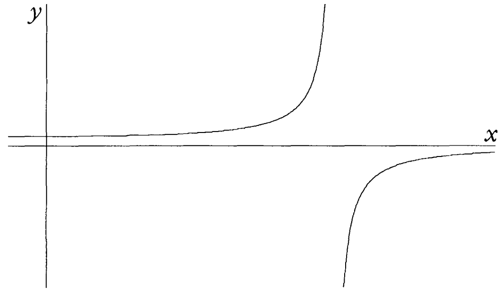

Figure 5. $f(x) = \dfrac{1}{1 - 2x^2}$.

Figure 5) the function $f(x) = 1/(1 - 2x^2)$ over the interval $[0, 1] \cap \mathbf{Q}$. The derivative is positive and even continuous on $[0, 1] \cap \mathbf{Q}$, but the function is not increasing.

There are no functions here more exotic than polynomials, rational expressions, and square roots—surely functions dealt with regularly by "analysts" and even ordinary users of the calculus. The familiar theorems on continuous functions fail, not because there is anything pathological about them, but because the domain has been changed from $\mathbf{R}$, which is complete, to $\mathbf{Q}$, which is not. These examples suggest that an attempt to separate the familiar properties of continuous functions from the underlying structure of the real line is unlikely to succeed.

The great theorems of the calculus are not necessarily "obvious"—otherwise it would not have taken nearly 2,000 years of mathematical effort to discover them or their proofs. To hide from our students the persuasive arguments by which we have come to believe them is to do them a disservice.

References

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2. H. Swann, Commentary on rethinking rigor in calculus *Amer. Math. Monthly* 104 (1997), 241–245.

A Far-reaching Formula

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It is well known that the formula for the area of a trapezoid—the average length of the parallel sides times the perpendicular distance between them—gives the areas of squares, rectangles, parallelograms, and triangles. In Figure 1, h is the distance between the parallel lines l and m.

It is not so well known that the formula can be used to find other areas. In Figure 2, the length of the radius of the circle is similar to h in Figure 1. We can consider the center and the circle to be the parallel sides so, as in Figure 3, the circumference of the circle is the base of the triangle and the area of the circle is, by the trapezoid formula, $(2\pi r + 0)r/2 = \pi r^2$.

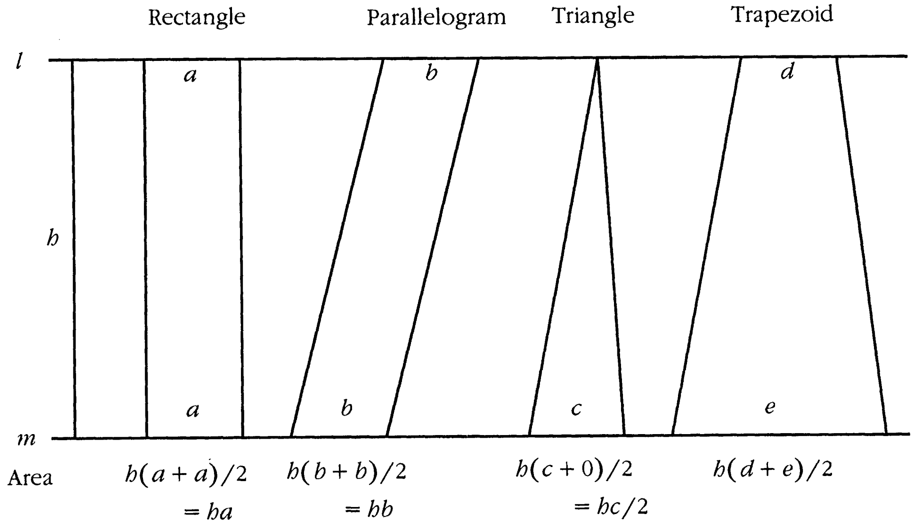

Figure 1

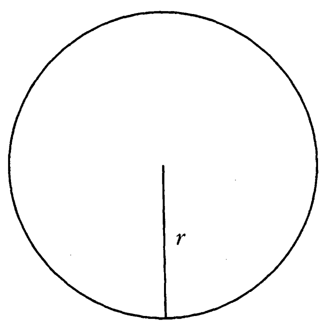

Figure 2

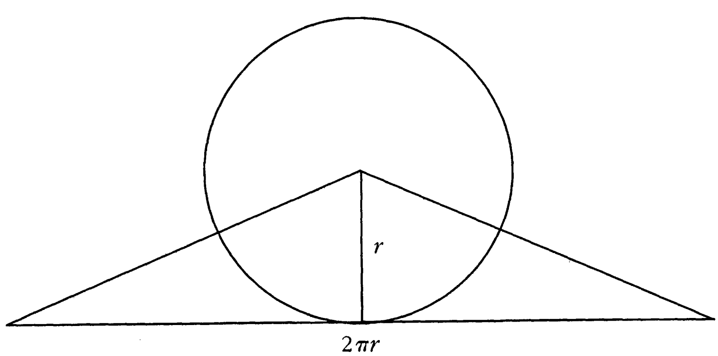

Figure 3

Similarly, the apex of a cone and the circle of its base can be considered as parallel sides and the slat height as the perpendicular distance between them. If r is the radius of the base and l is the slant height (see Figure 4), then the lateral surface of the cone is $(2\pi r + 0)l/2 = \pi rl$. Similarly, the lateral surface area of the truncated cone in Figure 5 is $(2\pi p + 2\pi q)l/2 = \pi l(p + q)$.

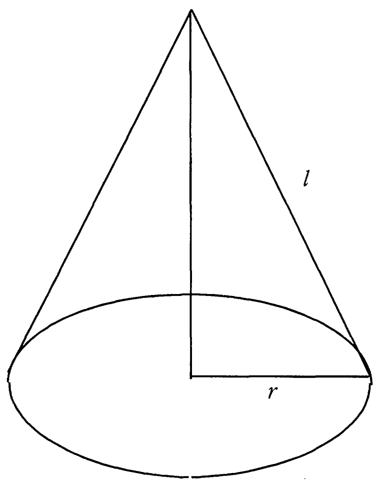

Figure 4

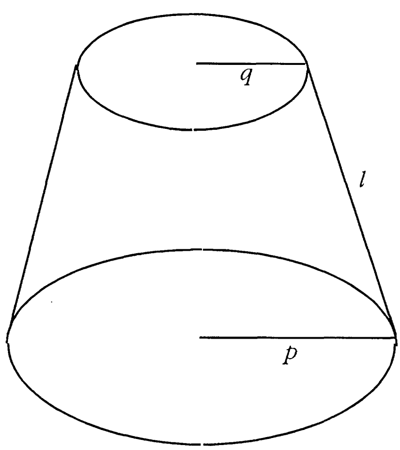

Figure 5

This principle has far-reaching applications. For example, the surface area of a highway can be found by multiplying the average length of its parallel sides by the perpendicular distance between them.

Students may appreciate seeing the connectedness among a variety of figures, and having only one formula to remember.