Connecting Mathematics with Reason

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We are often told to study history to learn from past mistakes, but this is a vain hope; history does not repeat itself such that we can use it to predict the future. But avoiding mistakes does not exhaust the ways history can be used to illuminate the present. One way in which it is most useful is as a check on how we organize and understand our world. Looking at history can help us see the hidden issues and assumptions that lie behind how we talk and think about that world. The arguments that in the past have justified the place of mathematics in education may shed light on the challenges we face today as we try to understand and further quantitative literacy.

Mathematics in Eighteenth-Century France

One of the defining moments contributing to our modern view of the educational value of mathematics occurred in France at the end of the eighteenth century. In the immediate post-revolutionary period, the education of the new French citizen was a major preoccupation. From the Enlightenment came the conviction that man was a rational animal; teaching mathematics was of major importance in the post-revolutionary educational program to strengthen the reason of the new *citoyen*, or citizen. A form of quantitative literacy that linked mathematical instruction with reason made mathematics an integral part of the curriculum in the *Écoles Centrales*, the short-lived revolutionary schools of the 1790s.

The rhetoric surrounding these schools was stirring, but its institutional manifestation was short-lived; by the time Napoleon came to power, the

Écoles Centrales were defunct. Napoleon did not abandon education, however. The system of *Lycées* that grew up under his administration persists to the present day. But the *Lycées* were more classically oriented than the *Écoles Centrales;* mathematics did not hold pride of place in their curriculum.

This does not mean that mathematics was not pursued in early nineteenth-century France, however. The subject was of central importance at the elite *École Polytechnique,* where Napoleon's engineers were educated. Mathematical tests decided admission to the school and determined rank within it. No longer an essential part of everyone's education, in the first decades of the nineteenth century knowledge of mathematics had become a mark of status and social differentiation.

There were real benefits to this elite status. In the first decades of the nineteenth century an extraordinary group emerged at the *École Polytechnique,* free to pursue mathematics without having to justify or explain their work to a large public. The first quarter of the new century saw Jean Victor Poncelet develop projective geometry, Pierre Simon LaPlace systematize probability theory, and Jean Fourier develop Fourier series.

Mathematics' change in status, from universal reason to intellectual sieve, did more than encourage research, however; it left its mark on the subject itself. This can be seen in the work of yet another *École* mathematician, Augustin Cauchy, who in his *Cours d'Analyse* of 1822 firmly established calculus on the rigorous basis of the limit. It was a signal achievement to bring rigor to a subject that had been poorly understood throughout the eighteenth century, and his work has long been hailed as a classic. Less well known, though, is that Cauchy's students rioted violently in protest against his work and his teaching. From their point of view, Cauchy's rigor was an assault on the humane mathematics that had been touted by the revolutionaries of the 1790s. The students argued that although Cauchy brought rigor to calculus, he did so at the cost of reasonableness. Another way to put it is that he introduced a course in rigorous mathematics at the cost of the kind of quantitative literacy that had been advocated and taught by an earlier generation of mathematicians.

In this particular case, the students may be said to have won the battle—Cauchy never published his projected second volume—but they certainly lost the war. Throughout the nineteenth century in France, as in Germany, rigor rather than reason was the preeminent value in mathematics. The case for mathematics as the route to reason was more solidly built in England, however, where throughout the century mathematics played a central role in the development of the idea of a liberal education.

Mathematics in Nineteenth-Century England

Much of the English discussion about mathematics and education was focused at Cambridge University where, until the 1860s, everyone who wanted a degree had to pass a major mathematics examination. Over the course of the century this examination, known as the Mathematical Tripos, became ever more grueling and competitive. This meant that during the 1830s, 1840s, and 1850s, much of England's intelligentsia spent their time in college pursuing evermore sophisticated mathematics; those who placed high in the Tripos were certainly the mathematical equals of those graduating from the *École Polytechnique*.

Beyond technical proficiency, however, comparison is difficult because mathematics was not the same subject in France and England. Ultimately the education at the *École Polytechnique* was more about training engineers than about educating either mathematicians or citizens. Cambridge University, for its part, was an institution whose major mission was to educate the clergy of the Anglican Church. Mathematics was pursued there as a form of quantitative literacy, a way to teach young men to understand and recognize the truth.

Cauchy's form of rigor was not clearly compatible with this goal; to this day there is no English translation of the *Cours d'Analyse*. This does not mean that the English were ignorant of his work, however; within two decades many had adopted his limit-based approach to calculus. The difference in the English and French understanding of that approach can be seen by contrasting Augustus De Morgan's 27-page chapter explaining the concept of the limit to Cauchy's two-line, operational definition. It can be further illustrated by the two men's approaches to divergent series. Because these often led to ambiguous results, Cauchy ruled them out of legitimate mathematics. De Morgan, however, was quite comfortable using such series: "Divergent series [were] nearly universally adopted for more than a century," he noted in 1844, "and it was only here and there that a difficulty occurred in using them" (Richards, 1987). In De Morgan's view, the creative possibilities of mathematics lay precisely in the challenges posed by this kind of ambiguity; to define them out of the subject because of a concern with rigor was nothing but counterproductive.

Over the course of the nineteenth century, the view of mathematics as reason remained strong in England, but it was challenged on at least two fronts. On the one hand, there was an internal challenge. It arose at Oxford University, which was, like Cambridge, devoted to educating the clergy. At Oxford, however, the curricular focus was on logic rather than mathematics. In the 1830s, the leading minds at these two universities faced off in a major battle over whether logic or mathematics was a better way to teach young men to use reason. The issue soon came down to whether the goals of education were better achieved by teaching the rules of reason, as in logic, or by practicing reason, as in mathematics—whether it was better to learn reason by precept or by practice.

When the battle was originally joined, neither side questioned whether mathematical arguments were quintessentially reasonable, or whether the forms of logic described that reason. Over the course of the next several decades, however, some began to ask whether even the time-honored mathematical proofs of Euclid were wholly satisfactory and, even if they were, whether Aristotle's logic was adequate to the reasoning they embodied. The easy connections among mathematics, logic, and reason on which rested the neo-Enlightenment program of mathematical and logical teaching became ever less clear over the course of the nineteenth century.

At the same time the English were questioning the relationships among mathematics, logic, and reason, they were adapting to a more external challenge in the form of professionalism. By the second half of the nineteenth century, the elite, self-defined communities of mathematicians found on the continent began to pose a powerful alternative to the gentlemanly ideals on which the English liberal education rested. By the end of the century, mathematics was beginning to be recognized as a specialized research subject in England. Thus, by early in the twentieth century, the problems inherent in identifying mathematics with reason were leading in England, as they already had on the continent, to the pursuit of "pure" mathematics and "formal" logic. By isolating their subject in this way, mathematical practitioners freed themselves from the confusions encountered at the interface of mathematics and reason.

This meant that practitioners could pursue their research in peace. But, at the same time, it strained their claims to model reason—whether by precept or by practice. Formal logic may be pure, but many were not willing to concede that it describes how we actually think; abstract mathematics may be rigorous, but many argued that it does not model human reason. As mathematics and logic were being redefined and purified in one context, they became increasingly irrelevant in another. By the beginning of the twentieth century, mathematics in England had become a research subject, but it no longer held pride of place in the Cambridge liberal arts curriculum.

The issues that face contemporary educators as they try to define and clarify the place of mathematics in the U.S. educational system are reminiscent of the ones that faced those defining the curricula at the *Écoles Centrales,* or at nineteenth-century Cambridge. But they are not the same. If a single lesson is to be drawn from the interrelated stories of mathematics and education in nineteenth-century England and France, it is that the link between mathematics and reason has always been a dynamic one, that both sides of the equation are highly susceptible to the vagaries of time and place. Both Cauchy's students and his English contemporaries recognized that mathematics as he defined it was not the same as that pursued by his mathematician forebears. Ideas of reason, as embodied in educational institutions, have changed as well. Although we, like our English predecessors, are focused on educating productive people, we are more likely to describe such people as functioning citizens or savvy consumers than as clear-thinking, moral gentlemen. What ties us to our past is the conviction that there is a powerful connection between mathematics and reason.

Quantitative Literacy

The concept of quantitative literacy is rooted in the connection between mathematics and reason. As described in "The Case for Quantitative Literacy," quantitative literacy is tied to two rather different concepts—numeracy and reason. With respect to the first, teaching quantitative literacy could perhaps be addressed relatively specifically and in a piecemeal fashion. With respect to the second, however, the current call for quantitative literacy harks back to the Enlightenment call for reason, echoes the challenges that faced the *Écoles Centrales* and nineteenth-century Cambridge, and radically expands the implications of teaching numeracy.

When teaching mathematics is seen as a way of teaching people how to think, it can no longer be isolated. Its implications spread throughout the curriculum and it has a place in every class.

In my view, that is where mathematics belongs: at the very heart of the educational project. But in advocating this position, I recognize that it places a significant responsibility on mathematicians and mathematics educators. De Morgan's defense of divergent series did not earn him respect within the mathematical community, but he stood his ground because, for him, mathematics models reason, and the ambiguities of divergent series were essential to his view of the way people think. We face similar choices as we consider what to include in a mathematics that models reason. We not only must reconsider the role of rigor but also determine the degree to which we are willing to allow machines to model reasoning for us. The historical record does not give answers to such questions, but it does place us in good company as we struggle to deal with them.

REFERENCES

Richards, Joan L. "Augustus De Morgan, the History of Mathematics, and the Foundations of Algebra." *Isis* 78 (1987), 184.