Undergraduate Research: How Do We Begin?

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"How do you begin to do undergraduate research?" This is becoming an important question as more and more mathematics professors and their institutions want to provide an undergraduate research experience for their students. In this article we provide some strategies for guiding undergraduate research that were shared by the participants of the 2008 Center for Undergraduate Research in Mathematics (CURM) workshop at Brigham Young University in Provo, Utah.

Like effective teaching, successful undergraduate research is based upon foundational ideas that should be adapted and expanded to fit the talents of the professors and the needs of the undergraduates.

Get students interested. A good visual aid can help students become interested, even enthusiastic, about a problem. For example, one faculty member uses soap bubbles on wire-frame cubes to illustrate minimal surface areas. Students might not be willing or able to tackle hard problems at first, but making their introduction to the project fun and exciting can make them more committed to overcoming the hard work that lies ahead. Ideally, they will start taking the initiative to solve more difficult problems on their own.

Present background material. The time a professor spends presenting background material depends upon the research topic. However, if a professor spends too much time, there will be little time for doing research. Generally, we find that the presentation of background material should not exceed 25% of the time.

There are several approaches to presenting the material. Some professors choose to prepare a set of notes for their students. They provide these notes to the first group of research students and extend or expand the notes each year with each new group of students. As it is important to carefully and accurately prepare anything the professor gives to his students, writing and revising such a set of notes may be very time consuming. However, one potential long-term benefit is that these extensive notes may become early drafts of a book or part of a book that the professor later intends to publish.

A second approach is to give the students articles to read. Some of the more experienced members of the workshop noted that research articles are written for professionals working in a given field and often lack details or allude to previous work within the field. Because of these facts, a student trying to read a research article may flounder and then become discouraged or disinterested in the project. The advisor may rewrite the article or parts of it to make it accessible to the students.

Another possible solution includes using primarily student-level journals, or material from books, which provide a sufficient amount of background. Entire books are usually too much for undergraduates to read within the time constraints of an undergraduate project. Instead, the students could be asked to read specific pages or sections of some selected textbooks that contain information pertinent to the students' research.

Work out details with the students. Meeting with the students and reading the article together helps develop students' ability to work independently. In this joint faculty-student reading session, the professor may question the students about what was read. It is important to emphasize that it is okay if students do not understand everything the first time. Depending on what the mentor's intent is for the students to gain from this process, the professor may emphasize different aspects of the article. For example, the faculty may want the students to learn a particular proof technique, or he may want them to understand certain theorems.

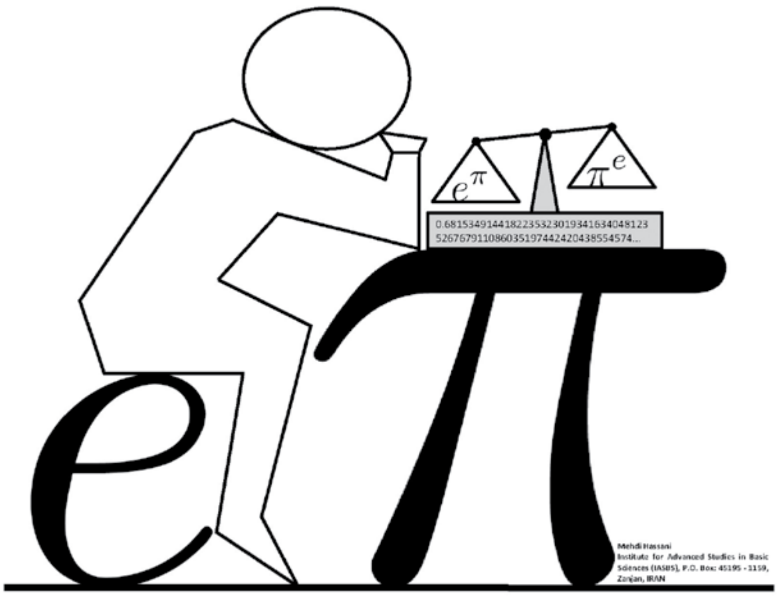
As the professor and students read over a theorem, some of the following questions may arise. Can you think of an example that illustrates this theorem? What purpose do the hypotheses of the theorem serve? If the theorem is not bi-conditional, is its converse true? For many theorems, this process helps the students understand the meaning of the theorem's statement. If some of the hypotheses are unnecessary, this could provide a possible avenue for students to explore. Otherwise, students may search for a counterexample.

Problem solving. The students should start working on basic problems early, and then dive into exploratory problems as soon as possible. Starting the students off at the basic level and allowing them to explore the area develops their background in the field. This can also help build students' confidence, enabling them to work more independently.

While it may be tempting to give students the whole problem at the beginning of the project, this may overwhelm and discourage them. Instead, over the course of time, the mentor may choose to give them several small problems that are part of a larger project, or even a series of problems that build or lead toward the project's goals. This process of problem solving provides students the insight to formulate and prove a theorem.

For example, if one of the goals of the project is to prove an existential theorem, the advisor may start with the students working in a simplified setting in which they can actually find or solve for the "object" that exists, thus giving the students the opportunity to realize that these "objects" always exist (or exist under certain conditions). This approach will lead them to discover, and ultimately prove, the result they were looking for.

Develop student independence. Continuous encouragement to pursue new avenues promotes student independence. Many students will be resistant to doing something new or something they are not "good" at. In one example given by a faculty member, the professor had a group working on a problem in discrete mathematics, and the students were making decent progress early on. However, after a week or two, the results stopped coming. He repeatedly urged the group to write a computer program to aid in solving the problem, but the students were extremely resistant. The faculty member was persistent and convinced them to write the code, after which, they gained the necessary insight to reestablish their progress.



Pondering

Preparing a list of expectations and presenting it to the students in the beginning of the research period is also beneficial. For example, the professor may want the students to know LaTeX well enough to write up their findings and effectively communicate with the research group via e-mail. The faculty member may also urge students to type their results as they work. This forces them to think carefully about their notation as well as the wording and logic of their proofs. It frequently leads to stronger, more eloquent proofs and can generate more research avenues. At the end of the project, these notes can be compiled into a final paper. Since the paper was prepared slowly over a long period of time, the overall quality will be higher than if the students waited until the end to write up the results. Regular presentation of their results, either to the advisor or to the entire research group also advances their confidence level and communication skills.

Moreover, the list of expectations may also include the average amount of time per week that each student is expected to commit to the project. The mentor may require his students to submit a brief report each week about their work and findings for that week. Writing such reports not only benefits the students, it also provides the professor insight into the students' understanding of the process.

Student frustration. At times, students may feel they are no longer making progress and become frustrated. Students should be told in advance that this will occur — that frustration is okay and happens to everyone. In trying to overcome frustration, the advisor may choose to work through hurdles with them, asking questions along the way. It is important that they become involved in this process and not just reply "yes, I understand" to the professor's questions. They should be able to explain their answers and their understanding of the important concepts and techniques. If the research problem is too complex for the student to solve, the professor may provide a simpler version of the original problem.

The research problem. As the mentor has worked through the previous steps, it is probably useful to point out potential research problems. For example, while reading through an article with the students, the professor may ask them "I wonder what would happen if we changed this condition. This could be a good research problem for you to explore." Once the students are ready to start working on a specific problem, this collection of potential research problems could be presented, and the students could choose which problem they would like to work on.

If appropriate, the students could work on a problem using more than one approach. For example, the students could use analytical techniques as well as computer simulations. If a student attempts to determine the validity of a conjecture, it may be insightful for them to alternate between trying to construct a counterexample and working on a proof.

Finally, it is beneficial for the students to compile their work and write a final report on whatever research they have done, even if they have only partial results. Communicating mathematics is an important skill, and writing about their work helps students improve this skill. Also, it brings a sense of closure to the project. In the end, be certain that your students have a positive experience. Point out how much they have learned and the significance of their work. Make sure that they are proud of their accomplishments and that they know that you are, too.

This workshop was run by the Director of CURM, Michael Dorff, (mdorff@math.byu.edu) *Brigham Young University. Authors for this piece comprise: Brad Bailey* (bbailey@ngcsu.edu) *North Georgia College & State University, Mark Budden* (mark.budden@armstrong.edu) *Armstrong Atlantic State University, and Urmi Ghosh-Dastidar* (ughosh-dastidar@citytech.cuny.edu) *New York City College of Technology. CURM is supported by the NSF grant DMS-0636648.*

"The Pea and Sun"

by Lawrence M. Lesser

(This may be sung to the tune of George Harrison's "Here Comes the Sun")

Banach-Tarski — come break a ball into six pieces:
Reassembled — that first ball sure increases!

The pea and sun,
A 2-for-1, and I say, it's all right.

Banach-Tarski — it is a mathematics wonder:
Banach-Tarski — could it maybe end world hunger?

The pea and sun,
A 2-for-1, and I say it's all right.