

The Licensure of Teachers for Quantitative Literacy: Who Should Be Entitled to Teach QL?

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The United Kingdom's Department for Education and Skills defines *numeracy*, otherwise known as *quantitative literacy* in the United States, somewhat broadly and imprecisely as follows:

Numeracy is a proficiency which is developed mainly in mathematics but also in other subjects. It is more than an ability to do basic arithmetic. It involves developing confidence and competence with numbers and measures. It requires understanding of the number system, a repertoire of mathematical techniques, and an inclination and ability to solve quantitative or spatial problems in a range of contexts. Numeracy also demands understanding of the ways in which data are gathered by counting and measuring, and presented in graphs, diagrams, charts and tables.

There is no inherent reason that the symbol systems associated with ordinary literacy and language would be intrinsically different from the system entailed in quantitative literacy as the mind seems to be equivalently disposed to support both language and numeracy. Wynn (1992), for example, argues that humans are innately endowed with arithmetical abilities, and she and others

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have found evidence of numeracy in infants as young as 3–5 months (equivalence of sets, ordinal discrimination, addition/subtraction, cross-modal numeric sensitivity). Basic numeracy (determining the number of objects without counting, ordinality, counting, number conservation, simple arithmetic) seems governed by innate developmental mechanisms, because numeracy emerges spontaneously in all cultures and in all typical environments. It seems also to be the case (Ferrari & Sternberg, 1998) that these basic actions are performed by rote initially, and only later, reinforced by the culture, do they acquire a conceptual representation along with some learned strategies (e.g., finger counting, min or count-on), procedural knowledge (it is illegal to divide by zero), translation of verbal statements into numerical propositions (dozen refers to 12), and certain beliefs (mathematics requires innate ability, is for men, or that there is only one right answer in mathematics).

Crosby (1997) makes a case that historically the well-understood and transforming effects of language on cognition are also found for quantification. His thesis is that in the late thirteenth century a new way of thinking emerged that quantified time (the first mechanical clocks), space (drawing in perspective), finances (double-entry bookkeeping) and visualized reality as aggregates of uniform quantified units—leagues, miles, pounds, hours, minutes, and musical notes—that expanded numbers from their nominal and ordinal properties, available beforehand in ordinary language, to the powerful additional properties of equal intervals and ratios, which are available to the “counting person” only when there is true quantification.

The concept, *quantitative literacy*, seeks an analog to ordinary literacy, crudely measured by whether a person can read and write, to something that transcends proficiency in four function arithmetic to a sensitivity to the quantitative dimensions of all experience, to an amalgam of skills, knowledge, dispositions that can be applied to the quantitative dimensions of experience, and to an interpretation of a new reality of the sort portrayed by Crosby’s thesis, but beyond the areas he described to all areas of experience. The question is what kind of teacher education program, supported by what kind of licensing requirements, license tests, national board certificates, etc. would support increased levels of quantitative literacy in the schools. To what course of study should the prospective teacher be exposed and how can that be captured in the state’s licensing protocols.

Take the following example¹ of quantitative literacy as it demonstrates the “developing confidence and competence with numbers and measures. It requires understanding of the number system, a repertoire of mathematical techniques, and an inclination and ability to solve quantitative or spatial problems in a range of contexts.” It is a lesson given by a teacher whose pupils had been

taught previously the standard definitional lesson on odd and even numbers and were now exploring patterns in them, such as *an even number plus an even number is always an even number*. One of the pupils, Sean, offered the conjecture that some numbers are both odd and even.

Of course there are no numbers that are both odd and even, and the teacher now has a dilemma that also goes to the heart of the question of how the teacher education program would have prepared a teacher for this dilemma, what knowledge would be needed, and where could it have been acquired in traditional higher education? The larger question of this paper is how the teaching licensing and program accreditation regulations could capture (or frustrate) what was needed to encourage the pursuit of quantitative literacy at this juncture in the lesson.

Should time be taken from the next topic in the state's prescribed curriculum to review this topic, a topic that would have only one or two items on the state's standardized test? Should the teacher tell Sean he is mistaken and correct him by simply restating clearly the odd-even numbers definition for him and be done with it? Or, should Sean's conjecture be pursued to some mathematical conclusion.

What kind of teacher education program would support the teacher's pursuit of Sean's assertion, nonsensical as it might seem? Quantitative literacy in fact is partly defined by a confidence to pursue the issue in this situation even though the teacher would surely never have come across such an idea in any teacher education program of study. It is equally certain, by the way, that that these numbers of Sean's conjecture would never be on any standardized test of number understanding. This fact only complicates the links between quantitative literacy, teacher education, and the licensing and accreditation regulations.

When Sean was asked, by the teacher how he thought some numbers were both odd and even, he replied with the novel observation that six was such an odd-even number because two went into it an odd number of times while eight was not such a number because two went into it an even number of times.

The teacher went to the heart of quantitative literacy instruction² by asking the class to consider whether Sean's conjecture had any merit. The class worked out the pattern that every other even number was one of these even-odd Sean numbers—six was, eight was not, ten was, twelve was not, but fourteen was and so forth. Others explored whether adding Sean numbers together gave another Sean number or gave non-Sean even numbers. Others noted that adding Sean numbers and non-Sean even numbers always yielded a Sean number. The same relationships held for subtraction while other outcomes held for the multiplication of odd, Sean and non-Sean even numbers.

After further exploration of the properties and patterns of Sean numbers, the class was asked whether these odd-even even numbers should be added to the list of numbers the class had studied (real, rational, integer, natural, complex, etc.) . How should they even decide whether Sean numbers should be added to the mathematics curriculum? Should they vote on it, or is democracy a poor mathematical procedure? And if they voted on it, should the decision be based on a simple majority, super majority, unanimity, should it be accompanied by confidence ratings, and so on? Each of these options demands some level of quantitative literacy.

This particular example showcases an unscripted and unplanned teaching event, what some might call a *teachable moment*, but it is more than that as it also represents the core of quantitative literacy—a confidence to tackle an uncharted quantitative matter, serviceable knowledge of mathematical procedure and knowledge, logical thought and problem-solving, an extension of the quantitative into the political and social, and so forth.

One of the problems with the assessment of quantitative literacy in these contexts is that standardized tests, favored by policy-makers who would hold teacher's accountable for their teaching, are quite distant from what goes on the class and are restricted to what was hoped went on in the class and what lends itself to easy measurement. A second problem is that the public really learns more about what did not go on in the school than what did, because the lower the test scores, the less we actually know about what went on in the classroom. The test would not reveal whether the teacher taught what was on the test poorly, or whether the teacher taught something else, like Sean's conjecture very well. A third problem is that the published standards, like the standards for numeracy, for the information that should be covered by the tests are usually so vague and abstract that almost nothing is ruled in or out of the curriculum (see Raths, 1999; Ohanian, 1999 & 2000).

There are at least two competing views in the nation, today and historically, about what teachers need to know and how they should be prepared. More about each is discussed later, but both views hold that teachers, regardless of what else they know, must know the subject matters they hope to teach their pupils. The academic major, the usual remedy, can be shown, however, to be an inadequate preparation in the subject matter the prospective teacher will teach. Mathematics majors, for example, are no better than non-math majors in creating real world examples of the division of one fraction by another³. By and large, the academic major does not induce in students the kind of penetrating understanding necessary to pursue Sean's conjecture or other novel notions in quantitative literacy. Efforts to reform the academic major have not abandoned the concept of the academic major itself, but rather have tried to make the

academic major more effective, especially for teachers, who seek to acquire a special kind of integrative subject-matter knowledge that would undergird complex areas like quantitative literacy. Proposed solutions have centered on new majors, interdisciplinary majors, and new types of courses within existing majors.

The case of the appropriate course of study for the elementary school, or social studies, or quantitative literacy teacher across the grades is particularly instructive because it illustrates the knotty problems that arise when teachers must acquire a wider range of knowledge than the typical academic major covers. In the case of the elementary school teacher it is difficult to see how the prospective elementary teachers would become well-grounded in mathematics, literature, writing, history, geography, the natural and social sciences, the fine arts, language—all subjects that are taught in grade school classroom. At the secondary level the matter is only a little less complicated for social studies or general science, which are informed by several distinct university subjects or majors, each of which is a full university course of study in its own right. The most promising preparation for quantitative literacy, an even wider domain than social studies or general science, is likely to be informed by proposals to solve the problem of the appropriate academic major for the elementary teacher. Six options are promising (see Murray, 1991 for an expanded account).

1. Interdisciplinary major. This option is a collection of reworked minors in the areas of the school curriculum: mathematics, foreign language, history and social science, English, natural science, and fine arts. Apart from the fact that each minor would be responsive to the unique requirements of the elementary school teacher, the interdisciplinary minor option is fairly conservative and administratively feasible. It is an honest approach insofar as each major area of the elementary school curriculum is addressed. A similar approach can be imagined for the teacher who is sensitive to quantitative literacy as the opportunities for this extension occur in each minor.

2. Philosophy of subject matter. In this approach the philosophy of each subject matter (e.g., philosophy of science, mathematics, etc.) is taken up, and essential and fundamental aspects of the structure of subject matter are covered. Teachers learn, for example, that there are no facts apart from theories or that “true” theories are not those that were proved, but only those that have failed to be disproved. Similarly, social studies education learn to view the history curriculum not so much as a chronology, or as the true view of the past, but as one of several possible stories of the past that could be constructed to make sense of the same historical events. Teachers learn of the similarities in the grammar and syntax of mathematics and language, and so on.

This approach is related to the so-called *structure of the disciplines* approach to curriculum reform that followed the Sputnik educational crisis some forty years ago. The underlying coherent principles or structures that hold academic disciplines together are the subject of the courses themselves in this approach. The separate natural sciences, for example, can be organized by the principle of evolution (evolution of species, matter, solar systems, societies and cultures, sub-atomic particles, chemical reactions, etc.). Or they can be organized by the principle of orders of magnitude (e.g., the powers of 10 device of relating sub-atomic structure, biochemistry, and celestial systems as well as the design constraints of other physical and animate structures that stem from their size alone). Whatever it is that makes a certain kind of study, *numeracy*, and not some other kind of study, is the subject of this approach to improving the teacher's capacity for teaching quantitative literacy.

3. Text approach or “great books” major. This approach entails an unusual course of study that contains a close reading of seminal texts, or founding texts, in each area (the “great books”) coupled with an examination of school textbooks for the assumptions they make about the discipline in question. The logic of this proposal, like the philosophy of the disciplines approach, is that the core structure of the discipline is addressed directly, and the “forest for the trees” problem that plagues most university study is minimized. The logic has a pedagogical dimension as well, because the student is introduced to the discipline in the way that approximates how world was first introduced to it.

The six topics (pattern, dimension, uncertainty, shape, and change) taken up by Steen (1990) are reasonable candidates for the numeracy “great books” topics along with the traditional mathematics notions, like number.

4. Genetic epistemology. This option entails the study of the developmental psychological literature from the perspective of the development of the concepts that make up the curriculum. In this approach the prospective teacher learns the relevant developmental constraints upon the pupil's acquisition of the curriculum and lays out, as an unavoidable part of the discussion, the nature of the subject itself. The story of how the young child develops the notion of number, for example, is valuable in its own right, but also reveals salient portions of number theory, the arithmetical algorithms, and other aspects of mathematics. Similarly, the account of the child's moral development reveals the principal issues in moral philosophy and political theory. The scientific account of how children form groups and gangs, establish rules for games, assign blame and praise, identify enemies, punish transgressions, acquiesce and conform to other's wishes and requirements, advance their position, and so forth are instructive for the counterparts of these and other issues in history and

political and social science. Thus, the prospective teacher acquires important knowledge about both the student's mind and the content of the discipline they hope to teach.

5. The cognitive psychology major. In this option the prospective teacher would study a reformed major in cognitive psychology in which the working of the mind in various domains becomes the specialization. The subject matter content would be picked up through the consideration of how the mind operates mathematically, aesthetically, historically and so forth. Like the philosophy of the disciplines or text approaches, this approach would provide a structure for the reformed minors in each subject area. Each area would be approached from the perspective of how we think about and know the content in question. The approach fits well with the current recognition in cognitive psychology that thinking is domain specific.

While our notions of number, transitivity, class inclusion, necessity, probability, and so forth are each objects of study in cognitive psychology, the study of the concept of *time* is illustrative. A well-developed notion of *time*, which supports, for example, an understanding of daylight savings time takes about eighteen years to acquire.⁴ The properties of the numbers that designate time, like the numbers assigned to years, are not appreciated by elementary school children who can be shown not to grasp order of these numbers or that the intervals between them are equal. A young child will argue that the taller of two trees planted at the same moment is older, that the corroded coin of two minted in the same year is older, and so on. Older children will argue that the clock actually runs slower and faster at certain times of the day and year. The point is that immediate time, let alone historical time, is a fragile concept for the child and the young adolescent. It cannot be merely assumed by the teacher that the order and intervals between dates, for example, have anything like the meaning they may have for the teacher.

6. The pedagogical content knowledge minor. This approach addresses the fact that teachers inevitably transform what they know into a teachable subject. They give the subject a new structure and meaning, one that is appropriate to their students' level of understanding. These structures can be studied and codified. Since this reformulation of the discipline is inevitable in teaching, one might as well address it directly and, as in the other approaches, use it as a way to structure and teach the academic disciplines. In teaching *Huckleberry Finn*, for example, the teacher inevitably interprets the book as a story of race relations, or generation gaps, or an historical period, or latent homosexuality on the frontier, or whatever. The academic major would explicitly address these pedagogical alternatives. As another example, many science teachers

attempt to clarify the nature of electric current by comparing it to the behavior of water currents in various sized pipes, and so forth. Is this a good way to think about electricity, particularly alternating current? How would one know? The answer to the question is not to be found in physics or in education, but in a qualitatively different kind of knowledge that will come from conversations between disciplinarians and pedagogues and one that builds upon genetic epistemology and cognitive science.

In the earlier example of the division of fractions ($1\frac{3}{4}$ by the fraction $\frac{1}{2}$), once the division by 2 error is cleaned up, are the following representations of the mathematics pedagogically equivalent and equally commendable—How many half slices are there in $1\frac{3}{4}$ pizza pies? How many 50 cent tickets can be bought for \$1.75? How many half pint cans of oil are needed to exactly fill of $1\frac{3}{4}$ pint engine? How many half yard ruffles are needed to cover the bottom of a dress of $1\frac{3}{4}$ yard's circumference? How could the equivalence and merit be determined?

This knowledge—the knowledge of what is a telling example, a good analogy, a provocative question, or a compelling theme—is a proper object of study and could yield a deep and generative understanding of the disciplines of quantitative literacy. To have multiple ways of representing a subject matter, to have more than one example or metaphor, to have more than one mode of explanation requires a high order and demanding form of subject matter understanding.

Once subject matter knowledge is in hand, there are still a number of obstacles to increasing the levels of quantitative literacy in the schools owing to tensions in the policy practices surrounding teacher education and licensure. Those most interested in increasing quantitative literacy in schools may themselves be divided on the degree to which they subscribe the adequacy or sufficiency of relying on naïve teaching to accomplish the task or the degree to which they would rely on the professional study and practice of teaching.

The Alternative of Reliance on Naïve Teaching

Nearly every reform report calls for an increase in the teacher's subject matter preparation at the expense of professional teacher education courses on the view that teaching is a naturally occurring human behavior, a wholly natural act that is an enduring and universal feature⁵ of the repertoire of human behaviors. Humans are, in other words, a teaching species, a species whose young cannot, and do not, survive unless they are taught, invariably by persons with no formal schooling in teaching or in teacher education. Ashley and Tomasello (1998) found evidence of teaching in children as young as three years old, and

Strauss, Ziv, and Stein (2002) found that children's style of teaching a new board game or building something changed from demonstration and modeling at 3–4 years to predominately verbal explaining at five and six years. Seven year olds can adapt their teaching on occasion to their perception of their pupil's proficiency and knowledge. They also introduce the new teaching strategy of asking the learners if they understood, and they then adapt their *teaching* to the learners' mistakes. Children's pedagogy is also influenced by schooling itself. Maynard (2004) found that older Mayan children (6–11 years), who had been to school, were also able to adopt "school-like" teaching with their younger siblings (didactic teaching at a distance) in place of indigenous teaching practices used in families for cooking and weaving (close-up interactive demonstrations).

J. M. Stephens (1967) catalogued the features of naturally occurring teaching in his theory of spontaneous schooling. His argument was that schooling, a feature of all known anthropological groups,⁶ was dependent on a set of natural human tendencies that some persons had in greater degrees than others. Those who had these tendencies in generous proportions would be seen, whether they intended to teach or not, as teachers by the members of their communities. Teaching and learning would take place naturally, spontaneously, non-deliberatively, and not necessarily with any particular motive or intention to benefit the pupil. They would occur merely because the tendencies, which fundamentally serve only the teacher's needs, led incidentally and inevitably to learning in those persons in the teacher's company. Teaching, in other words, was natural and spontaneous; it occurred whenever a person with these tendencies was with any other person for a protracted period, and it occurred to satisfy some need of the teacher, not some need of the student. It is not important that Stephen's speculations on the specific character of the natural or spontaneous tendencies are correct in every detail, but only that there are natural teaching abilities and that these seem to be adequate to account for most of the features of contemporary teaching and schooling.

The natural teaching view is also reinforced by the fact that many effective private school teachers have not taken education courses, nor have professors, who were trained only to research, not teach, their subjects (Judge, Lemosse, Paine, & Sedlak, 1994) and seemingly meet their teaching responsibilities satisfactorily without the benefit of engaging the content of education courses.

Some policy-makers raise the related question: Even if formal teacher education can refine and improve natural teaching somewhat, can the nation's needs for teachers still be met, less expensively and adequately, by the natural teaching techniques and styles we all seem to possess coupled with study in an appropriate academic major?

An Alternative to Naïve Teaching: Reliance on Pedagogical Education

The theory of spontaneous schooling, and the view of teaching that is based on it, have a number of problematical consequences for contemporary schooling because schooling now takes place on larger scales than that found in families and other anthropological groups, and because schooling increasingly takes place in circumstances where the teacher and the pupils have less and less in common. As a result, reliance on the theory of natural teaching can be expected to lead to serious pedagogical mistakes for both weak and superior students. Quite apart from the matter of scale and the degree of similarity between the teacher and the pupil, the theory promotes a direct mode of instruction that is unduly limiting in terms of modern views of cognition and cognitive development that support advances in quantitative literacy. Finally, the theory provides insufficient guidance for the solution of difficult and novel problems in schooling that go beyond the natural teacher's exclusive reliance on "*showing and telling*," the core of the natural style of teaching found in children and adults.

Low Expectations. When the teacher and the pupil are not alike and when the teacher may have lower expectations for the *different* pupil, the natural tendencies lead to very unfortunate consequences (Brophy & Good, 1986; Evertson, Hawley, & Zlotnick, 1985). When the teacher and the pupil have dissimilar backgrounds, we can expect the natural teaching mechanisms that support familial instruction will not operate to benefit the student.

Natural teaching leads to a predictable number of pedagogical mistakes that novices, and regrettably some licensed teachers, make unless they also have had the opportunity to learn and practice extensively some counterintuitive and *unnatural* teaching techniques. For example, it is certain that the natural teacher, well-meaning and well-read with good college grades, will still make the following pedagogical mistakes with their pupils for whom they have low expectations, regardless of how benignly they came to have these expectations. They will treat these pupils not as individuals but as a group, seat them further away and outside the classroom zone of frequent teacher-pupil interaction, look at them less, ask them low-level questions, call on them less often, give them less time to respond, give them fewer hints when they are called upon, and give them less praise and more blame than other pupils. And the natural teachers will do all this out of a mistaken sense of kindness that is seemingly oblivious to the pedagogical harm their undisciplined actions have caused their pupils (Hawley & Rosenholtz, 1984; Murray, 1996).

This untrained, natural, and kind person, believing the pupil does not know very much, will not want to embarrass the pupil by calling on the pupil often,

will ask *appropriately* easy questions when the pupil is called upon, will give fewer hints and less time when the pupil fails to respond as it would be unkind to prolong the pupil's embarrassment and so on. The educated teacher, like all professionals, and in contrast with the *spontaneous* or *natural* teacher, must discipline many of his or her kinder instincts and implement an equitable and disciplined professional approach to bring about high levels of achievement from those pupils for whom the teacher would otherwise have low expectations (Oakes, 1985). These professional actions are frequently counterintuitive and as a result require extensive practice so that they can be performed by *second nature*.

Higher-order Forms of Learning Needed for Quantitative Literacy. Kantor & Lowe (2004) argue persuasively that historically the schools, with a few exceptions that proved the rule, were inattentive to quality education and higher order subject matter understanding of the sort expected in quantitative literacy. Teaching was largely *showing and telling* coupled with rapid fire teacher questions and student recitation and memorization, which while useful for the rote learning of some quantitative facts, limited higher level achievement and was a hallmark of the natural teaching regime.

A further limitation of the natural teaching regime, apart from the harm caused to weaker pupils over time, is that it does not take the superior pupil much beyond the kind of information that can be told and demonstrated and conforms to the stimulus-response and imitative forms of learning. While such declarative knowledge is important, the forms of quantitative literacy that are constructed by the pupil, not merely transmitted to the pupil, are increasingly seen as key to the student's performance at the advanced levels of the disciplines (Murray, 1992; Ogle, Alsalam, & Rogers, 1991). A pupil can be told and shown, for example, that A is greater than B , and that B is also greater than C , but an essential ingredient of quantitative reasoning, the knowledge that A *must be* greater than C , and that one could know that without ever looking directly at A and C , cannot be simply given to the pupil. Not only is A truly greater than C , but more than that, it *has to be* greater. The quantitative literacy notion of necessity has its origins elsewhere and outside the definitions in mathematics. Showing and telling have not been found, except in very unusual circumstances, to be effective means of "teaching" necessity (Beilin, 1971; Murray, 1978 & 1990; Smith, 1993). It is one thing to know that a statement is true, but quite another to know that it *must be* true. The origins of necessity, and other pivotal concepts, like irony or justice, seem to lie in *dialectical* instruction, which demands intellectual action on the part of the teacher and the student. While more demanding on the student, dialectic or maieutic teaching,

is a less direct and more subtle form of instruction than that supported by the natural “*show and tell*” teaching tendencies.

The Naïve Theory of Mind. Along with the natural teaching techniques there often comes a naive and serviceable, but limited, theory of the human mind (Heider, 1958; Baldwin, 1980). The pupil’s school performance in the naive or common sense theory is tied to four common place factors—ability, effort, task difficulty, and luck. With these four factors, the natural teacher can explain completely the pupil’s success or failure on school tasks by attributing the level of the pupil’s performance to his/her ability or effort, or to the difficulty of the school task, or to plain luck. The problem with naive theory, apart from the circularity in the four factors, is that more sophisticated theories have been developed in which it can be shown that ability, to take only one example, is not fixed or stable, and that it varies from moment to moment interactively with many other mental factors, not just the few in the naive theory (Baldwin, 1980; Murray, 1991). Naive theories, for example, see forgetting as the inevitable decay of stored knowledge, when the educated view is that forgetting is an active thinking process of interference and reorganization (Rose, 1993). Similarly misconceptions in science and mathematics are seen as the result of misinformation or forgetting when the educated view is that they stem from the lively interaction of the earlier, more primitive and well-established conceptual frameworks with later information (an imperfect balance between assimilation and accommodation in the Genevan sense (Baldwin, 1980)).

Naïve Pedagogy. Natural teaching is essentially *showing and telling* (see Olson and Bruner (1996) for an account of folk pedagogy). Naïve pedagogy is based upon a *transmission of intact packets of information* model of teaching. Strauss and Shilony (1994) interviewed experienced and novice science and humanities teachers about how they would teach a topic of their choosing to children of various ages (7-17 years). Both novice and regrettably many experienced teachers in each discipline conceptualized teaching only as the flow of information from their heads to their pupils’ heads, acknowledging their own role was only to devise manageable and interesting ways of entry into the student’s mind so the information could be stored and anchored appropriately. The student is passive, a receptacle waiting to be filled, and if the information fails to flow to its destination, the receptacle was taken to be too small and/or the student was inattentive.

Astington and Pelletier (1996) catalogued the following tenets of naïve pedagogy: (1) children are born with abilities and capacities that unfold linearly in time, (2) instructional sequences should match developmental sequences, (2) learning occurs sequentially within a hierarchy of skills, and (3) student

errors are attributable to incomplete learning or inattention. When the pupil needs to do something, the teacher need only demonstrate or model it, and when the pupil needs to learn something, the teacher need only tell the pupil what they need to know.

On the whole, these folk or naïve pedagogical techniques and beliefs frustrate the modern pedagogies based on dialectic, discovery, invention and collaboration that would be tuned to enhancing quantitative literacy. They are also at variance with some contemporary research findings: developmental pathways, for example, are rarely linear and often show fits and starts, oscillations, and even reversals, particularly when performance is at an optimal level or when a new skill is being developed (Fischer & Bidell, 1998).

These naive views of how the mind works coupled with equally naive views about the nature of the academic subject matters as received and objective truth further limit the benefits that can be expected from nonprofessional or natural teaching (see Amsler and Stotko (1996), for examples of the possible and legitimate variations in what constitutes correct subject matter knowledge). The naive view of subject matter also shows itself principally in the area of assessment of the student's understanding of a subject matter.

Classroom Assessment. The natural or naïve teacher's evaluation of the pupil's correct and incorrect responses provides a telling and targeted arena for distinguishing naive and educated teachers. A student's reasoning may look illogical to a naive teacher, while the educated teacher will see that the student's reasoning is intact, but has operated on different premises from those of the set problem. The naive teacher will be distressed when a pupil who had pluralized *mouse* correctly suddenly pluralizes it as *mouses*, while the educated teacher will see the new plural, not as an unfortunate regression, but as a positive sign of cognitive advancement in which the pupil is exhibiting a newly developed appreciation of a linguistic rule that is merely over-generalized in this instance.

Other decrements in performance may also indicate educational progress; some six year old pupils not only maintain incorrectly that the longer row of two rows of five beans has more beans, but also maintain that the longer row must have more beans and would always have more beans. These errors occur even after the pupil has just counted the equal number of beans in each row. It happens that the error, "there *must* be more beans," which seems the more serious error, is indicative of more developed reasoning than the error, "there are more beans" (Murray & Zhang, 2005). Naturally, it is very difficult for the naive or natural teacher to accept any error or poor performance as a marker of progress, yet the failure to see some errors as markers of progress is another

serious pedagogical mistake that stems from the naive theory of teaching and learning (see Bruner, 1961 on *creative errors*).

The student's superior performance may also be misinterpreted by the naive teacher (see Strauss & Stavey, 1982 for examples where correct performance actually rests on immature and incorrect reasoning). Murray (1990) found that young children's success on a developmentally advanced quantitative reasoning task (the classic wine and water mixture problem) was, despite the appearances, not an indication of the same level of cognitive development as older children's success on the same task. The adolescents seemingly and inappropriately coded the problem as a probability problem and reasoned to an indeterminate conclusion when in fact outcome is a matter of necessity—there must be as much wine in the water in one glass as water in the wine of the other glass.

If a child arrives at the correct answer to a multiplication problem through serial addition, how would the naive teacher score the response—as superior or inferior to the response of a child who arrives at an incorrect answer through multiplication? Do college students, who correctly calculate the mean, median, and mode, operate at different standards of sophistication if their reasoning is based on a calculation algorithm, a mechanical model of balance, an algebraic deduction, or a special case in the calculus? Upon what theory, and by what means, would the naive teacher determine whether some solutions are more sophisticated, elegant, significant, and so forth, than other solutions. By what criteria would the teacher even see his/her teaching as successful and/or high quality (see Fenstermacher & Richardson, 2005 on these distinctions)?

The naive or educated teacher's mistakes in subject matter knowledge and its assessment are a problem under any view of teacher employment. Additional study in the subject matter would seem the obvious remedy, and nearly every reform initiative in teacher education, as noted earlier, recommends additional and deeper subject matter preparation. The exact nature of the study, however, has been shown to be complex (Wilson, Floden, & Ferrini-Mundy, 2001; Rice, 2003; Floden & Meniketti, 2005). Generally more preparation in the subject, particularly mathematics, is positively related the state's assessment of student learning, but there are inconsistencies in which additional subject matter preparation sometimes weakens student learning (Rice, 2003). The state assessments are, of course, about relatively narrow and easily scored concepts of quantitative literacy.

The Problem of Abbreviated Study. The research on the efficacy of pedagogical courses is weaker than that for subject matter courses, but also shows some positive association with student teaching (Rice, 2003). It is

doubtful that a sufficient level of pedagogical training can be reached in a short period. For example, on a simple reading of Skinner, as might be found in a survey course in education, prospective teachers could believe that positive reinforcement (or reward) is an effective and preferred way to increase the likelihood of desirable pupil behavior. Without an awareness of the important exceptions and qualifications in which rewards actually weaken a response (the *over-justification phenomenon*), teachers will make mistakes by implementing procedures that run counter to their intentions (Cameron & Pierce, 1994)⁷.

Similarly, upon a quick reading, the prospective teacher could come to believe that student grades should be normally distributed or that reliability is a property of a test rather than a property of those who took the test. These professional lessons cannot be easily abridged or rushed because many educational innovations are counterintuitive and subtly tied to hidden factors.

For example, it makes a difference whether addition problems, like $8+5=$ __, are presented horizontally or vertically. While a seven-year-old pupil, to take another example, may understand that the amount of clay in a ball would be unaffected if the ball were flattened into a pancake, she would more than likely believe incorrectly that the same pancake would weigh more and take up less space, despite the fact that the she had claimed the ball and pancake have the same amount of clay. In fact it is only in adolescence that she would understand that the volume of the ball and pancake were the same. Furthermore, it is now acknowledged that many research findings are inherently provisional and must be qualified by context and the cohort or generation of pupils who participated in the study, as different results are obtained from different cohorts and contexts on such basic questions as whether intellectual performance decreases after a certain age. Thus, having studied the research literature at one time is not a guarantee that the results can be applied at a later time with regard to such nagging and recurring issues as social promotion, skipping grades, ability grouping, optimal class size, delayed instruction, and so forth. Current and deeper study is required throughout the teaching career.

The Problem of Insufficient Time. Sudden or effortless changes in behavior are taken by developmental psychologists as a sign that the change was not fundamental, but rather a temporary change, caused by a peripheral mechanism (e.g., fatigue, inattention, misperception, etc.), and not authentic. Protracted and extended practice and experience is needed to overcome the acquisitions of a prior stage of development or of the naïve or natural teaching regime, which seems to be deeply rooted in behavior.

Smith (1989) has shown that highly motivated, knowledgeable, and experienced teachers were still unsure and shaky after ten months of practice in their

efforts to implement a *conceptual change* science teaching technique, even though they practiced the new technique extensively under ideal teacher education conditions (extended coaching one on one). Such protracted experience is rarely afforded in the traditional teacher education program. Despite having practiced the technique in a variety of settings, having video and stimulated recall analysis of their teaching performance, and having personal feedback of their efforts, the teachers regressed to their earlier teaching style whenever the lesson took an unusual and unexpected turn (Smith & Neale, 1990). Their regression to *show and tell* sometimes undermined the entire point of quantitative reasoning itself, because the teachers would deny or ignore an unexpected outcome in a demonstration in favor of the outcome that was supposed to have happened. In a light and shadow lesson, when a single shadow was expected and predicted, but a double shadow appeared, the teacher would deny it or ignore it and continue with the demonstration as if the single predicted and expected shadow had appeared, all in opposition to the new teaching technique.

The classic defense against this kind of *regression under stress* to the more primitive and older strategies is *over-learning* or practice well-beyond what is needed to simply learn the new skill or approach. Regrettably, few teacher education programs can make the necessary provisions for over-learning.

Olson and Bruner (1996) conclude that the shift from the simplest pedagogies of natural teaching to the more sophisticated ones available in scholarship entails a focus on what the student, not the teacher can do, on what the student thinks, on the student's view of teaching itself, and on knowledge as an emergent event in the dialectic between the teacher and the student.

Weak Protocols for Quality Assurance in Teaching. Ironically, the calibration of teacher certification and more ambitious goals for teachers of quantitative literacy is held back by the very fact that teaching has all the attributes of the other professions—accreditation, professional associations, standardized tests, licenses and credentials, advanced degrees, and so forth. While there is some fragile evidence for the efficacy of the license in the teaching field and advanced degrees (Rice, 2003), none of these requirements, all demanding in their appearance, has much credibility within or outside the profession as each is routinely waived when there are shortages of otherwise qualified persons for the public schools. In the case of the private schools, many states typically set and require no standards at all, a practice that only reinforces the lack of standing the current standards have.

To take one example of the low regard in which these bureaucratic standards are held, the National Board for Professional Teaching Standards (NBPTS), departing from the practice of other professional national boards, elected *not* to

require a degree in teacher education, a state teaching license, or study at an accredited institution for those permitted to sit for its certification examinations. National Board certification, itself, is not even required for advancement in the field or promotion to higher levels of professional responsibility.

Since 1951 all states must give their permission (a license) for a person to teach in a public school, and a few also require the license for private school teaching. Historically and continuing to current time, three factors have been relied on in granting this permission—(1) an assessment of the prospective teacher's character, (2) the prospective teacher's tested knowledge (particularly of the teaching subject), and/or (3) the prospective teacher's teaching skill as attested to by the completion of a higher education program of study in pedagogy.

The state's granting of formal permission to teach is meant to be based on indicators that permit an overall prediction that a particular candidate will perform safely and satisfactorily in the complex situation of teaching. Upon which of the three factors the various states elect to award their license depends on whether they see teaching as a profession based exclusively upon specialized university-level study, or whether they see teaching as little more than a civil servant's line of work that can be taken up by nearly any well-meaning person who has mastered a subject matter.

One issue, both historically and at present, centers on these competing views of teaching—how a person learns to teach, who should be entitled to teach, and more importantly who should be prohibited from teaching, whether teaching is inherently moral or technical, an art or a science, and so forth. Recently, policy-makers have wanted also to know whether any of these factors influence the performance of the prospective teacher's students on the standards-based assessments the state makes of pupil and student achievement. No matter the basis of the license award, the students of licensed teachers generally perform slightly higher on state tests than the students of non-licensed teachers.

A second issue centers on who has the authority and expertise to grant the license. While it is essentially a settled matter today that the states have this authority, it was not always a settled matter because local communities, the profession, individuals, and the colleges have battled for the right to make the determination, and today some argue that licensure should be granted only at the national level in accordance with national standards.

From colonial times onward, parents, school boards, personnel directors, state superintendents, policy-makers and their counterparts have sought an answer to the question of what will predict who, among all the available candidates for a teaching position, is likely to succeed and perhaps more urgently who is likely to fail in the position.

Regrettably, all that is really available to them in their quest of a way to determine who should be permitted to teach is a set of flawed measures. A collection of flawed and incomplete indicators can be useful and one of the most correlated measures, *IQ*, is itself little more than a systematic combination of other limited measures, like memory span, word meaning and fluency, spatial relations, manual dexterity, classification, numerical reasoning, reaction time, common knowledge, which by themselves have little predictive validity. Each is weak and flawed in its power to account for very much of human behavior, although the compilation of these separate component measures is significantly related to nearly all aspects of human intellectual accomplishment.

Unfortunately, the historical and current desire for a simple, single, and inexpensive measure of teaching potential yields risky answers that would be as silly as using spatial relations skill or reaction time as the sole indicator of intelligence.

Historically, the prevailing 18th century tests of good character (basically interviews with local clergy) were supplanted by local tests of subject matter and pedagogy in the 19th century, and when these proved to be biased, invalid, and easily corrupted, they in turn were replaced by diplomas and degrees from programs of study in the newly emerging normal schools in the late 19th and early 20th centuries. The 1980's saw a resurgence of tests of basic skills and subject matter coupled with the academic degree, but some, like the American Board for the Certification of Teacher Excellence (ABCTE), promote a stand alone subject matter test as a sufficient basis for gauging teacher competence and potential.

The recurring dissatisfaction with the nation's schools, prompts non-reflective policy-makers and some of the reform-minded, to simply reject the indicator of the moment—the education degree on one occasion is replaced or supplemented with a standardized test because schools of education, even accredited ones, give degrees to some academically weak students. Tests of uncertain psychometric merit are then relied upon, and historically were relied on exclusively until the 1920's, when it was clear then as now that the tests' validity coefficients are low or absent. Direct classroom observation of a sample of the candidate's teaching, while closer to the predictive task at hand, proves unduly burdensome when properly done. Because the number of observations needed to reach acceptable reliability levels is nearly identical with the entire first year teaching assignment itself, predictive observational measures of teaching are essentially redundant with the very behavior they were put in place to predict.

Currently, there are about eight potential ingredients in the nation's system of quality assurance, all admittedly insufficient by themselves, but collectively

they permit tolerable levels of accurate prediction of teacher success, each one speaking to a different aspect of teacher quality. These eight interrelated ingredients are: the degree in teacher education, accreditation⁸, state program approval, the teaching license, national board certification, tenure, license tests, and the achievement of the teacher's pupils itself. While some of these eight indicators or measures may by themselves lead to a correct prediction of effective teaching, each is subject to known distortions that may lead to an inaccurate prediction; that is, they may indicate that a person can teach well or at an acceptable level, when in fact the person will prove to be inept with some pupils in some challenging circumstances.

In matters of importance, where mistakes have significant societal costs, prudence and common sense commend systems of checks and balances supported by multiple measures. A sound prediction that a prospective teacher will succeed might rest on the person's completion of a state approved degree program from an accredited institution coupled with performance on standardized tests of subject matter and pedagogy coupled with demonstrations of teaching that incorporated measures of pupil performance. The prediction is enhanced through interview techniques that seek to establish that prospective teacher possesses attitudinal characteristics, values, beliefs, and expectations that align with those possessed by veteran successful teachers.

The logic of convergence as a strategy for building a credible prediction out of individually weak and flawed measures requires that the measures contributing to the prediction be multiple and independent. Efforts that conflate measures or have one substitute for the other introduce unwarranted risk. The initial driver's license, for example, requires a road test in which the candidate demonstrates proficiency in the task itself. There would be considerable risk if the road test were waived solely on the basis of good grades in accredited or approved driver's education courses, or high marks on a written test of knowledge about driving. Rather, the state seeks to reduce the risk of granting a license to substandard drivers by requiring independent and multiple sources of evidence about the candidate's driving (a sample of driving behavior, a written test of driving knowledge, and driver's education or experience).

Very nearly the opposite approach has evolved for teaching. First, the license is often not required for certain teaching assignments – for private school teachers, or tutors, or others who work outside the public schools. It is waived now for about 5% of the public school workforce. It would be unthinkable to waive the drivers' licenses or require it only for those who drive publicly owned vehicles or require medical licenses only for those physicians who work in public hospitals and clinics. Program approval and accreditation are frequently collapsed into one assessment and in some scenarios the license test

may substitute for all other indicators together, thereby losing all the power of convergence.

The effort to increase the levels of quantitative literacy in the schools will surely fail unless each of these elements in the quality assurance system is addressed and coordinated. Change in education, historically, can come through the manipulation of one or two of these quality assurance devices, but these changes are typically short-lived and disappointing. Lasting change begins with a clear conception of the measurable features of *numeracy*, the establishment of a course of study along the lines of the options for a new academic major described earlier, the specifications of new requirements for the teaching license, the redesign of license tests, recognition in the accreditation and state approval standards, and incorporation in the state's curriculum assessments. Without this clear conception, the policy levers provided by teacher education, licensing, credentialing, accreditation are relatively powerless to provide a structure that will directly encourage and reward a teacher who has the capacity to pursue Sean's conjectures.

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Endnotes

¹ Based on an episode in Deborah Ball's teaching as a professor of mathematics education in the Michigan State University School of Education who also taught third grade mathematics each day in East Lansing.

² It requires understanding of the number system, a repertoire of mathematical techniques, and an inclination and ability to solve quantitative or spatial problems in a range of contexts

³ Ball, D. (1991) found that not only were mathematics and non-math majors no better at finding examples of the division of $1\frac{3}{4}$ by the fraction $\frac{1}{2}$, but their offered examples were often wrong mathematically. They tended to give examples where the division was by 2 rather than $\frac{1}{2}$.

⁴ Prior to this level understanding, adolescents will argue that while setting the clock ahead in the spring makes one truly older, the effect is cancelled later when the hour is lost, so age is in the end unchanged.

⁵ Draper (1976) and Konner (1976) show there are complex limits to universality of teaching in anthropological groups. In some cultures children are taught to eat but not to sit and to walk and vice versa in other cultures.

⁶ Premack & Premack (1996, p. 315) point out that "pedagogy is not an official anthropological category: no catalogue lists the pedagogical practices of different groups ... the anthropology of pedagogy is largely nonexistent; its proper study has yet to begin."

⁷ See further comment on the over-justification phenomenon in the Spring, 1996 issue of the *Review of Educational Research*, 66, No. 1, 1–51.