Precalculus in Transition: A Preliminary Report

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Abstract. This report summarizes our initial investigation into low student achievement in our five-unit precalculus course. We investigated issues related to course content, student placement, and student success. As a result, we have streamlined the course content, we are planning to implement a required placement test, and we are planning a 1–2 week preparatory workshop for students whose knowledge and skills appear to be weak. Further study is ongoing.

Background and Goals

San Jose State University is a large metropolitan university in the statewide California State University system. The student body, composed of over 30,000 undergraduate and graduate students in eight colleges is one of the most ethnically diverse in the country, with a large percentage of freshmen being first generation college students.

For several semesters, the San Jose State University Mathematics Department has been concerned about low student achievement in its precalculus course, Math 19. In each of the past several semesters, 40-45% of the 400-500 students who took this five-unit course earned Ds or Fs. All of these students must repeat the course if they wish to take calculus or some other course for which Math 19 is a prerequisite. The financial implications of this outcome to the university are significant: it costs money, time and space to accommodate such a large number of repeat attempts to earn at least a C– in the course.

Three main questions arose. Are the scope and sequence of topics appropriate? Are students being inappropriately placed in this class? What characterizes successful students in this course? Our assessment of factors influencing low student achievement in Math 19 began by addressing these questions.

Description: What Did We Do?

Scope and Sequence of Topics. There are really two issues revolving around the scope and sequence of topics. First, we revisited the topics to be included in the course itself. Second, we wanted to very clearly establish prerequisite knowledge and skills.

Course Topics. To examine the scope and sequence of topics in Math 19, we began by establishing the main purpose of the course. We had always assumed that most students take our precalculus course as preparation for calculus I. To determine whether this was true, we surveyed Math 19 students in Fall 2003 (474 students enrolled, 376 respondents) asking about their intended major, their reason(s) for taking Math 19, and their intention to take calculus. Overwhelmingly, the survey responses indicated that the vast majority of our precalculus students (80%) intended to take calculus I. Of those intending to take calculus, 47% cited some area of engineering as their intended major and 39% cited some area of science as their intended major. (Survey data from Spring 2004 (231 students enrolled, 153 respondents) had similar results.) Based on these results, we believe it's reasonable to focus the topics of Math 19 on preparation for calculus I.

To refocus Math 19 on preparation for calculus I, we streamlined the topics to emphasize depth over breadth. We retained only those topics we felt were absolutely crucial to success in calculus I: functions and their graphs, polynomial and rational functions, exponential and logarithmic functions, trigonometric functions, analytic trigonometry, applications of trigonometry, polar coordinates, analytic geometry, and systems of equations. In several of these broad topics, we eliminated or de-emphasized some elements that seemed to take up extensive class time without contributing substantially to the main goal of preparing students for calculus I. In particular, we eliminated scatter diagrams and data analysis; complex zeros and the fundamental theorem of algebra; simple harmonic motion and damped motion; vectors and operations on vectors; rotation of axes, polar equations, parametric equations of conics; matrices, determinants, and systems of inequalities. In addition, we chose to de-emphasize rational functions, applications of exponential and logarithmic functions, and angular measures in degrees.

By streamlining the topics, we were able to build considerable leeway time into the syllabus. In fact, 30% of the class time is now considered "leeway" and left to the instructors to use as they see fit. This permits them to design in-depth study of topics that are particularly difficult for students. In an end-of-semester survey of spring 2004 Math 19 instructors, we received some indications that they perceive this new approach to be more efficacious than the previous approach. One instructor commented, "The elimination/reduction of some of the topics from the old syllabus (e.g., rational functions, synthetic division, etc.) allowed more time to investigate core topics in a deeper fashion. I would not recommend any additional major changes to the new syllabus."

Prerequisite Knowledge and Skills. To establish the prerequisite knowledge and skills, we gathered information from several sources. First, we looked at the precalculus textbook's (Sulivan's *Precalculus*, 6th ed.) review material contained in the appendices. Second, we looked at the topics covered in the California State University entry-level math requirement[1]. Third, we looked at the topics covered in an online mathematical analysis readiness test produced by the CSU Mathematics Diagnostic Testing Project.[2] These three sources allowed us to create a document listing all prerequisite knowledge and skills that students should have upon entering Math 19, which can be handed out to students on the first day of class.

Independently of our construction of a prerequisite skills list, we asked Math 19 instructors to each provide a list of topics they felt their students should know but did not know upon entering the class. Overwhelmingly, instructors felt that their students' knowledge and skills pertaining to fractions, order of operations, and algebraic expressions were very weak. We intend to use our prerequisite knowledge and skills list along with our instructors' impressions of students' greatest weaknesses to create a one or two week intensive preparatory workshop for Math 19 to be conducted in the week(s) immediately preceding the start of each semester.

Placement Practices. Current placement procedures for calculus I (Math 30, 3 units) require students to achieve a sufficiently high score on a calculus placement exam (CPE).[3] We also have a calculus I with precalculus review course (Math 30P, five units) in which students can enroll by achieving a sufficiently high score on the calculus placement exam or passing precalculus (Math 19) with a C– or better. Students who prefer not to take the CPE are not allowed to take Math 30 or Math 30P; instead, the highest class they are allowed to take is Math 19, assuming they qualify. To qualify to take Math 19, they must satisfy the California State University entry-level math requirement. This can be satisfied with sufficiently high scores on the ACT (23) or SAT (550), or with a sufficiently high grade (C or better) in a transferable college-credit math course taken at a community college, or with a passing score on the CSU entry-level math exam (ELM).

We began to investigate the effectiveness of these placement practices by analyzing Math 19 course grades versus our current entry-level mathematics (ELM) exam scores. These data are summarized in Table 1. The overall percent passing Math 19 in Fall 2003 (66%) was higher than in previous semesters, but it still seems low. Students exempt from the ELM exam (about 50% of the Math 19 enrollment) seemed to do best (72% passed, mean grade of 2.2), which makes sense, since they are likely better prepared than students who are required to take the ELM exam. Students having passed the ELM exam did seem to have a significantly better chance of succeeding in Math 19 (66% passed, mean grade of 2.0) than those who failed the ELM exam (46% passed, mean grade of 1.3). The low grades of stu-

[1] The California State University has a system-wide placement testing program in basic mathematics skills that consists of the Entry Level Mathematics (ELM) examination. Further information can be found at www.calstate.edu/AR/FOM.pdf.

[2] See mdtp.ucsd.edu/test/.

[3] SJSU's calculus placement exam is provided by the California State University/University of California Mathematics Diagnostic Testing Project (MDTP). Further information can be obtained at mdtp.ucsd.edu.

	Number of Students	Mean Grade[b] in Math 19	Percent Passing (at least C-)	Correlation with Math 19 Grade
Overall	474	2.0	66	N/A
Exempt from ELM Exam	232	2.2	72	N/A
Took ELM Exam	242	1.9	60	0.29
Passed ELM Exam	168	2.0	66	0.28
Failed ELM Exam[c]	74	1.3	46	-0.04

[a] Grades are reported here as grade points, where an A = 4.0, A– = 3.7, B+ = 3.3, etc.
[b] Mean grades and passing percents exclude grades of W.
[c] Students who failed the ELM exam met the entry-level math requirement by either completing developmental mathematics coursework at SJSU or completing a transferable college-credit math course at a community college with a sufficiently high grade (C or better).

Table 1. Math 19 Grades[a] versus ELM Exam Scores (Fall 2003)

dents who met the entry-level math requirement by other means are a concern. Such developmental mathematics coursework might not be enough to prepare students for Math 19. Perhaps an additional intensive review before the beginning of Math 19 is necessary for these students. Perhaps a better screening criterion or a reorganization of the Math 19 content is needed. We will continue to monitor the effect of our new syllabus.

As a second step in our investigation of placement practices, we analyzed Math 19 course grades versus calculus placement exam (CPE) scores, for those students who took the calculus placement exam (about 12% of the Math 19 enrollment). These data are summarized in Table 2. Students who took CPE before attempting Math 19 did seem to perform significantly better in the course (87% passed, mean grade of 2.7). It's possible that this is due to a biased sample of students taking the CPE. Students who have met the entry-level mathematics requirement may choose to take the calculus placement exam. Given that the exam costs $20, it's reasonable to assume that many of those students who opt to take the CPE are fairly certain of their calculus readiness, hence are likely to be among the better-prepared students. The sample size of students who took CPE was relatively small, and results might not be too credible. For example, it is surprising that students recommended for Math 30P in fact did worse than those prevented from enrolling in Math 30P. Since there were only four students in the former group, this could just be a statistical anomaly. Further study should be done before attempting to interpret the data.

To gain a different perspective on how well our students were prepared for Math 19, and as an independent check on our placement system, in Fall 2003 we gave our Math 19 students a test during the first week of classes. The test was strictly for diagnostic purposes and did not count towards students' grades.

The test was necessarily short (six multiple choice questions), so as not to demand too much time from instructors or students. Students were asked to compose three functions and identify the graph of the composite function, find the

	Number of Students	Mean Grade in Math 19	% Passing (at least C–)	Correlation with Math 19 Grade
Overall	474	2.0	66	N/A
Did not take CPE	419	1.9	54	N/A
Took CPE	55	2.7	87	0.18
Took CPE, recommended for Math 30P	4	2.6	75	-0.71
Took CPE, recommended for Math 19, prevented from enrolling in Math 30P	32	3.1	94	0.24
Took CPE, recommended for College Algebra and Trigonometry (Math 8), allowed to enroll in Math 19	19	2.2	84	-0.23

Table 2. Math 19 Grades versus CPE Scores (Fall 2003)

1. Let $f(x) = x - \pi/2$, $g(x) = 4x$, and $h(x) = \sin(x)$. Which of the following graphs represents the composition $ghf(x)$?*
2. Let $f(x) = (12x^2 - 7x - 12)/(x - 2)$. Find the zero(s) and vertical asymptote(s).
3. Solve for x: $\log_{10}(x + 1) + \log_{10}(x - 2) = 1$.
4. A guy wire 80.0 feet long is attached to the top of a radio transmission tower, making an angle of 30° with the ground. How high is the tower, to the nearest tenth of a foot?
5. Find the vertex of the parabola $y = -4x^2 + 18x - 13$.
6. Solve the system of equations $y = x^2 - 6x + 9$, $y - x = 3$.

* Each item had five answer choices.

Figure 1. Diagnostic Test Items

zeros and asymptotes of a rational function, solve a logarithmic equation, solve a right triangle problem, find the vertex of a parabola, and solve a system of two equations (one linear, the other quadratic). (Figure 1 contains these diagnostic test items.) For each item, students were given five answer choices, including at least one "main distractor", that is, an answer that would result from an "almost correct" solution attempt. For example, the solution of the logarithmic equation introduces an extraneous answer, so one of the answer choices for this item was $x = -3$ or 4, which could be obtained if a student skipped the final check.

We scored the test two different ways. First, we calculated a raw score: 1 point for each correct answer, 0 points for each incorrect answer. Then we calculated a partial credit score: 1 point for each correct answer, 1/2 point for each item in which a main distractor was selected, 0 points for all other responses.

For the 346 students (out of 474 enrolled) who completed the test[4], the mean score was 1.5 out of 6, with a standard deviation of 1.2. Allowing partial credit for the main distractors, the adjusted mean score was 2.1 out of 6, with a standard deviation of 1.2. We did not expect students to do very well on this test, because some of these topics would be covered in Math 19. We found that the main weakness seemed to be in solving item 3 (solving a logarithmic equation) and item 5 (finding the vertex of a parabola). For item 3, very few students seemed to have any understanding of how to cope with the logarithmic expressions. For item 5, the most popular response was the answer choice indicating the y-intercept of the parabola.

To discover how much these students had learned in Math 19, we gave exactly the same test to students in Math 30/30P (calculus) in Spring 2004, most of whom had taken Math 19 the previous semester. Again, the test was strictly for diagnostic purposes and did not count towards students' grades.

For the 259 students (out of 328 enrolled) who completed the test[5], the raw and adjusted results were increased to 2.3 out of 6 with a standard deviation of 1.2 and 2.9 out of 6 with a standard deviation of 1.1, respectively. The increase was expected, but the magnitude of the increase was somewhat discouraging. Since the topics had just been covered in the previous semester, we had expected the increase to be more pronounced. We are hoping that our revised precalculus syllabus, which emphasizes depth over breadth, will help prepare students better for calculus. Specifically, we found our calculus students to have particular difficulty (still) with item 3, solving a logarithmic equation.

In a survey of Math 19 instructors at the end of the spring 2004 semester, they noted that, while the topic of logarithms was difficult for students, they did not feel that any more time (in the now-streamlined syllabus) was needed on this topic. We intend to repeat the test in our Fall 2004 calculus courses to assess whether the additional depth of study afforded by the streamlined syllabus in Math 19 appears to have an impact on student's knowledge and abilities.

Characteristics of Successful Students. At the end of the diagnostic test given to our calculus students in Spring 2004, we asked them to write some words of advice to Math 19 students on how best to succeed in Math 19. A total of 216 students responded to this open-ended question, providing a total of 348 individual suggestions. Nearly all (333 total) of the suggestions focused on behaviors and perspectives students should adopt and actions students should take to help themselves succeed. We were struck by how positive most of the responses were. Overall, they indicated that these students (most of whom had successfully completed Math 19) did understand why they succeeded or what their mistakes were in precalculus. A summary of the responses is given in Table 3.

Insights: What Did We Learn?

Spring 2004 is the first semester in which we used the streamlined syllabus. Early feedback indicates that the instructors felt the new syllabus was better suited to the needs of the students and the goal of preparing students for calculus. To monitor this in future semesters, we will conduct surveys of precalculus and calculus instructors.

[4] Precalculus instructors were strongly encouraged, but not required to participate in the diagnostic test. Out of 15 sections, data were collected from 13 sections. For two of these sections, it was obvious that the instructor had given the diagnostic test as a take-home assignment, rather than as an in-class test. Data from these two sections were thrown out.

[5] Out of 10 sections, data were collected from 7 sections.

Numb. of Students	Suggestion
120	Do all the homework
52	Study hard
30	Attend all classes
23	Ask questions/go to office hours or tutoring
18	Pay attention/participate in class
15	Take good notes
14	Keep up
11	Practice
9	Review constantly
9	Read the book
8	Learn algebra/trig/everything
7	Understand concepts; don't just memorize
5	Take Instructor A (a specific instructor at SJSU)
4	Get into a study group

Table 3. Advice on How Best to Succeed in Math 19

Placement remains a difficult issue. There is sentiment in our department to implement a mandatory placement exam for Math 19, but the data from Fall 2003 suggest that success in Math 19 does not depend entirely or even mostly on a single score on a placement exam or competency exam. Data from our diagnostic test showed that students have quite a few weaknesses in their knowledge and skills upon entering Math 19, despite the fact that many of them have taken trigonometry, precalculus, and/or calculus already. At the same time, most of these students intend to take calculus and to major in science or engineering. In fall of 2004, we will implement a course-wide diagnostic exam in Math 19 that is similar to our calculus placement exam (and produced by the same organization). Data from this new diagnostic exam will be much more thorough than that obtained with our 6-item test, and we hope will inform a decision on how to proceed with a mandatory placement exam.

Our investigation of student success has really only just begun. While we now have a sense of what successful students consider to be the keys to their success, we don't have a good sense of what is causing the remaining students to fail. We have much more work to do in this area. In fall 2004, we hope to expand the items on our student survey to include questions about students' outside commitments, their experience with college level expectations, and their initial perceptions of what they think will be the keys to their success in Math 19.

Finally, we are beginning to realize that ongoing assessment is crucial to distinguishing between real problems and anomalies. It is our hope that further assessment efforts will lead to meaningful change for our Math 19 course.

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