the real world, while simultaneously developing worthwhile mathematics." In addition, it is my own belief that, for mathematics educators who prepare future teachers, modeling can also serve as a means for keeping current and updating their own knowledge of mathematics.

In my own integrated content/methods classes for prospective K-8 teachers, I first introduce math models as they appear in selected elementary math curricula so that it is easy for students to appreciate the power of models, even when they are neither very sophisticated nor complex. An illustration is provided below with a probability context taken from some of my previous work with the Comprehensive School Mathematics Program [CEMREL, St. Louis], entitled *What's Fair?*:

> Alice and Bruce are a sister and brother who share responsibility for doing the supper dishes every evening. One day, Bruce proposes the following game to Alice as a means of determining who will do the dishes: Alice will take three marbles – two red and one blue – and place them behind her back. Without looking, and after shaking them up in both hands, she will select exactly two of the marbles and, holding them in her closed fist, bring that hand around in front of her. Bruce must guess whether the marbles in Alice's fist are the same color or different colors. If he guesses correctly, Alice must do the dishes. If Bruce is wrong, he must do the dishes. Do you think the game is **"fair"**? Why or why not?
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> In an elementary classroom [say, fifth grade] how would you decide the answer to the fairness question? [Certainly one way would be to conduct an experiment with students in pairs, each pair conducting perhaps ten trials, and keeping track of how many times matching colors occurred and how many times different colors occurred.]
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> This is a good point at which to introduce a simple **model** for the problem Just draw three colored dots for the marbles and connect them pair wise with line segments or chords which may then be labeled "S" for same and "D" for different. It should be clear that different colors occur 2/3 of the time. Compare this theoretical probability with the earlier experimental probability.
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> Now invite students to discover some combination of two colors which, keeping all other rules the same, will produce a "fair" game. The most frequently suggested combo – two of each color – is <u>not</u> correct and yields some very *surprising* results when the corresponding model is examined. [There are actually infinitely many combos that <u>will</u> produce a fair game, and if students find a few of these, they may see a pattern which will generate all the rest.]

Once students understand what a mathematical model is, have seen some nice elementary examples and experienced firsthand the power such models provide, I try to give them an opportunity to <u>create</u> such a model themselves, by presenting them with a problem whose solution requires the creation of just such a model. At the MAA's *PREP*

Workshop on Math Modeling in Portland, Oregon, I suggested the following modification to a well known problem, entitled *Shunda's New Stand*:

> Shunda is a newspaper girl. Between 4 and 6 PM each day, she stands at the corner of Hamilton Street and Euler Avenue and sells newspapers to people passing by. Shunda buys newspapers from a dealer for 20 cents each and sells them to her customers for 40 cents each. Because the number of customers Shunda has varies from day to day, the dealer agrees to buy back the papers she doesn't sell, but for only 10 cents each. [e.g., if her supply of papers on a particular day is 22 and she sells only 10 papers, her **GAIN** is $2, her **LOSS** is $1.20, and her **PROFIT** is $0.80, if she is making a balance sheet for that day.]
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> On a day when she expects lots of customers [e.g., after a big game, or perhaps around Election Day], she might buy a much larger supply, say, 35 papers. On a slow news day, she might purchase a smaller number than 22. Shunda started her newspaper business as an apprentice, but her learning and training period will end before very long. After that, she will have to follow stricter rules, and the daily supply of newspapers she buys will need to remain **CONSTANT**: the same number every day.
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> Question: How can Shunda discover the **BEST** supply to buy from the dealer in order to maximize her profit when operating under the stricter rules?

Many of my students recognized that there was a need either for historical data or for creating hypothetical sales. Some actually did the latter, but there appeared to be little rhyme or reason to how the data were generated. In particular, no one mentioned using any kind of distribution. Some of my colleagues have suggested that a nice way to do this would be to use a Poisson distribution and use technology [e.g., *Minitab* or *Fathom*] to generate some sample data. But my prospective elementary teachers apparently have not studied enough mathematics to know about such an approach. Some were able to determine a break-even function [sales equal one-third of supply], but did not know what to do with it. Quite a few recommended finding the mean, median or mode of some collected data but did not seem to have reasons for what they wanted to do. Others mentioned using such devices as stem-and-leaf plots or box-and-whiskers diagrams based on sample data, but again did not have reasons. A few had such useful advice as "Shunda should purchase enough papers during her apprentice period so that she <u>never</u> sells out – so she can learn what the true demand is." One thought that it made sense to buy a relatively large number of papers, say 30. Then, find the break-even point [in this case, 10]. Then, once you've sold that many, start to discount so you don't have to return any papers. This was not really permitted by the parameters of the problem but did show some good thinking, I thought.

Because my initial open-ended question was not really resulting in models, I decided to give some additional information as a follow-up project:

> Here is what Shunda actually did. She decided to collect some **DATA** on which to base her decision. So, let's suppose that Shunda decides to keep a record of the daily **DEMAND** for newspapers during, say, an experimental 20-day period. Here's the list for that period of time: 17,23,31,8,15,17,22,27,15,17,22,26,5,35,29,31,17,23,25,35
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> Shunda will try to determine what would have been the best constant **SUPPLY** during that experimental period. For example, if she had purchased a constant supply of 35 papers, she could have met the demand every day and would have **SATISFIED** all of her customers every day. Is that the best solution? On the other hand, had she purchased a constant supply of just 5 papers, she would **NEVER** have had to **RETURN** any unsold. Perhaps that's the best solution? Gee, maybe she should just average 35 and 5? Average them all? Find the median? The mode? Help!
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> Question: How can Shunda use the experimental demand data to decide on the constant daily supply that would have produced the **MAXIMUM TOTAL PROFIT**?

Some students tried to apply their original suggestions to this set of data, but many did not. The first time I gave this extension, only a few students actually discovered the optimum number of papers that should have been purchased, and they did so only because they actually calculated all possible profits from utilizing all possible supply numbers. In subsequent trials, many more students were successful because they began to use spreadsheet technology. One student decided to throw out the two lowest days for being "uncharacteristically low." But, then, she reasoned that she must also throw out the two highest days as well. Working with the remaining data, she found the mean and standard deviation and decided that the number of papers she'd be most likely to sell would be within one standard deviation, or roughly between 17 and 27. Since Shunda will "make more money selling more papers, go with the highest number in that range: 27." Her guiding principles were that Shunda makes more money when she sells more papers and sells everything she buys. Therefore, she should buy "the <u>maximum</u> number she is <u>likely</u> to sell."

Some students either made <u>calculating</u> errors which led them to erroneous conclusions, or misunderstood what the "break-even" point was [they thought it was half rather than a third of the number of papers bought]. Others determined a supply number that would always give them a profit but forgot that they were seeking the <u>maximum</u> profit. Some did their calculations correctly yet reached conclusions that contradicted those calculations. A few did not know what to do with demand numbers that exceeded their chosen supply number, so even if they produced elaborate charts, the charts became meaningless.

I did have one student who tried a **demand staircase** approach to the problem. Because this was quite close to the approach taken by the Belgian mathematician Georges Papy when he was consulting with our Math and Science Group at CEMREL, I then shared with the students his complete solution. I also demonstrated this technique at the PMET Mini-Conference at Appalachian State [July, 2006] as well as the Annual Meeting of the Association of Mathematics Teacher Educators in San Diego [January, 2004]. Interested readers can find the complete solution in the *CSMP Library of Math Story-Workbooks*, available from McREL [Aurora, CO]. Mathematicians will, of course, recognize this as a classic type of problem in operations research [an optimization problem].

One final idea I tried was to have a colleague [Nat Miller] from the PMET *Workshop on Math Modeling* write up his suggested solution approach and then have my students attempt to carry it out. I thought it would be a good experience in "reading mathematics" for them, an opportunity to see a different approach to problem solving, and another chance to practice some different computational skills. Nat chose to take a <u>marginal profit</u> approach [from buying n rather than n-1 papers]. He managed to work in the idea of a distribution, a decreasing function, some graphing, and so forth. So, it proved to be a challenging but worthwhile mathematical experience for my students. I distributed copies of his approach as well as some of the students' solutions at the PMET Mini-Conference. Space restrictions preclude including that information here.

For those interested in pursuing the inclusion of math models within the methods class, I would especially recommend the work of Sharon McCrone at Illinois State University as well as the excellent volume: *Mathematics Methods and Modeling for Today's Mathematics Classroom*, by Dossey, Giordano, McCrone, Weir and COMAP [Brooks/Cole, 2002].

The nature of the methods class will need to change in order to accommodate the inclusion of mathematical modeling. Meshing methods with modeling experiences will require a rethinking of the roles of content, pedagogy, and clinical experiences. Serendipitously, however, mathematics educators will now have a natural built-in means and opportunity to improve and increase their own mathematics knowledge.