

## THE GENIUS OF EULER

Leonhard Euler is one of the greatest and most prolific scholars in history. Born in Basel, Switzerland in 1707, he became a towering figure in the scientific world of the European Enlightenment. This year, we celebrate the 300th anniversary of Euler's birth and his many contributions to mathematics and the sciences. During his lifetime, Euler wrote hundreds of works in mathematics, not only contributing to every branch known at that time, but also developing entirely new fields of research. In the sciences, Euler investigated a wide variety of topics, including mechanics, naval science, cartography, astronomy, acoustics, optics, and more.

“The Genius of Euler” will present a selection of Euler’s “greatest hits.” Among other results, we’ll examine his summation of the reciprocals of the squares, his proof that the series  $\sum_{p \text{ prime}} 1/p$  diverges, his discovery of hitherto unknown amicable numbers, his quadrilateral theorem, and his derivation of “Euler’s Identity.” In all cases, we shall consider the original arguments with the mathematical detail intact. Although just a tiny fragment of his output, this material should easily convince any participant that Euler deserves the high esteem in which he is held.

Each day participants will receive problems tied to the presentations. Some of these will reinforce Euler’s work, and some will extend it. Among the latter are such results as Ivan Niven’s elegant demonstration of the divergence of reciprocals of the primes or J. J. Sylvester’s exploration of odd perfect numbers.

The last few sessions will be devoted to participant presentations of results from the Eulerian canon. Participants will have access to Series I of Euler’s *Opera Omnia*, consisting of 29 volumes and (roughly) 14,000 pages of his contributions to pure mathematics (In any mathematician’s life, there should be at least one opportunity to graze through this extraordinary collection.) Participants, working in small groups, can use the *Opera Omnia* as the source of material for their presentations.

The workshop will run from Monday morning until Friday at noon. Each day will consist of two morning sessions separated by a break, with a third session after lunch – except for Wednesday afternoon, which will be free time to explore Washington, DC.

A tentative schedule of topics appears below.

## SCHEDULE

### Monday

1<sup>st</sup> Morning Session: Getting started. Brief Euler bio. A survey of mathematical concepts bearing Euler’s name (e.g., the Euler polynomial, the Euler brick) and a few of his discoveries that could – but do not – carry his name (e.g., Venn diagrams, gamma function)

2<sup>nd</sup> Morning Session: Infinite series before Euler. The Basel Problem. Euler’s

approximation of  $\sum_{k=1}^{\infty} 1/k^2$ . His proof that  $\sum_{k=1}^{\infty} 1/k^2 = \pi^2/6$ .

Afternoon Session: Euler's alternative evaluation of  $\sum_{k=1}^{\infty} 1/k^2$  using arcsines. His general attack on  $\sum_{k=1}^{\infty} 1/k^n$  for even  $n$ .

## Tuesday

1<sup>st</sup> Morning Session: Some of Euler's calculus. His approximation of  $\pi$  via arctangents, his concept of differentials, and his "proof" of l'Hospital's rule.

2<sup>nd</sup> Morning Session: An overview of Euler's number theory. The totient function and its application to finding "new" amicable pairs. His inductive proof of the Little Fermat theorem.

Afternoon Session: More of Euler's number theory. The  $\sigma$  function and its use in proving the Euclid-Euler theorem about even perfect numbers.

## Wednesday

1<sup>st</sup> Morning Session: Euler and the fundamental theorem of algebra.

2<sup>nd</sup> Morning Session: A taste of Euler's geometry: his proof of Heron's formula and his quadrilateral theorem.

## Thursday

1<sup>st</sup> Morning Session: Miscellaneous topics: Euler and binary representations, his first theorem on partitions, his proof of Euler's Identity

2<sup>nd</sup> Morning Session: Participant presentations

Afternoon Session: Participant presentations

## Friday

1<sup>st</sup> Morning Session: Participant presentations

2<sup>nd</sup> Morning Session: Participant presentations

## ABOUT THE ORGANIZER

William Dunham, Koehler Professor of Mathematics at Muhlenberg College in Allentown, PA, is the author of four books with a mathematical/historical flavor: *Journey Through Genius: The Great Theorems of Mathematics* (Wiley, 1990), *The Mathematical Universe* (Wiley, 1994), *Euler:*

*The Master of Us All* (MAA, 1999), and *The Calculus Gallery: Masterpieces from Newton to Lebesgue* (Princeton, 2005). Most recently, he has edited a book in honor of Euler's 300th birthday titled *The Genius of Euler: Reflections on His Life and Work* (MAA, 2007).

## **HOUSING**

Accommodations will be in the dormitories of George Washington University Potomac House 2021 F St NW, Washington, DC 20052

Check-in June 17th starting from 3 PM  
Check-out June 22nd before 11AM

More info on the Potomac House at

[http://gwired.gwu.edu/hc/ViewResidenceHalls/ViewHall/merlin-cgi/building\\_id/90](http://gwired.gwu.edu/hc/ViewResidenceHalls/ViewHall/merlin-cgi/building_id/90)

Workshop will take place at MAA Carriage House 1529 18th street NW Washington, DC 20036. MAA is located close to the corner of the 18<sup>th</sup> street and Q street.

## **MAA CARRIAGE HOUSE LOCATION**

[Map of 1781 Church St Nw Washington, DC 20036-1358](#)

## **DIRECTIONS FROM THE POTOMAC HOUSE TO THE CARRIAGE HOUSE**

1. You can take a brisk 30-minute walk from the dormitory to the Carriage House; directions can be found at:

[Driving Directions from 2021 F St Nw, Washington, DC to 1781 Church St Nw, Washington, DC](#)

2. You can use the metro (plan about 45 minutes for the walk to the metro, the ride, changing the lines, and the walk from the metro). Foggy Bottom Metro station (2301 I Street NW) is just a few blocks away from the dormitory. Take the orange line towards New Carrollton or blue line towards Largo Town Center. Ride three stops until the Metro Center and change to a red line (upstairs) towards Shady Grove. Ride two stops and exit at Dupont Circle. From there, the MAA Carriage House is just a few blocks away at the corner of the 18<sup>th</sup> street and Q street. The cost of the fare is \$1.35 and you can buy farecards from every station. More information is available at <http://www.wmata.com/metrorail/buyingfarecrd.cfm>.

You can also choose the Farragut West metro station (900 18<sup>th</sup> street) instead of Foggy Bottom and continue as previously described.

The link to the homepage of Metro <http://www.wmata.com/default.cfm>

The map of all the metro stations <http://www.wmata.com/metrorail/systemmap.cfm>

## **MEALS**

### **Monday, June 18**

Breakfast-at the dorm  
Lunch, dinner-Carriage House

### **Tuesday, June 19**

Breakfast-at the dorm  
Lunch-Carriage House  
Dinner-at the dorm

### **Wednesday, June 20**

Breakfast-at the dorm  
Lunch-Carriage House  
Dinner-on your own. Enjoy the wonderful opportunity to explore the restaurants in downtown Washington, DC

### **Thursday, June 21**

Breakfast-at the dorm  
Lunch-Carriage House  
Dinner-at the dorm

### **Friday, June 22**

Breakfast-at the dorm  
Lunch-Carriage House